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OF

CIVIL ENGINEERS.

(INSTITUTED 1852.)

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AMERICAN SOCIETY OF CIVIL ENGINEERS.
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TRANSACTIONS.

Paper No. 911.

EXPERIMENTS AT DETROIT, MICH., ON THE EFFECT OF CURVATURE UPON THE FLOW OF WATER IN PIPES.*

By Gardner S. Williams, M. Am. Soc. C. E.; Clarence W. Hubbell, Assoc. M. Am. Soc. C. E., and George H. Fenkell, Jun. Am. Soc. C. E.

With Discussion by Messrs. B. C. Murphy, Hiram F. Mills, Irving P. Church, Charles H. Tutton, J. L. Campbell, Charles W. Sherman, D. Farrand Henry, George Y. Wisner, Edgar C. Thrupp, Rudolph Hering, James A. Seddon, J. P. Frizell, Henry Bazin, E. Kuichling, Walter Ferris, E. Sherman Gould, H. W. Brinckerhoff Edward S. Cole, E. E. Haskell, Wager Fisher, Robert E. Horton, W. M. White, A. V. Saph, E. W. Schoder, Arthur Adams, W. E. Wilson, Gardner S. Williams, Clarence W. Hubbell and George H. Fenkell.

*Presented at the meeting of September 4th, 1901.

Introduction.

The history of the experiments to be described may be considered as properly beginning in the summer of 1893, when the writers became associated in the Engineering Department then organized by the Board of Water Commissioners of the City of Detroit, Mich., the Regulations governing which prescribing, as among the duties of the Civil Engineer, the head of that Department, that "He shall particularly have charge of the Distribution System, and shall make it his especial study and care that the said system shall be as perfect as possible."

Beginning with such crude instruments as Bourdon gauges, and gradually working up past the mercury column to the more delicate apparatus to be described later, many valuable and interesting data were obtained in the next three years, which would compare not unfavorably with work done by others, and which, from time to time, has been presented before this and similar societies; but the opportunity of taking up this particular investigation systematically, though long looked forward to, did not offer itself until the fall of 1896, when the Engineering Department recommended, and the Board ordered, the laying of a 30-in. main through a distance of about 5 000 ft. to connect two older mains. The course selected for this main embraced ten right-angled deflections, including the two at the beginning and end, the deflections being alternately to the right and left. In the design of this work, which was immediately constructed, provision was made for the experiments under consideration. Further constructions in 1898 enabled the investigations to be extended to 16-in. and 12-in. pipe, which work has been carried on since at intervals as opportunity occurred. Descriptions of these constructions will be found in this paper under the headings, "Thirty-Inch Cast-Iron Pipe," "Sixteen-Inch Cast-Iron Pipe" and "Twelve-Inch Cast-Iron Pipe."

Object of the Experiments.

The purpose of the investigation was to determine the effect of curvature upon the flow of water in closed circular conduits under pressure, and, particularly, to examine the effect of changes in radius of curvature upon the losses of head in those cases occurring in ordinary water-works distribution systems. In carrying out this purpose it has become necessary to investigate various allied problems, and to devise

and construct special apparatus to further its accomplishment. Some of these secondary questions have proved of almost, if not quite, as great interest as the main question, but the fact is not to be lost sight of that the effect of curvature was the subject chiefly under consideration.

Having already satisfied themselves that there were no data extant that an engineer was warranted in applying to the cases occurring in ordinary practice, notwithstanding the unanimity of opinion expressed in hydraulic treatises, the writers have considered it unnecessary to present any *résumé* of the meager data supposed to bear upon the question in hand, nor do they propose in the body of this paper to devote attention to any experiments which have already been published.* The intention has been to observe existing conditions as accurately as possible, and to present the results first as they were observed, and then with such reductions as careful study of them suggested; and to discuss nothing with which a familiarity has not been gained by actual contact.

The writers do not pretend by any means to have exhausted the subject, and, as these investigations cover only curves of 90°, and do not reach the curves of very long radius occasionally used, there is still a very interesting and extensive field left untouched.

Conditions of Experiment.

The Lines Investigated.—The pipe lines experimented upon were in every case a part of the distribution system of the Detroit Water-Works, and were required to perform their designed function, either wholly without interruption, or with very brief intermissions. It became necessary, therefore, to operate at such times and in such a manner as not to interfere with the regular service. For this reason, the work of the first series of experiments upon the 30-in. pipe was generally done at night, and, in order that the additional consumption of water should prove the least serious at the pumping station, Saturday night was usually selected, although the first observations were made on Memorial Day, May 30th, 1898. The other series have been carried out in the day time, but velocities have been necessarily restricted and variations frequent.

* A discussion, of the previous investigations upon this subject, by one of the writers, may be found in *The Technic* for 1899, published by the Engineering Society of the University of Michigan, Ann Arbor.

The Force Employed.—The party was made up of the employees of the Engineering Department, with an occasional assistant from outside.

In addition to the writers, who have each handled every instrument and each had charge of the party, the following men have been engaged as observers and recorders in these investigations:

Eugene A. Rummler, Draftsman, Eng. Dept., Observer on Pitot tube and resistance gauge.

W. Edgar Hewitt, Draftsman, Eng. Dept., Observer on Pitot tube and resistance gauge.

J. Edward McGrath,* Draftsman, Eng. Dept., Observer on Pitot tube and resistance gauge.

J. T. Wagner, Rodman, Eng. Dept., Observer on Pitot tube and meters.

Herbert O. Hanford, Draftsman, Eng. Dept., Observer on resistance gauge and Recorder on Pitot tube.

Edward A. Kirby, Clerk, Eng. Dept., Observer and Recorder on Pitot tube.

Anthony F. Dierkes, Draftsman, Eng. Dept., Observer on resistance gauge, and maker of instruments.

George G. Gibson, Clerk, Iron Pipe Dept., Recorder on Pitot tube.

Robert N. Burnham, Draftsman, Eng. Dept., Observer and Recorder on resistance gauge.

This force does not include men required to operate gates, in making shut-offs and controlling the discharge.

Dates and Duration.—The dates of these investigations were approximately as follows:

30-in.	investigations of	1898, May 30th, to September 21st.
16-in.	"	" September.
12-in.	"	" November.
30-in.	"	1899, June.
12-in.	"	" June.

Since the field work, in each case, the available time of the writers and their assistants has been spent in the discussion of the data obtained, but no estimate can be given of the time actually spent in the investigation as a whole, which has now extended over a period of more than three years, from preliminary experiments on 12-in. pipe, in November, 1897, to the last work on the 30-in. line in January, 1901.

Scope of Investigations.—The investigations embraced simultaneous observations of the loss of head in at least two sections of the line

* Died January 6th, 1901, at Detroit.

being investigated, and a determination of the velocity in the pipe. The ranges of mean velocity experimented with have been:

For 30-in. pipe, zero to 3.5 ft. per second.
" 16-in. " " 5.8 ft. "
" 12-in. " " 4.8 ft. "

They have also embraced examinations of the distribution of velocities within the pipes, and of pressures around the circumference, and some experiments upon the flow of water in 2-in. and 5-in. brass pipes.

Incidentally, they include the rating and standardizing of the instruments used, and a study of the differences of condition between instruments moving in still water, and instruments stationary in moving water.

Reductions of Observations.—Owing to the removal of one of the writers from the scene of the investigations before the completion of the experiments it has been necessary to divide the work of the preparation of the results for publication, and, while frequent consultations have been held and all have been kept fully informed of every part of the work as it progressed, both in the experimental stage and in the digesting of the gathered material, the writers feel some regret at not being able to spend together the time devoted to the computations and consideration of results, as some points might be brought out more clearly by a free discussion of the unreduced data, but this work, like most of its kind, is only an incident among the many, and, therefore, has had to take a subordinate place, other matters being of more importance.

Nomenclature.

The following abbreviations and terms are used:

A = Area;
a = Coefficient;
b = Constant;
c = Coefficient in Chezy formula;
d = Diameter of a circular pipe;
G = Gauge;
g = Acceleration due to gravity = 32.2 ft. per second;
H = Head;
H_f = Head lost in friction and other resistance between two points in a closed pipe of uniform section;
H_v = Head producing velocity;
H_{vc} = Head producing velocity at the center of the pipe;
l = Length;
m = Coefficient or exponent;
n = Coefficient or exponent;
p = Pressure;

$P.\ C.$ = Point of Curvature = Beginning or up-stream end of a curve in the pipe line;
$P.\ I.$ = Point of Intersection = Intersection of two tangents to a curve in the pipe line;
$P.\ T.$ = Point of Termination = Termination or down-stream end of a curve in the pipe line;
Q = Quantity of discharge, in cubic feet per second;
R = Radius of curvature of axis of pipe;
r = Radius of a right section of circular pipe;
s = Slope $\frac{H_f}{l}$;
T = Tube;
Tan. = Tangent;
U = Mean velocity, in feet per second, in pipe, determined by weight of water discharged;
V = Velocity, in feet per second;
V_c = Velocity, in feet per second, at the center of pipe;
V_m = Mean velocity, when determined other than by weight;
V_t = Velocity at any point, determined by the Pitot tube;
V_{max} = Maximum velocity, in feet per second, observed in a pipe;
W = Weight, in pounds avoirdupois.

"Reading" is the term used to designate a single reading of the scale of an instrument.

"Observation" is used to designate such a combination of readings, one or more, as form a complete unit in the result.

"Experiment" is used to designate a set of observations made under practically constant conditions.

"Series" is used to designate a number of experiments made upon the same section of the line.

"Traverse" is a term used to describe the operation of measuring velocity with the Pitot tube by taking observations consecutively at different points along a diameter of the pipe.

"Position," when applied to a "traverse," designates the result of the set of observations taken at a single setting of the instrument, *i. e.*, at one point on the diameter traversed.

"Piezometer" designates the opening or openings in the pipe wall or the side of a chamber by which the pressure of the water flowing past is communicated to a gauge.

Pitot Tubes.

Description of Pitot Tubes and Their Use.

Forms of Tubes.—A Pitot tube, when complete, consists essentially of two pipes, one of which terminates in an orifice which may be so directed as to receive the impact of a flowing stream, while the other terminates in an orifice which may be set at right angles to the direc-

tion of the flowing stream and may then communicate its pressure. The former is called the velocity or impact tube or opening or simply the point, and the latter the pressure opening or the side. The latter may terminate in an orifice or orifices in the pipe wall, or may open into the body of the stream, and it may be so arranged as to communicate something less than the pressure in the stream, in which case the instrument is enabled to be used at lower velocities than when the full pressure is communicated. In connection with these investigations there have been used eleven different tubes which are designated as follows:

Nos. 1, 2, 3, 4, 5 and 6; *A*, *B*, *C*, *D* and *E*.

These tubes were of brass, and are shown on Plate I.

Tube No. 1 was the first built, and was used in some preliminary investigations upon 12-in. pipe made in the fall of 1897, as was also Tube No. 2 which was made from it by increasing the size of the interior pipe, connecting to the pressure opening, to avoid the possibility of the loss of head in the small internal pipe affecting the reading of the gauge. These tubes which may be called the "Bazin form," from their resemblance to the instrument used by M. Henry Bazin in his studies of the liquid vein, had, as shown, a single pressure opening in the bottom.

Although the results obtained with them were very satisfactory, it was thought that even better results could be obtained with the form represented by Nos. 3, 4 and 6, in which the pressure openings were four, located 90° apart and 45° from the knife-edge, around the sides of the bulb, in the point of which was the impact opening. The bulb formed an equalizing chamber to the four pressure openings. This form is somewhat similar to one discovered by the writers, in a German work on hydraulics, many months after theirs were constructed. These three tubes differed only in the size and form of the bulb and sustaining knife-edge, Tube No. 3 having the finest and No. 6 the coarsest lines.

Tube No. 5 was made from Tube No. 2 by plugging the original pressure opening, and drilling, at right angles to its axis and to that of the impact opening, a hole entirely across the instrument connecting with the original pressure tube, and filing away the heel of the instrument.

Tubes *A*, *B* and *C* were as nearly alike as a skilled workman could make them. They may be called the "Freeman form," from a resem-

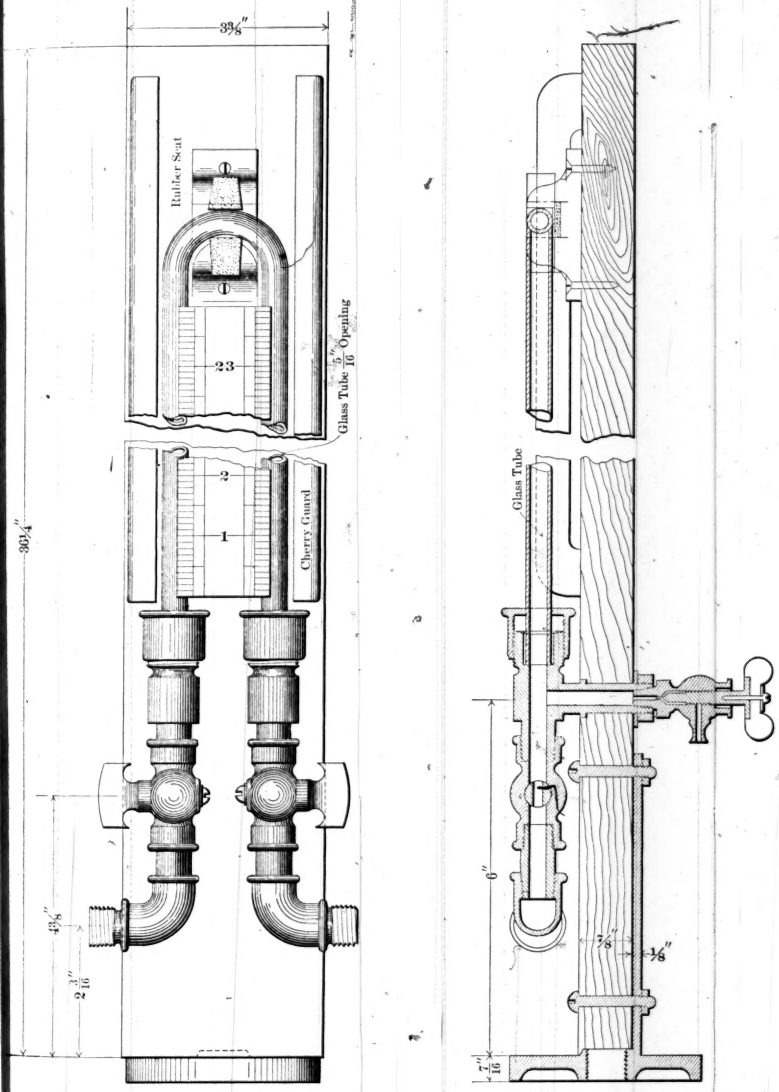
blance to an instrument built by John R. Freeman, M. Am. Soc. C. E., although they possess a marked difference in the form of the point.

Tube *D* was built for use in very small pipe, and it is quite similar to Tube No. 5, except that its lines are finer and the impact opening cuts away the knife-edge, causing it to appear notched in the side view. It has also a down-stream opening which has not been used. Tube *E* was designed for similar use, and contains only an impact opening, the pressure being supplied from piezometers in the side of the pipe. Its impact opening is very similar to those of Tubes *A*, *B* and *C*, but much smaller.

The Gauges.—For the purpose of observing with the Pitot tube, it is necessary to connect each tube with a gauge by which the difference in pressure can be observed. The most satisfactory form consists of two parallel glass pipes connected at the top and having a scale between or behind them. The upper portions of these pipes remain filled with air when the water rises in the lower portion, and the difference of head is then read in water upon the scale. Six gauges were used for this purpose, and designated as Nos. 4, 5, 8 and 9, and "*C* double" and "*C* triple." Numbers 4 and 5 were duplicates, and consisted of inverted glass U-tubes about $\frac{5}{16}$ in. in internal diameter. Numbers 8 and 9 were duplicates, and consisted of straight tubes of $\frac{1}{8}$ in. internal diameter, joined at the top by a brass casting. The construction of Gauges Nos. 5 and 9 is shown in Figs. 1 and 2. The scales of the *C* gauges were divided in double centimeter units and subdivided to tenths, the smallest division being 2 mm. The scales of Gauges Nos. 4, 5, 8 and 9 were divided into inches and tenths. For work in the field, these gauges were mounted upon tripods or hung on the side of the gate-wells in which the observations were taken.

Connections Between the Gauges and the Tubes.—The tubes were connected to the gauges by rubber hose, $\frac{3}{8}$ in. in internal diameter and 10 to 20 ft. long, equal lengths being used on each side, and care being taken to expel the air before connecting up.

Method of Inserting the Tube in the Pipe.—These tubes have all been designed for insertion into the pipe through a 1-in. round-way corporation cock, such as may be inserted in any main while under pressure, with a Mueller, Payne, Lennox or Walworth tapping machine. The cock being set in the main at the place of observation, the follower and tail piece is removed, and, in the latter's place, is



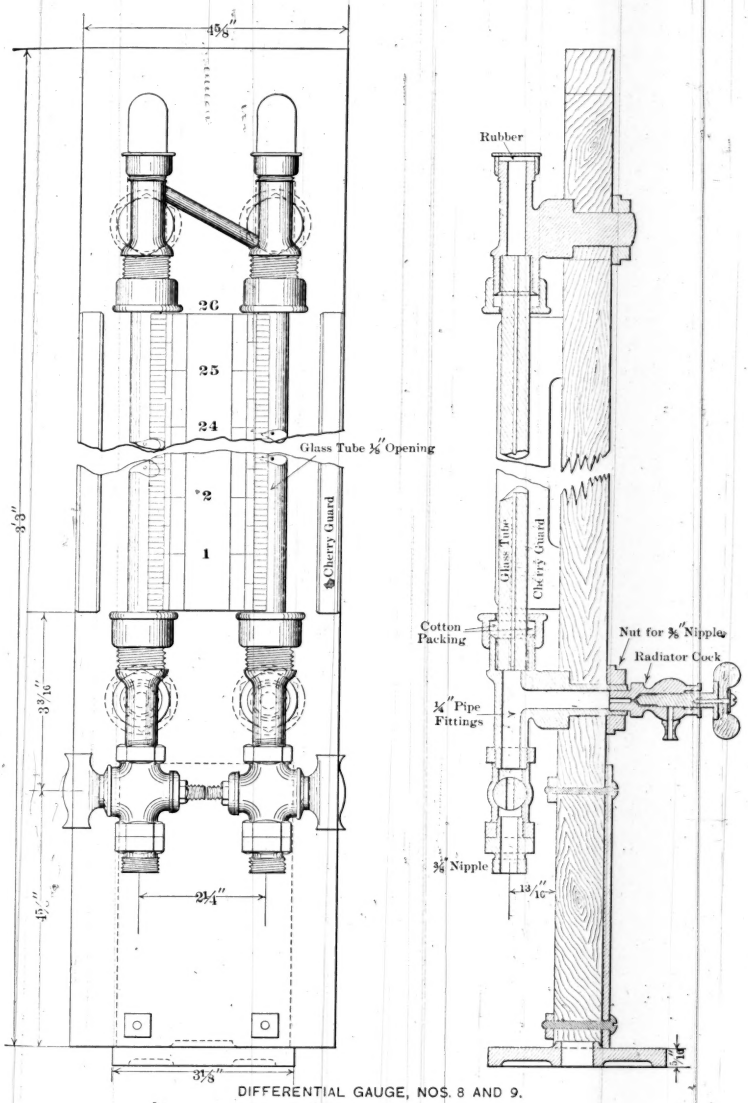
DIFFERENTIAL GAUGE, Nos. 4 AND 5.

FIG. 1.

substituted a stuffing-box, shown in Fig. 4, sufficiently long to contain the portion of the tube below the cylindrical casing. This stuffing-box being fastened to the cock, with a gasket between them, by the follower, the tube is then inserted in it and the nut at its top screwed down against a rubber packing which grips the tube. The cock is then opened, and the tube may be shoved into the interior of the pipe and set at any desired position. The pointing of the tube is adjusted by aligning the arm at the top of the instrument with the direction of the pipe.

Method of Measuring the Diameter of the Pipe.—In order to measure accurately the internal diameter of the pipe experimented upon, the same stuffing-box was used with a cap having an eccentric opening. Through this was passed a rod terminating at the bottom in a hook somewhat like the letter J, and having a heel projecting about $\frac{1}{8}$ in. at the bottom. This was made of such size that, when the cap was on, the hook would pass through the cock in one position, but, when reversed after entering the pipe, the point of the hook would locate the inside of the pipe next to the cock. The rod was pushed in until the heel located the farther side, then reversed and withdrawn until the point located the near side, when the distance measured on the rod extending from the stuffing-box, between the two positions, plus the distance from the heel to the point, gave the diameter. By turning the rod around again it could be withdrawn from the pipe. Allowing for reasonable roughness of the pipe wall, the diameter could be determined by this means within less than $\frac{1}{32}$ in.

Method of Observation.—In part owing to the pipes experimented upon being supplied by direct pumping, a pulsation in the water columns was noticeable on the Pitot tube gauges. Where glass tubes, considerably in excess of the diameter of the orifices in the Pitot tubes, were used, as for example those of Gauges Nos. 4 and 5, these pulsations were not rapid, and of comparatively short amplitude; but when the smaller columns of Gauges Nos. 8 and 9 were used, the number of pulsations became quite high, 10 to 20, and even 100 per minute being observed, and their amplitude sometimes was several inches. While the larger columns were more easily read, some preliminary experiments with tubes of $\frac{3}{4}$-in. diameter showed that a considerable length of time (several minutes) was required for the columns to come to equilibrium after a change of velocity, and, as it would



DIFFERENTIAL GAUGE, NOS. 8 AND 9.

FIG. 2.

not be possible to maintain steady velocities, it was considered best to use a small column which could be depended upon to respond promptly to such changes as might occur. Therefore, particularly in traversing, Gauges Nos. 8 and 9 were generally used with the tubes.

In reading, the practice was to read the two extremities of the vibration in each column, the two upper extremities being read simultaneously by a single observer and then the two lower, the readings being called to a recorder who entered the time and the readings in a notebook. It was found that an observer could read the four positions to the smallest division of the scale, which was graduated to tenths of an inch, once every 15 seconds, and this was the rate of reading practiced when the tube was traversing the pipe, after the first few experiments in which the observations were taken 30 seconds apart. When the instrument was on the center at the completion of or between traverses, the four readings were usually taken every 30 seconds. A person recording for himself would be able to get the four readings about once in 40 to 50 seconds.

In measuring the discharge of a pipe with a Pitot tube, it was necessary at first to traverse the diameter of the pipe with the instrument, *i. e.*, to take readings at various points along the diameter. In order to simplify the reductions and give the positions equal weight, the area of the pipe was divided by a series of concentric circles into a central core and concentric rings of equal area and a position was taken at the center, at the edge of the pipe, as nearly as the tube could approach, and on each intersection of the dividing circles with the diameter. At each location the four readings were taken three times, which occupied 45 seconds, and the instrument was then moved to the next position, so that a position per minute was occupied. In the earlier traverses, when observations were 30 seconds apart, five were taken at a position. The first position was the center, then the side of the pipe was taken and then the points consecutively across the pipe, returning to the center at the close. This complete operation was called a "Traverse." To locate the point of the instrument, an observing stick was used corresponding in its divisions to those of the diameter of the pipe, which, when rested against the top of the stuffing-box, gave the setting by a collar on the tube.

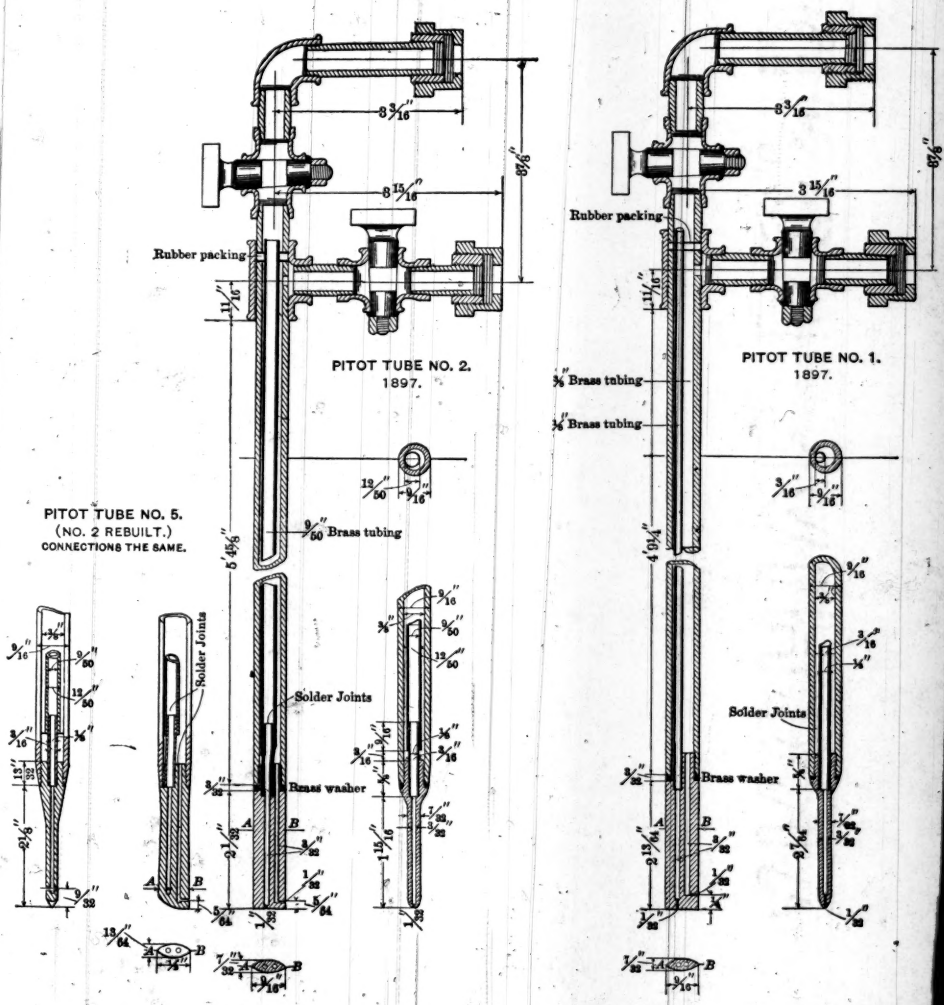
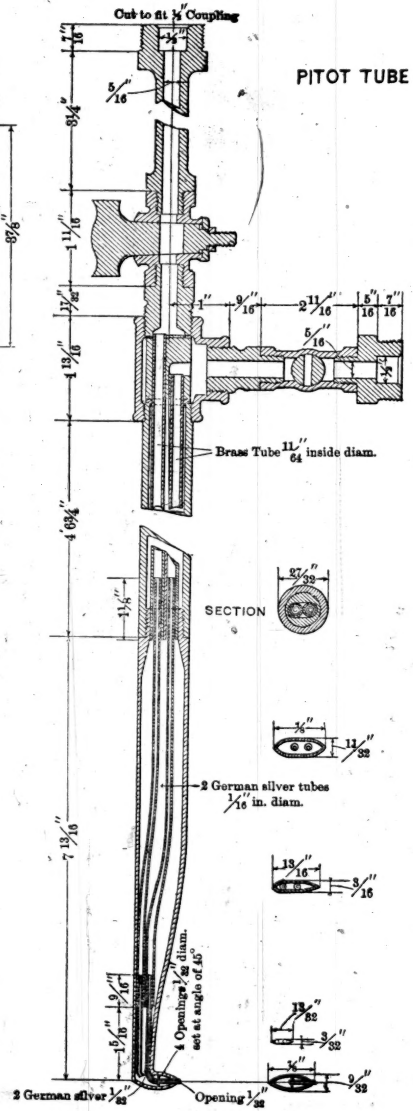
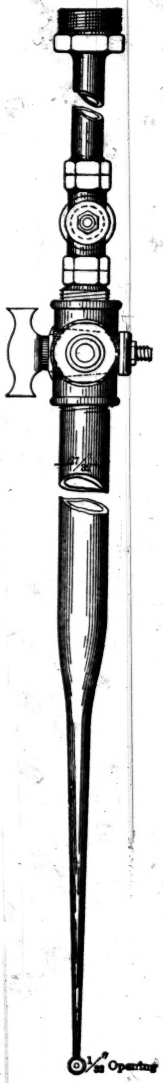
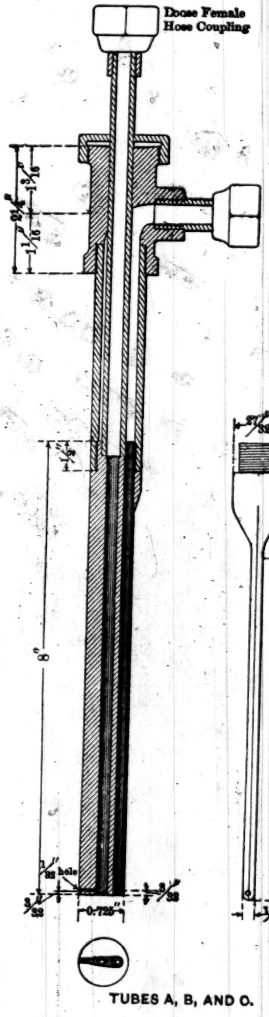

PITOT TUBE NO. 2.
1897.
Rubber packing
Brass tubing
Solder Joints
Brass washer
PITOT TUBE NO. 1.
1897.
Rubber packing
Brass tubing
Brass tubing
Solder Joints
Brass washer
PITOT TUBE NO. 5.
(NO. 2 REBUILT.)
CONNECTIONS THE SAME.
Solder Joints

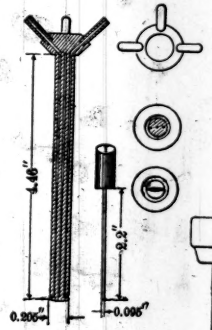
PLATE I.
TRANS. AM. SOC. CIV. ENGRS.
VOL. XLVI I, No. 911.
WILLIAMS, HUBBELL AND FENKELL
ON FLOW OF WATER IN PIPES.



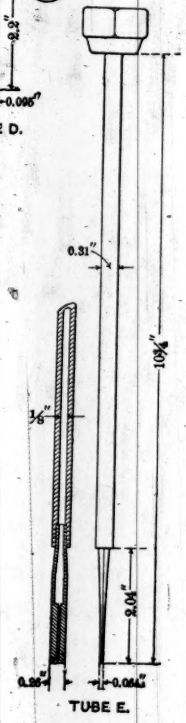




TUBES A, B, AND C.



TUBE D.



TUBE E.

Method of Reducing Pitot Tube Observations.—Inasmuch as the point of the tube is directed so as to receive the impact of the flowing stream, it has been argued, on theoretical grounds, that the difference of head observed is $2\,H_v$, from the law of fluid impact which, as given by Weisbach, is that:

"The normal impulse of water against a plane surface is equal to the weight of a column of water, the cross-section of whose base is equal to that of the stream, and whose height is twice that due to the velocity $\left(2\,H_v = 2\,\frac{V^2}{2\,g}\right)$."

Some, however, prefer to consider the head observed as the velocity head, and so have proposed, for reducing, the formula $H = \frac{V^2}{2\,g}$, not $H = \frac{V^2}{g}$.

Practically, it makes very little difference which form of expression is adopted, as the change is provided for in the coefficient of the instrument. For example, having a certain known velocity, U, the tube shows a velocity, $V_1 = \sqrt{2\,g\,H}$, by the velocity-head theory, and $V_2 = \sqrt{g\,H}$ by the impact theory. There then results $U = m\,V_1 = n\,V_2$, where m and n would be coefficients of the tube, and might, theoretically, be greater or less than unity, according to the form of the instrument. From the foregoing relation there also results $m\,\sqrt{2\,g\,H} = n\,\sqrt{g\,H}$, or $m\,\sqrt{2} = n$, whence, if m = unity, $n = \sqrt{2} = 1.4142$; and if n = unity, $m = 0.7072$.

The writers prefer the velocity-head theory, for the reason that they have been able to design an instrument which, according to this theory, has a coefficient of nearly unity and others having coefficients as low as 0.75, but they have as yet been unable to produce one having a coefficient approaching unity, on the impact theory, without introducing what they interpret as other effects than those of simple impact. Accordingly, the difference in height of the two columns, as determined from the means of the extremities of the vibrations, has been treated as the head to which the velocity is due, or $H_v = \frac{V^2}{2\,g}$.

As the gauge scales were graduated in inches and tenths, there results V, in feet per second, $= \sqrt{\frac{2\,g\,H_v}{12}} = \sqrt{\frac{64.4}{12}\,H_v} = 2.32\,\sqrt{H_v}$.

The velocity corresponding to the mean H_v for each position is

determined according to this equation. The value at the center, to be used in determining the mean, is obtained by taking the sum of the velocity of the first and third positions there and twice that of the second and dividing it by four. Bearing in mind that the points of observation divide the pipe into rings of equal area, the mean velocity is determined by the summation of half the extreme, or side, velocities, twice the center velocity, as just obtained, and the intermediate velocities, all divided by the number of positions. The center velocity, being observed at the beginning, middle and end of each traverse, has been used twice in computing the mean velocity, to compensate for the deficiency occurring from the summation of the several velocities. The effect of this is to give a value of V_m, about 0.6 of 1% higher than would be obtained by integrating the volume of revolution generated by the mean curve drawn through the observations when 21 positions are used.

Effect of Pressure and Temperature upon Gauge Readings.—To be strictly accurate, the difference in head represented by the two columns of air in the differential gauge should be subtracted from the difference of the two water columns. The value of this difference of the air columns changes with both temperature and pressure, and may be readily computed from the "Law of the Expansion of Gases." Inasmuch, however, as the experiments described have been made under pressures having a range of from 15 to 35 lbs., or only about one atmosphere, excepting the tank ratings, and the temperature range has been only about 40° Fahr., the mean being near 70° Fahr., except on the 12-in. work, when the mean was about 40° with a minimum of of 32°, and the rating of the tubes in the pipe has been conducted under pressures between 16 and 25 lbs., and a temperature of about 70° Fahr., no attempt has been made to correct the tube observations for either temperature or pressure. In comparing the tank and pipe ratings, the effects of pressure would be apparent, though very small.

Calibration of the Tubes.

In Open Tank and Still Water: Apparatus.—In order to establish the ratio existing between the true velocity and that deduced from the readings of the tube gauge by the foregoing reduction, the method of calibration usually adopted with current meters, *viz.*, that of moving the instrument through still water at known velocities, was first tried. For this purpose a circumferential trough, rectangular in section, 9

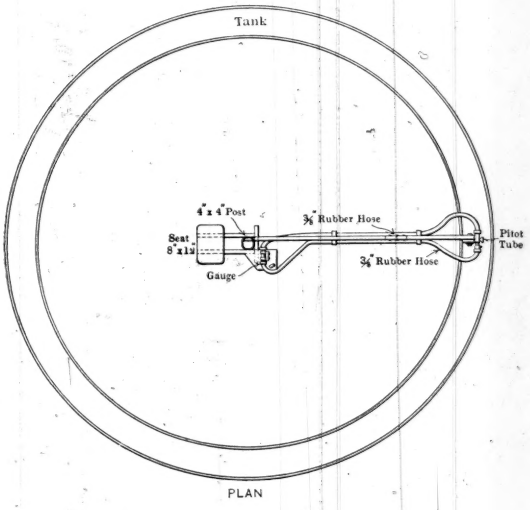

Tank
4" x 4" Post
3/8" Rubber Hose
Seat
8" x 12"
Pitot Tube
Gauge
3/4" Rubber Hose
PLAN
Ceiling
Brass Plate
1/2" x 4 1/2" Steel Pin
screwed in post
2" Block, 8" x 20"
4" x 4" Pine Post
APPARATUS FOR RATING PITOT TUBES
IN STILL WATER.
2" Screws
12' 8"
4" x 1" Pine
Pitot Tube
Gauge
1/2" Bolt
Strap
4-3/16" Bolts
Gauge support
Observer's Seat
3/4" Hose
Strap
3" x 1"
Strap
3" x 1"
1/2" Bolt
Footrest, 12" 3" x 1"
1/2" x 4 1/2" Steel pin
Brass cup
2" Block, 8" x 20"
Point of Tube
Tank, circular,
8' deep, 9' wide
ELEVATION
5' 11"
Floor

FIG. 3.

ins. wide and 8 ins. deep, was built of galvanized iron, as shown in plan and sectional elevation in Fig. 3. The diameter of its center line, which was made the path of the tube, was 11 ft. 10 ins. The tube to be rated was supported upon an arm attached to a central shaft which was free to revolve in bearings on the floor and ceiling, and which also supported the gauge and a seat for the observer. The gauge was connected with the tube by the usual rubber hose, the point of the tube being 3 ins. below the surface of the water in the trough.

In Open Tank and Still Water: Experiments.—The arm carrying the tube was revolved by a man walking behind it and allowing it to simply rest against, and not be grasped by, his hand; and the time of each revolution was taken, from a watch reading to $\frac{1}{5}$ second, when the arm passed a certain point on the trough. As soon as a revolution was completed, the time recorder called out the seconds consumed by it as a guide to the maintaining of a uniform speed. A separate recorder was found necessary for the gauge readings, and these were taken as in the field-work, but as frequently as they could be read, often every 5 seconds. The velocities were maintained as nearly uniform as possible for periods of at least 5 minutes, and the results so obtained were plotted as a log, with times as the abscissas, and gauge readings and true velocities as the ordinates, and from these plottings periods of uniform condition were selected, to be averaged and used for each experiment, which averages are presented in Table No. 1.

As the gauge was working under less than atmospheric pressure, it being located above the water surface, considerable skill was required to expel the air from the connecting hose and keep it from working in. Frequent observations were taken with the tube still, by which defects in the conditions could be easily detected. To prevent the passage of the tube from generating a velocity in the water of the trough very thin paddles of sheet tin were inserted at several points, filling the cross-section, and these were carefully withdrawn as the tube approached and replaced after it had passed. The watch used was compared with a well-regulated ordinary watch and its rate determined at the beginning and end of the experiments, and the correction applied in the reductions.

These experiments were made in the Water Office at Detroit, in January, February and July, 1899, the room being artificially heated during cold weather.

TUBE NO. 3, IN 2 IN. PIPE
STUFFING BOX, TUBE NO. 3 AND PIEZOMETER.
TUBE NO. 5, IN 2 IN. PIPE.

1
2
3
4
5 INCHES
AIR CHAMBER
HOSE TO GAUGE
STUFFING BOX,
TUBE NO. 6, IN 2 IN. PIPE
TUBE C IN 2 IN. PIPE
TUBE D
TUBE E
TUBE NO. 3, IN 2 IN. PIPE
STUFFING BOX, TUBE NO. 3 AND PIEZOMETER.
TUBE NO. 5, IN 2 IN. PIPE.

In Open Tank and Still Water: Results.—Tubes Nos. 3, 5 and 6 were calibrated in the open tank as just described, and the results of each single revolution are plotted on Plate II, and in Table No. 1 the experiments have been grouped according to the velocity, as computed from the tube readings, which is called the apparent velocity. The number of experiments in each group is shown in the first column. The ratio of the true to the apparent velocity, which is the coefficient of the tube, is also given. Rejecting those experiments in which the true velocity was less than 1 ft. per second, the arithmetic mean gives the following values for the several tubes and gauges:

			Coefficient.
Tube No.	3	Gauge No. 9	0.9142
"	6	" 9	0.9658
"	5	" 9	0.8506
"	5	" 5	0.8351

From the plottings of the individual observations, taking the slopes of the mean lines, which have been located by stretching a thread and averaging its position by the eye, the ratios obtained are:

			Coefficient.
Tube No.	3	Gauge No. 9	0.926
"	6	" 9	0.950
"	5	" 9	0.859
"	5	" 5	0.859

By the grouping, it was expected that the effect of velocity, if there were such an effect, would be more clearly shown, but, after a study of the results, it was concluded that the effect of velocity on the coefficient of the tubes was insignificant, except in the case of Tube No. 5, the plottings of the ratings of which are notable, in that they show, for the experiments with both gauges, a peculiar sinuosity of the locus which has no apparent counterpart in the ratings of Tubes Nos. 3 and 6.

In Closed Pipes, with Flowing Water: Apparatus.—As it was possible to imagine conditions which might produce different coefficients for these instruments when rated in still water, as just described, from those which should be used in the reduction of the field observations, it was determined to attempt a rating in a closed pipe. For this purpose the resources of the Hydraulic Laboratory of the College of Civil Engineering of Cornell University were kindly placed at the disposal of the writers by Professor E. A. Fuertes, M. Am. Soc. C. E., Director of the College, and experiments have been carried on there by the

TABLE No. 1.—Tube Ratings in Still Water.

Tube No. 3; Gauge No. 9.—Jan. 16, 1899.

No. of ratings.	Pitot tube; ap. vel.	True vel.	True vel. ap. vel.
1	0.803	0.675	0.841
2	0.863	0.712	0.820
1	0.956	0.701	0.733
4	1.081	0.9405	0.869
3	1.543	1.432	0.928
3	1.656	1.451	0.876
2	1.684	1.486	0.882
1	1.78	1.738	0.975
2	1.85	1.763	0.953
1	1.88	1.738	0.924
2	1.93	1.842	0.954
3	2.00	1.832	0.916
1	2.09	1.960	0.938
6	2.209	2.059	0.932
1	2.332	2.275	0.975
1	2.445	2.143	0.868
2	2.759	2.548	0.923
1	2.887	2.459	0.821
4	3.000	2.713	0.904
2	3.069	2.751	0.8963
5	3.121	2.829	0.9063
4	3.155	2.925	0.927
4	3.216	2.962	0.921
3	3.370	2.895	0.859
3	3.639	3.235	0.8889
4	4.122	3.822	0.921
3	4.123	3.840	0.931
3	4.214	3.876	0.924
7	4.291	3.873	0.903
3	5.196	4.895	0.942
2	5.366	4.983	0 929
2	5.675	5.269	0.928
1	5.777	5.265	0.911
1	6.089	5.584	0.917
3	6.144	5.642	0.918
3	6.22	5.532	0.889
2	6.36	5.781	0.916
2	6.572	5.851	0.890
2	6.611	6.043	0.914
1	6.731	6.142	0.913

Mean 0.913

Tube No. 5; Gauge No. 5.—Feb. 10, 1899.

No. of ratings.	Pitot tube; ap. vel.	True vel.	True vel. ap. vel.
6	1.01	0.847	0.836
5	1.05	0.904	0.864
6	1.13	1.056	0.777
4	1.69	1.237	0.720
3	1.77	1.283	0.725
3	1.77	1.320	0.746
1	1.88	1.31	0.691
1	1.95	1.32	0.677
1	2.73	2.26	0.828
2	3.09	2.47	0.798
3	3.19	2.916	0.810
5	3.27	2.720	0.830
3	3.31	2.61	0.788
1	3.40	2.77	0.815
1	3.48	2.94	0.845
6	3.99	3.515	0.880
4	4.05	3.607	0.891
4	4.04	3.657	0.905
8	4.10	3.72	0.907
11	4.14	3.763	0.909
10	4.185	3.688	0.881
1	4.68	4.08	0.871
2	4.79	4.32	0.902
5	4.86	4.30	0.883
4	4.95	4.53	0.916
5	5.00	4.334	0.867
2	5.14	4.385	0.853
3	5.24	4.513	0.860
2	6.30	5.07	0.804
7	6.40	5.356	0.837
4	6.54	5.655	0.865
5	6.66	5.640	0.847
5	6.76	5.802	0.858
2	6.83	5.78	0.846
4	6.92	5.825	0.842
3	7.02	6.05	0.862
1	7.02	6.12	0.871
1	7.14	6.12	0.857

Mean 0.8851

Tube No. 5; Gauge No. 9.—Feb. 4, 1899.

No. of ratings.	Pitot tube; ap. vel.	True vel.	True vel. ap. vel.
3	0.74	0.468	0.6305
12	0.87	0.641	0.7396
4	1.14	0.877	0.7691
1	1.29	0.967	0.7496
3	1.35	0.967	0.7160
3	1.42	1.034	0.725
2	1.63	1.165	0.713
8	1.70	1.201	0.705
2	2.16	1.838	0.849
6	2.25	1.911	0.848
4	2.30	1.925	0.834
2	2.35	2.041	0.869
2	2.73	2.534	0.928
2	2.85	2.625	0.921
4	2.95	2.776	0.939
2	3.01	2.784	0.925
3	3.17	2.989	0.942
4	3.24	3.019	0.930
2	3.43	2.940	0.856
4	3.69	3.260	0.883
2	3.75	3.311	0.883
2	3.78	3.315	0.877
2	3.85	3.320	0.861
6	3.99	3.670	0.919
3	4.09	3.780	0.924
2	5.03	4 159	0.827
2	5.12	4.377	0.855
2	5.18	4.646	0.897
4	5.24	4.299	0.820
3	5.31	4.232	0.797
2	5.40	4.427	0.819
2	5.53	4.427	0.801
5	5.61	4.500	0.802
2	5.70	4.424	0.777
2	5.77	4.715	0.817
4	5.93	4.90	0.826
3	6.01	4.90	0.815
2	7.37	6.224	0.875
3	7.46	6.543	0.878
3	7.50	6.625	0.883
4	7.57	6.441	0.851
2	7.68	6.286	0.819

Mean 0.8506

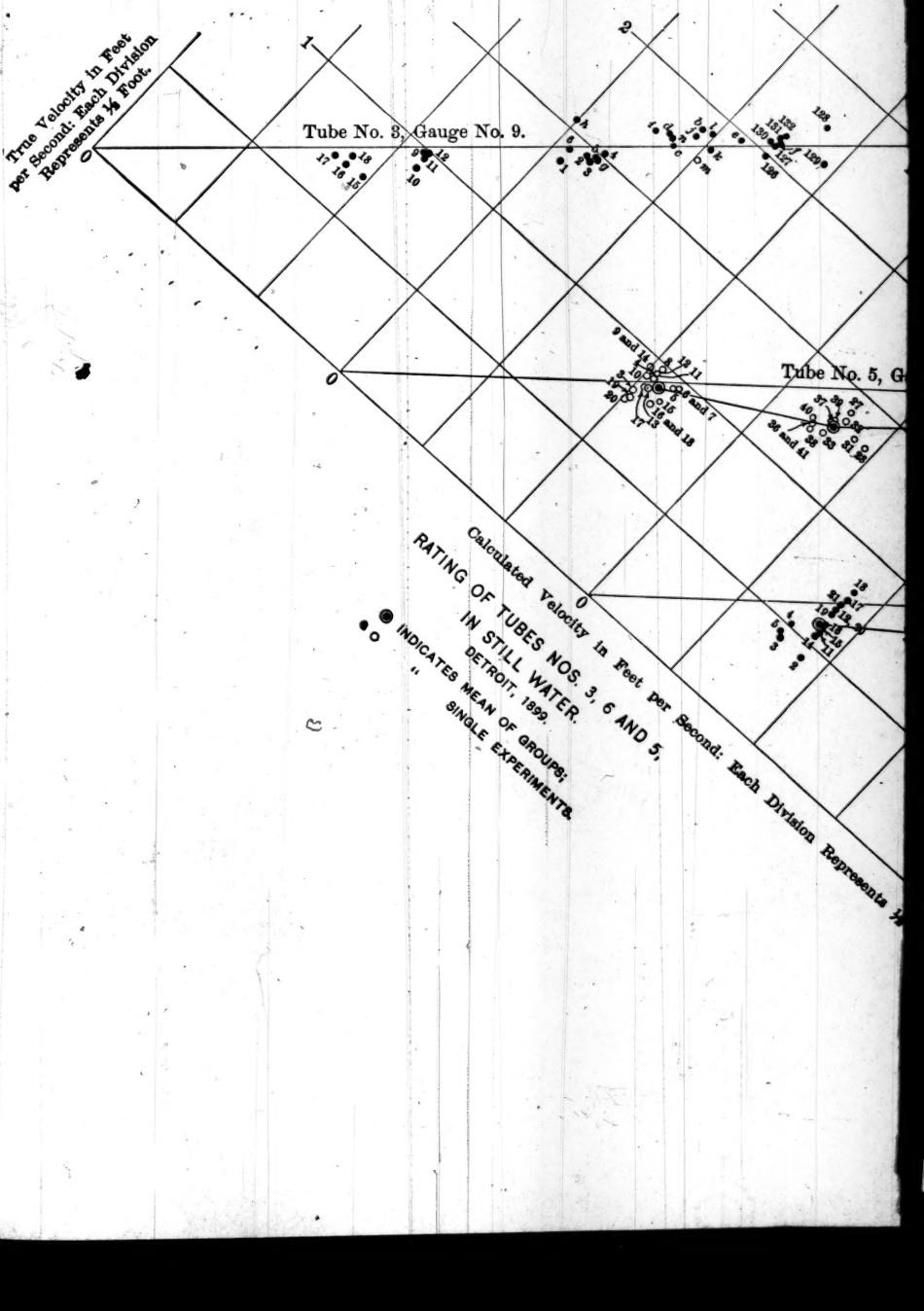
Tube No. 6; Gauge No. 9.—July 1, 1899.

No. of ratings.	Pitot tube; ap. vel.	True vel.	True vel. ap. vel.
1	0.33	0.42	1.237
3	0.38	0.425	1.155
1	0.44	0.45	1.023
1	0.48	0.47	0.979
4	0.62	0.768	1.241
4	1.02	1.154	1.089
4	1.11	1.185	1.070
3	1.13	1.150	1.018
4	1.60	1.55	0.967
1	1.67	1.60	0.952
9	1.73	1.672	0.967
4	1.78	1.745	0.981
1	2.07	2.04	0.985
3	2.10	1.963	0.933
7	2.91	2.747	0.942
2	2.99	2.755	0.923
4	3.10	2.926	0.939
2	3.16	3.00	0.963
1	3.29	3.20	0.9726
6	3.35	3.263	0.9714
2	3.39	3.42	1.009
1	2.15	2.10	0.977
8	2.20	2.40	1.091
4	2.24	2.19	0.957
2	2.29	2.045	0.895
3	2.33	2.23	0.960
1	2.38	2.19	0.937
3	2.81	2.62	0.934
1	3.56	3.42	0.960
1	3.60	3.42	0.950
5	3.66	3.41	0.930
2	4.15	3.97	0.958
5	4.20	3.904	0.929
8	4.25	3.915	0.922
6	4.31	4.102	0.952
3	4.36	4.08	0.937

Mean 0.9666

writers, beginning in August, 1899, and continuing to March, 1901, in which they have had the assistance of M. S. Darrow, Jun. Am. Soc. C. E., Assistant in the Hydraulic Laboratory, and Mr. W. Edward Wilson, student in civil engineering.

For these experiments a system of piping located in the basement of the College of Civil Engineering was utilized. This may be supplied directly from the University mains or from a tank in the top of the building, the water surface in the latter being about 47 ft. above the pipe used. The inlet to the system being only a 2-in. pipe, the size of the experimental pipe was necessarily limited to the same, in order to provide for a suitable range of velocity. This 2-in. supply pipe was led axially into a cast-iron cylindrical receiver 2.17 ft. long and 1.135 ft. in diameter. A screen, consisting of a brass plate perforated with $\frac{1}{8}$ and $\frac{3}{32}$-in. holes was located 0.5 ft. from the inlet end. The outlet of this receiver was a brass cycloidal mouthpiece connecting to a seamless drawn-brass pipe 0.416 ft., or 5.013 ins., in diameter. The diameter of the generating circle of the cycloid was one-fourth that of the following pipe, and it was rolled parallel to the axis of the pipe, the cycloid at its middle becoming tangent to the pipe wall. This brass pipe, from the entrance of the mouthpiece, was 20.97 ft. long, and terminated in a similarly proportioned cycloidal mouthpiece connecting to a line of seamless drawn-brass tubing 0.1746 ft., or 2.096 ins., in diameter. At a point 68.23 ft. beyond the entrance to the second mouthpiece there was arranged a connection for the insertion of the tubes, which is shown in Fig. 4. For the work with Tubes *D* and *E* the preceding length was changed to 68.30 ft. Beyond this point the pipe extended straight for 46.3 ft., then turned 180° and returned to a point near the inlet where the discharge took place through a 1-in. valve and nipple. The jet from this outlet passed into a funnel of galvanized iron having a vertical diaphragm in its center, one side of which delivered to the waste drain, and the other to a scale tank of 2 500 lbs. capacity. The scale's error was determined to be less than $\frac{1}{10}$ of 1 per cent. The funnel was suspended so as to swing freely, and the jet could be thus almost instantaneously thrown into or out of the tank, the time required being less than $\frac{1}{2}$ second. For these observations, the time was taken with an ordinary watch, and the temperature of the water weighed was taken with a standardized thermometer having an error so small as to be neglected.


True Velocity in Feet per Second: Each Division Represents ½ Foot.
Tube No. 3, Gauge No. 9.
Tube No. 5, G
Calculated Velocity in Feet per Second: Each Division Represents ½
RATING OF TUBES NOS. 3, 5 AND 5, IN STILL WATER. DETROIT, 1899.
INDICATES MEAN OF GROUPS;
" SINGLE EXPERIMENTS.

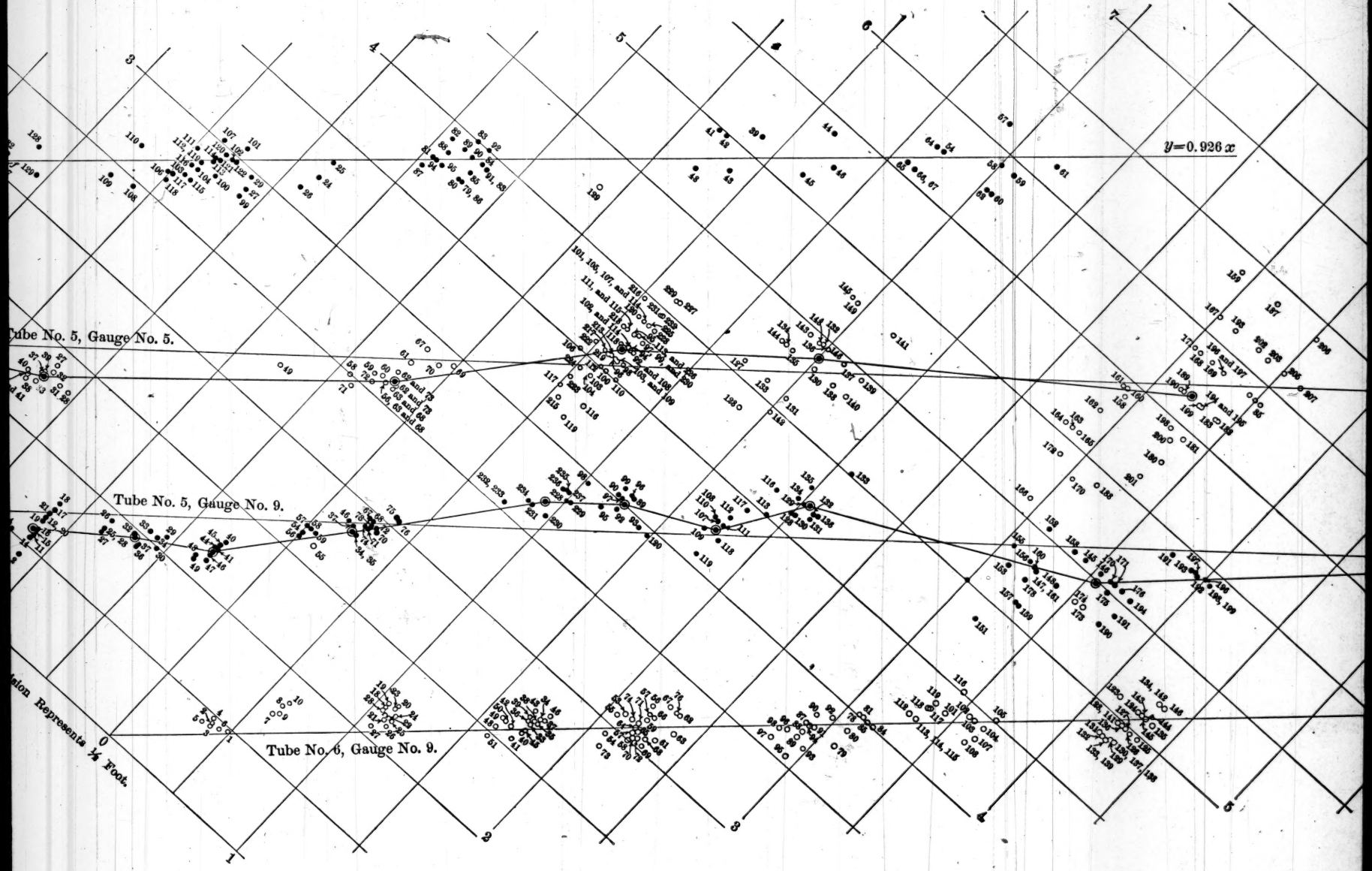
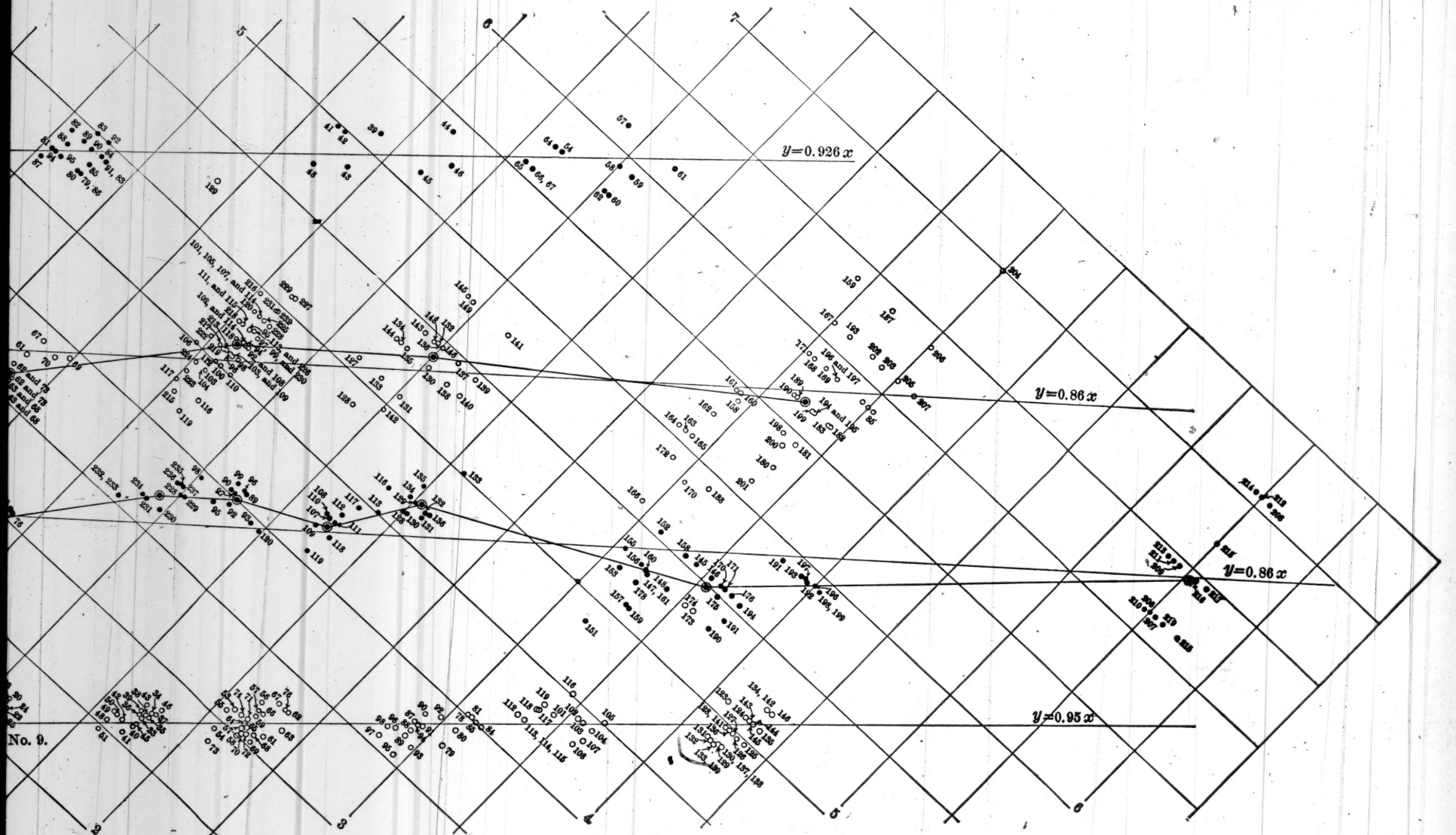

y=0.926 x
Tube No. 5, Gauge No. 5.
Tube No. 5, Gauge No. 9.
Tube No. 6, Gauge No. 9.
ision Represents ¼ Foot.

PLATE II.
TRANS. AM. SOC. CIV. ENGRS.
VOL. XLVII, No. 911.
WILLIAMS, HUBBELL AND FENKELL
ON FLOW OF WATER IN PIPES.



In Closed Pipes, with Running Water: Experiments.—The method of experimentation was first to make traverses of the pipe with the tubes, a uniform velocity being maintained and frequent weights being taken. Traverses were made at the above point with Tubes Nos. 3 and 5, and *D* and *E*. With Tube No. 3 and Tube *D* the triple gauge was used, and the third column connected to the circumferential chamber, which communicated with the interior of the pipe by four $\frac{1}{16}$-in. holes, 90° apart, drilled at right angles to the axis of the pipe.

The data of these traverses are presented in Table No. 2, and the velocities are plotted on the upper part of Plate III. With Tube *E*, traverses were taken on two diameters at right angles, and, as will be seen from the plate, all traverses except that with Tube No. 5 show the velocity to be a maximum at or very near the center of the pipe.

Previous experiments (those on the pipe lines at Detroit) having demonstrated that when the maximum velocity was at the center of the pipe a fairly constant ratio existed between the mean velocity and that at the center, and these traverses showing this condition to be approximately established at the calibrating point, advantage was taken of the two conditions to determine the constancy of the tube coefficients for different velocities. The tubes were set upon the center and the water was caused to flow past them at velocities ranging from about $\frac{1}{4}$ ft. up to nearly 6 ft. per second, the discharge being measured by weight. The results of these experiments are presented in Table No. 3 and the lower part of Plate III, and show that a practically constant ratio exists between the mean velocity as obtained by weight and the center velocity as obtained by the tube, and this, with the two preceding conditions, establishes the constancy of the tube coefficients. If, now, the ratio of the mean velocity to the center velocity be established, these observations furnish the desired rating of the tubes, provided that the presence of the tubes themselves has not, by reason of the obstruction they present, caused a disturbance of the distribution of the velocities. The traverse made with Tube No. 5, Plate III, shows clearly by its lack of symmetry that the area of this tube is so great that such a method of calibration with it is out of the question, and the same may be expected with Tube *C;* but, comparing the traverse made with Tube No. 3 with those made with Tubes *D* and *E*, it seems warrantable to apply such a method of rating to Tube No.

3, and Tubes *D* and *E*, and it has also been tried with Tube No. 6, the area of which, at the pressure openings, is less than 50% greater than that of Tube No. 3.

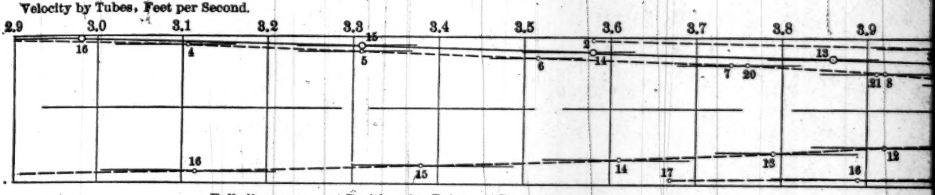
In Closed Pipes, with Running Water: Reductions of Rating Experiments.—The first step in this application is the determination of the constant ratio, $\frac{\text{Mean velocity}}{\text{Central velocity}} = \frac{V_m}{V_c}$.

Table No. 4 shows the data used in this determination.

Out of the thirteen traverses in Table No. 4, as shown by the last column, three have been rejected, *viz.*, *f*, *i* and *c*. Those marked *f* and *i* are rejected because they were made at a velocity below what seems, from other experiments, to be a critical one for the pipe experimented with, or, at any rate, because the traverse itself shows that normal conditions were not established. They are two halves of the same traverse, and in one case the disturbance seems to have affected the pressure openings, and in the other the ring. A peculiar disturbance or change of condition is noticeable (Plate III) whenever the true mean velocity approaches 0.7 ft. Abundant evidence has been gathered to prove that a point of critical condition exists in this pipe at about the velocity 0.7 ft., so that the writers feel no hesitation in rejecting Traverses *f* and *i* on these grounds.

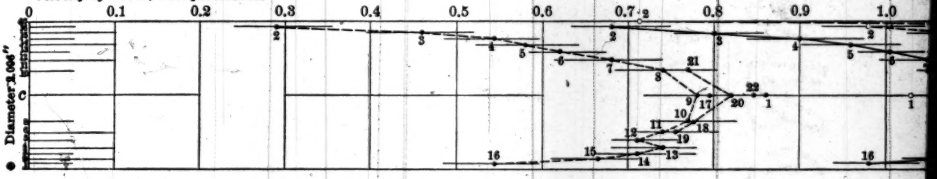
Traverse *c*, made with Tube No. 5, is rejected because, as already stated, and as may be seen from the upper diagrams on Plate III, the presence of the tube seriously disturbed the arrangement of the velocities.

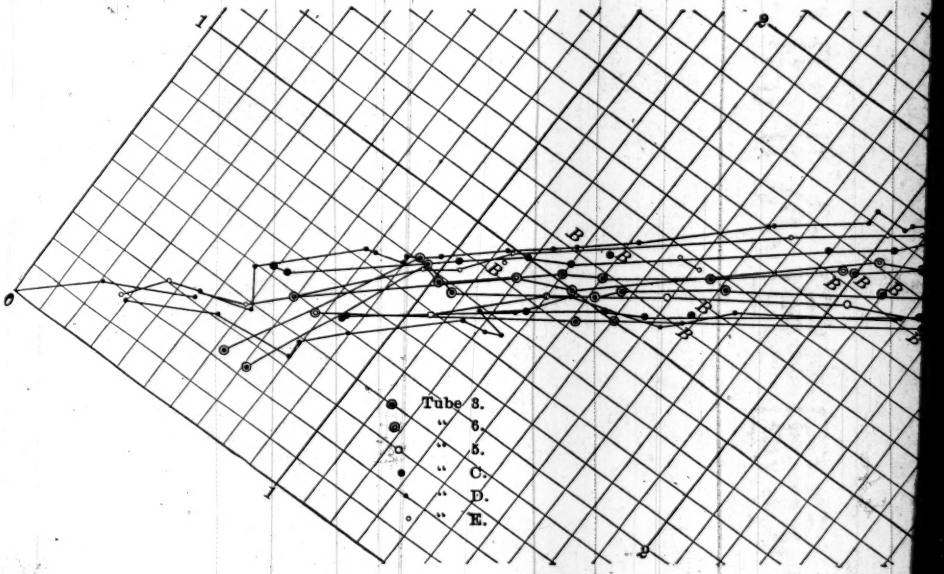
As to the accuracy of the value 0.8139, obtained for the ratio $\frac{V_m}{V_c}$, it is to be considered that the maximum velocity may not be exactly in the center, in which case it would not be measured, and the resulting error would be an increase in the value of the ratio, V_c being small as compared with V_m. Again, it is difficult, even impossible, to measure the velocity at the side of the pipe, on account of the size of the point of the instrument, and therefore a higher velocity is measured than that exactly at the side, which error again tends to increase V_m, and so increases the ratio. As there seems to be no counterbalancing probability of a decrease of the ratio, it is assumed that the foregoing value is in excess of the true one, and the value 0.81 is assumed for use with this pipe at this location.

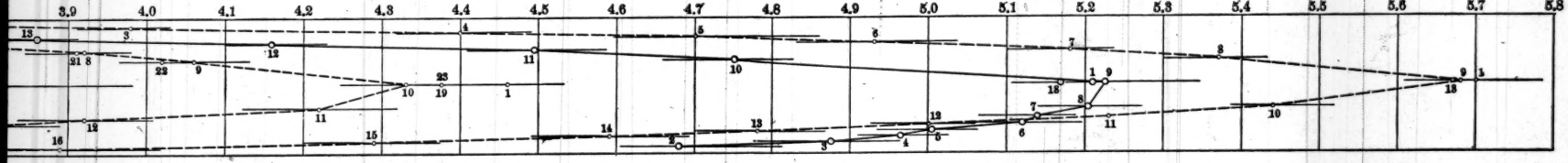


Full lines represent Positions by Point and Pressure.
Broken " " " " " " Ring.
● represent Positions by Tube 3.
○ " " " " 5.
○ " " " " D.
• " " " " E.

Each Position is the Mean of 4 to 20 Observations taken 15 seconds to 1 minute apart.

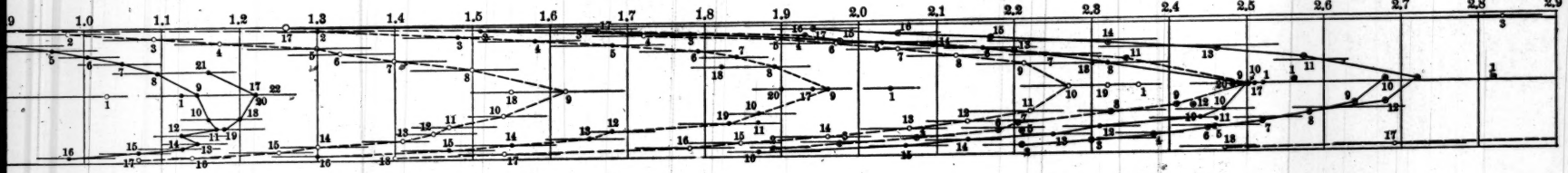


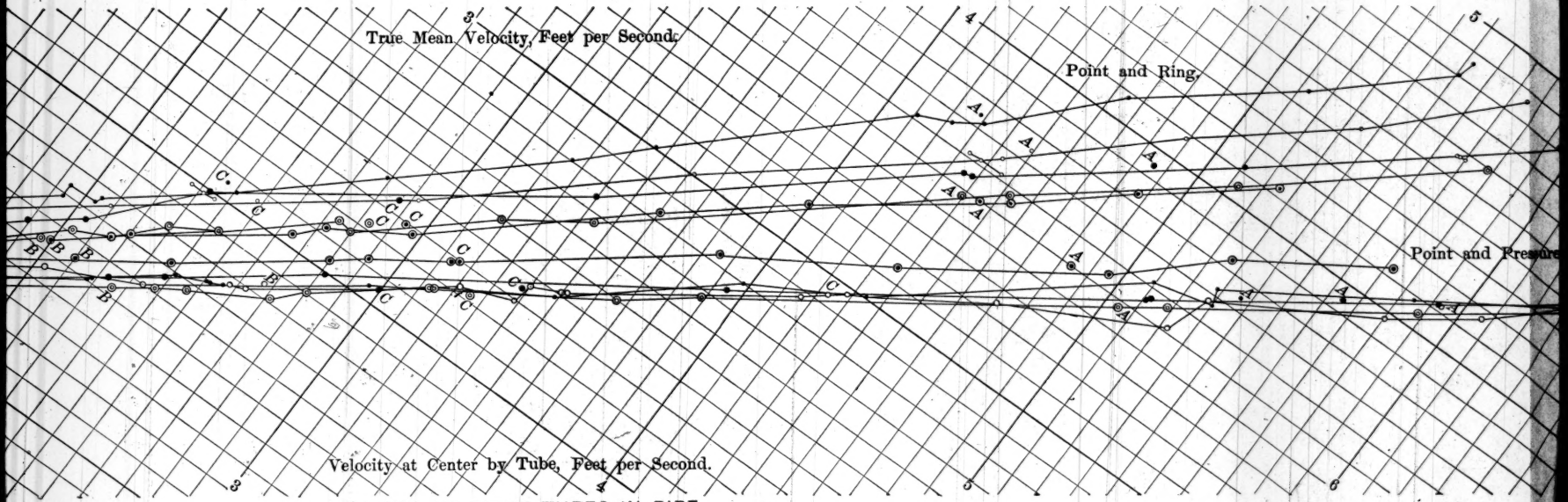




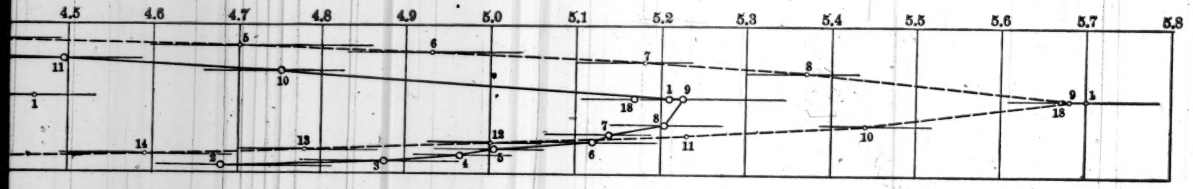
TRAVERSES IN 2-IN. BRASS PIPE WITH PITOT TUBES

MADE AT THE HYDRAULIC LABORATORY OF THE COLLEGE OF CIVIL ENGINEERING OF CORNELL UNIVERSITY, DEC. 18TH AND 20TH, 1899, AND FEB. 8TH TO MARCH 8TH; 1901.



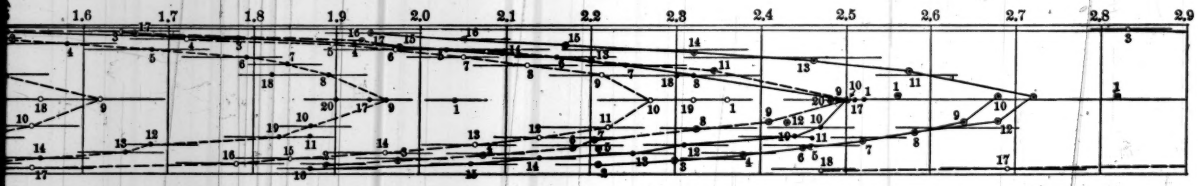


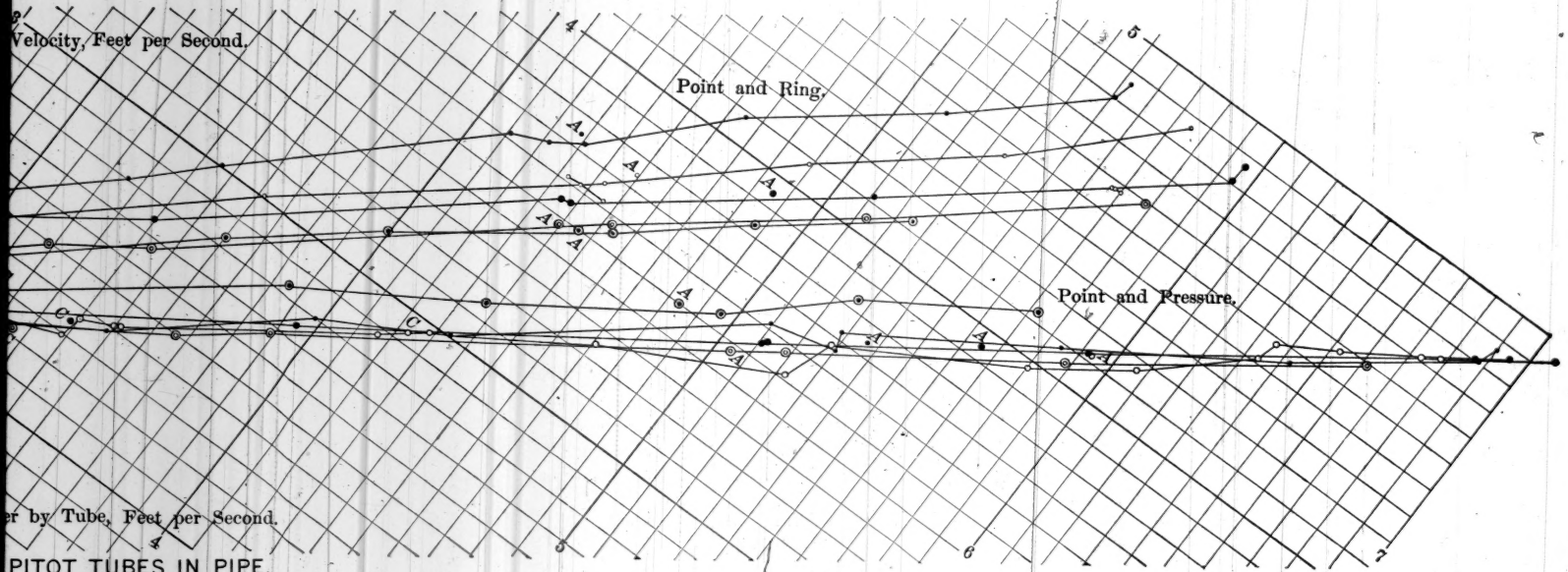
RATING OF PITOT TUBES IN PIPE.



TRAVERSES IN 2-IN. BRASS PIPE WITH PITOT TUBES

ADE AT THE HYDRAULIC LABORATORY OF THE COLLEGE OF CIVIL ENGINEERING OF CORNELL UNIVERSITY,
DEC. 18TH AND 20TH, 1899, AND FEB. 8TH TO MARCH 8TH; 1901.





PITOT TUBES IN PIPE.

TABLE No. 2.—Rating Traverses In 2-Inch Brass Pipe.

Tube No. 3, Horizontal.—December 18th, 19th, 1899.

Position.	No. of observation.	Point and Pressure. *a.* Head, H_v, double centimeters.	Velocity, feet per second.	Velocity used.	U, feet per second.	Point and Ring. *b.* Head, H_v, double centimeters.	Velocity, feet per second.	Velocity used.	U, feet per second.
Center	1	1.88	2.82	.....	2.0114	1.555	2.56	.....	2.0114
1	.....	.....	.....	(0.90)		.....	.....	(0.80)	
2 West	2	1.15	2.21	2.21		0.80	1.89	1.89	
3	3	1.25	2.30	2.30		0.925	1.975	1.98	.....
4	4	1.34	2.38	2.38	.....	1.02	2.075	2.07	.....
5	6	1.42	2.45	2.45	2.0064	1.125	2.18	2.18	2.0064
6	7	1.50	2.52	2.52	.....	1.15	2.205	2.21	.....
7	8	1.58	2.58	2.58	1.9984	1.28	2.325	2.32	1.9984
8	9	1.66	2.64	2.64		1.38	2.41	2.41	
C	10	1.75	2.72	2.74	.....	1.48	2.50	2.51	.....
				2.74				2.51	
10	11	1.57	2.575	2.58	.....	1.295	2.345	2.35	.....
11	13	1.43	2.46	2.46	.....	1.145	2.20	2.30	.....
12	14	1.27	2.32	2.32	.....	1.04	2.095	2.09	.....
13	15	1.11	2.17	2.17	1.9978	0.925	1.975	1.98	1.9978
14	16	1.00	2.05	2.05		0.885	1.93	1.93	
15 East	17	0.89	1.94	1.94	.....	0.65	1.66	1.66	.....
16	.....	.....	.....	(1.80)	.....	.....	.....	(1.59)	.....
17	.....	.....	.....	.....	.....	.....	.....	.....	.....
C	18	1.705	2.68	.....	2.0005	1.475	2.48	.....	2.0005
5	5	1.43	2.46	.....	.....	1.15	2.21	.....	.....
8	12	1.70	2.68	.....	.....	1.40	2.43	.....	.....

* Center, in Traverses *a* and *b*, is 0.0647 in. east of true center.

Note.—Quantities in parentheses are estimated.

Tube No. 5, Horizontal.—December 18th, 19th, 1899.

Position.	No. of observation.	Point and Pressure. *c.* Head, H_v, double centimeters.	Velocity, feet per second.	Velocity used.	U, feet per second.
Center	1	6.31	5.17	.....	.....
1	.....	.....	.....	.....	.....
2	2	5.20	4.68	.....	3.572
3	3	5.65	4.875	.....	
4	4	5.85	4.965	.....	.....
5	5	5.95	5.005	.....	3.575
6	6	6.20	5.12	.....	
7	7	6.25	5.14	.....	.....
8	8	6.40	5.205	.....	3.574
C	10	6.45	5.225	5.21	3.574
10	11	5.35	4.75	4.75	.....
11	12	4.775	4.495	4.50	3.569
12	13	4.10	4.16	4.16	.....
13	14	3.55	3.86	3.86	.....
14	15	3.05	3.58	3.58	3.566
15	16	2.60	3.31	3.31	.....
16	17	2.10	2.98	2.98	.....
17	18	0.375	1.26	1.26	3.566
C	19	6.40	5.21	.....	3.570
2	9	5.40	.....	.....	.....

TABLE No. 2—(*Continued*).

Tube *D*, Horizontal.—February 8th, 1891.

Position.	No. of observation.	Point and Pressure. *d*. Head, H_v, double centimeters.	Velocity, feet per second.	Velocity used.	*U*, feet per second.	Point and Ring. *e*. Head, H_v, double centimeters.	+0.10*	Velocity, feet per second.	Velocity used.	*U*, feet per second.
Center	1	1.51	2.52	.....	1.741	0.88	0.98	2.04	.....	1.741
17	..	.....	.....	(0.65)		.....	.....	.....	(0.60)	
16 E.	2	0.54	1.51	1.51		0.30	0.40	1.30	1.30	
15	3	0.75	1.78	1.78	.....	0.42	0.52	1.48	1.48	.....
14	4	0.865	1.92	1.92	.....	0.486	0.586	1.58	1.58	.....
13	5	0.965	2.03	2.03	.....	0.563	0.663	1.68	1.68	.....
12	6	1.10	2.16	2.16	.....	0.65	0.750	1.79	1.79	.....
11	7	1.19	2.24	2.24	.....	0.70	0.800	1.84	1.84	.....
10	8	1.263	2.32	2.31	.....	0.743	0.843	1.89	1.85	.....
C	9	1.48	2.50	2.51 / 2.51	.....	0.81	0.910	1.96	1.99 / 1.99	.....
8	10	1.445	2.47	2.47	1.739	0.73	0.830	1.87	1.87	1.739
7	11	1.432	2.46	2.45		0.72	0.820	1.86	1.82	
6	12	1.26	2.31	2.31		0.57	0.67	1.68	1.68	
5	13	1.20	2.25	2.25		0.542	0.642	1.65	1.65	
4	14	1.088	2.14	2.14	.....	0.471	0.571	1.55	1.55	.....
3	15	1.00	2.06	2.06	.....	0.41	0.510	1.47	1.47	.....
2	16	0.88	1.87	1.87	.....	0.30	0.400	1.30	1.30	.....
1 W.	..	.....	.....	.....	.....	.....	.....	.....	.....	.....
C	17	1.492	2.51	.....	1.736	0.797	0.897	1.94	.....	1.736
10	18	1.25	2.30	.....	.....	0.686	0.786	1.82	.....	.....
7	19	1.415	2.44	.....	.....	0.69	0.790	1.83	.....	.....
C	20	1.47	2.49	.....	1.736	0.76	0.860	1.90	.....	1.736

Tube *D*, Horizontal.—February 14th, 1901.

Position.	No. of observation.	Point and Pressure *n* and *i*. Head, H_v, double centimeters.	Velocity, feet per second.	Velocity used.	*U*, feet per second.	Point and Ring, *f* and *g*. Head, H_v, double centimeters.	Velocity, feet per second.	Velocity used.	*U*, feet per second.
Center	1	0.306	1.125	.....	.....	1.75	0.86	.....	.....
17 East	..	.....	.....	(0.30)	.....	.....	.....	(0.10)	.....
16	2	0.11	0.68	0.68	.....	0.02	0.29	0.29	.....
15	3	0.15	0.80	0.80	.....	0.05	0.46	0.46	.....
14	4	0.19	0.90	0.90	.....	0.07	0.545	0.545	.....
13	5	0.22	0.96	0.96	.....	0.08	0.58	0.58	.....
12	6	0.24	1.005	1.005	.....	0.092	0.62	0.62	.....
11	7	0.26	1.05	1.05	.....	0.11	0.68	0.68	.....
10	8	0.28	1.095	1.095	.....	0.13	0.74	0.74	.....
C	9	0.31	1.145	1.135	1.22	0.145	0.78	0.78	0.817
8	10	0.32	1.16	.....	1.20	0.14	0.77	.....	0.775
7	11	0.326	1.17	.....	1.18	0.13	0.74	.....	0.755
6	12	0.30	1.125	.....	(1.16)	0.12	0.71	.....	(0.745)
5	13	0.31	1.145	.....	1.145	0.13	0.74	.....	0.74
4	14	0.30	1.125	.....	1.125	0.12	0.71	.....	0.71
3	15	0.27	1.07	.....	1.07	0.105	0.665	.....	0.665
2	16	0.23	0.98	.....	0.98	0.07	0.545	.....	0.545
1 West	..	.....	.....	.....	(0.40)	.....	.....	.....	(0.25)
C	17	0.35	1.22	.....	.....	0.15	0.795	.....	.....
8	18	0.34	1.20	.....	.....	0.142	0.775	.....	.....
7	19	0.33	1.18	.....	0.7439	0.135	0.755	.....	0.7439
C	20	0.35	1.22	.....	.....	0.16	0.82	.....	.....
10	21	0.32	1.16	.....	.....	0.14	0.77	.....	.....
C	22	0.35	1.22	.....	.....	0.17	0.845	.....	.....

* Correction for initial error of gauge, with no flow.

TABLE No. 2—(*Concluded*).

Tube *E*, Horizontal.—February 15th, 1901.

Position.	No. of observation.	Point and Ring. *j.* Head, H_v, double centimeters.	Velocity, feet per second.	Velocity used.	U, feet per second.	Point and Ring, *k.* Head, H_v, double centimeters.	Velocity, feet per second.	Velocity used.	U, feet per second.
Center	1	0.25	1.03	....	.....	7.68	5.70	....	4.575
17 East	..	....	....	(0.45)	.....	....	....	(1.20)	.....
16	2	0.23	0.98	0.98		3.04	3.58	3.58	.....
15	3	0.28	1.09	1.09		3.76	3.98	3.98	.....
14	4	0.33	1.175	1.175	1.272	4.58	4.40	4.40	.....
13	5	0.40	1.30	1.30		5.23	4.70	4.70	.....
12	6	0.42	1.33	1.33		5.76	4.93	4.93	4.579
11	7	0.46	1.40	1.40		6.35	5.18	5.18	
10	8	0.53	1.50	1.50	.....	6.83	5.37	5.37	.....
C	9	0.62	1.62	1.59 1.59	.....	7.63	5.68	5.68 5.68	.....
8	10	0.56	1.54	1.54	.....	6.98	5.44	5.44	.....
7	11	0.51	1.47	1.47		6.46	5.23	5.23	.....
6	12	0.50	1.45	1.45		5.90	5.00	5.00	.....
5	13	0.47	1.41	1.41		5.41	4.78	4.78	.....
4	14	0.40	1.30	1.30	1.272	4.99	4.59	4.59	.....
3	15	0.37	1.25	1.25		4.34	4.29	4.29	
2	16	0.31	1.14	1.14		3.59	3.89	3.89	4.578
1½ West	17	0.27	1.07	....	.....	3.20	3.67	....	.....
C	18	0.57	1.55	....	.....	7.62	5.67	....	.....

Tube *E*, Vertical.—March 8th, 1901.

Position.	No. of observation.	Point and Ring. *m.* Head, H_v, double centimeters.	Velocity, feet per second.	Velocity used.	U, feet per second.	Point and Ring. *l.* Head, H_v, double centimeters.	Velocity, feet per second.	Velocity used.	U, feet per second.
Center	1	4.69	4.46	....		1.32	2.36	....	1.986
17 Top	2	0.205	0.93	0.93	3.615	0.12	0.712	0.71	.....
16	3	1.896	2.83	2.83		0.64	1.645	1.65	1.986
15	4	2.28	3.105	3.10	.....	0.70	1.72	1.72	.....
14	5	2.60	3.31	3.31	.....	0.85	1.895	1.89	1.979
13	6	2.94	3.515	3.52	.....	0.92	1.97	1.97	.....
12	7	3.31	3.74	3.75	.....	1.00	2.05	2.05	
11	8	3.65	3.92	3.92	.....	1.07	2.125	2.13	1.986
10	9	3.92	4.06	4.04	.....	1.15	2.21	2.21	.....
C	10	4.43	4.33	4.40 4.39	3.612	1.215	2.27	2.31 2.31	.....
8	11	4.21	4.22	4.22	.....	1.16	2.22	2.22	.....
7	12	3.65	3.92	3.92	.....	1.08	2.14	2.14	
6	13	3.42	3.79	3.79	3.598	1.004	2.065	2.06	2.020
5	14	3.10	3.61	3.61	.....	0.902	1.96	1.96	1.913
4	15	2.71	3.38	3.38	.....	0.804	1.845	1.84	1.986
3	16	2.30	3.115	3.12	3.644	0.75	1.78	1.78	.....
2	17	1.72	2.69	2.69	3.608	0.56	1.54	1.54	
1½	18	1.444	2.47	....	.....	0.462	1.40	....	1.977
C	19	4.515	4.375	....	3.591	1.275	2.32	....	1.979
12	20	3.35	3.76	....	.....	....	....	....	.....
11	21	3.64	3.91	....	3.604	....	....	....	.....
10	22	3.84	4.02	....	.....	....	....	....	.....
C	23	4.52	4.375	....	3.617	....	....	....	.....

TABLE No. 3.—Experiments with Pitot Tubes on Center in 2-In. Brass Pipe.

Tube No. 3, 68.23 ft. from Inlet.

Number.	Date, etc.	Duration. Of experiment. Minutes.	Duration. Of weighing. Seconds.	Head = H_v Centimeters. Observed.	Correction.	Corrected.	Velocity, Feet per Second. By tube, at center. V_c	By weight, mean. U	Ratio $\frac{U}{\sqrt{V_c}}$	Temperature of water. Fahrenheit.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
	Dec. 18, 1900.			Point and Pressure.						
..	No flow			0.000	0.0	.....	0.000	0.000	.....	.....
1		15	305	15.472	.....	.....	5.717	4.130	0.722	36°-37°
2		15	300	9.028	.....	.....	4.367	3.151	0.722	36°-37°
3		15	300	3.714	.....	.....	2.801	2.042	0.729	36°-37°
4		14	300	4.648	.....	.....	3.134	2.280	0.728	36°-37°
5		15	300	2.667	.....	.....	2.374	1.720	0.725	36°-37°
6		14	300	4.002	.....	.....	2.908	2.128	0.732	36°-37°
7		15	300	7.035	.....	.....	3.855	2.835	0.735	36°-37°
8		10	300	0.950	.....	.....	1.416	1.025	0.723	36°-37°
9		15	140 200	1.027	.....	.....	1.472	1.084 1.090	0.738	36°-37°
10		10	250	1.400	.....	.....	1.720	1.273	0.740	36°-37°
11		8	200	0.538	.....	.....	1.066	0.782	0.734	36°-37°
..	No flow	.....	.....	0.000	.....	.....	0.000	0.000	.....	.....
12		10	200	13.098	.....	.....	5.260	3.834	0.729	36°-37°
13		17	300	11.610	.....	.....	4.952	3.555	0.718	36°-37°
..	No flow	.....	.....	0.000	.....	.....	0.000	0.000	.....	.....
..	No flow, Dec. 19th, 1900.	.....	.....	— 0.060	+0.06	0.000	0.000	0.000	.....	.....
14		24	600	0.780	+0.06	0.240	0.712	0.2307	0.324*	36°-37°
15		16	600	0.400	+0.04	0.440	0.958	0.8107	0.846*	36°-37°
..	No flow	.....	.....	— 0.040	+0.04	0.000	0.000	0.000	.....	.....
	Dec. 18, 1900.			Point and Circumferential Piezometer.						
..	No flow	Time, duration and weights same as for point and pressure.	.....	— 0.10	+0.10	0.000	0.000	0.000	.....	.....
1			.....	12.924	0.10	13.024	5.245	4.130	0.787	36°-37°
2			.....	7.449	0.10	7.549	3.993	3.151	0.789	36°-37°
3			.....	3.222	0.10	3.322	2.649	2.042	0.771	36°-37°
4			.....	4.095	0.10	4.195	2.977	2.280	0.766	36°-37°
5			.....	2.204	0.10	2.304	2.206	1.720	0.780	36°-37°
6			.....	3.414	0.10	3.514	2.725	2.128	0.781	36°-37°
7			.....	6.062	0.10	6.162	3.608	2.835	0.786	36°-37°
8			.....	0.700	0.10	0.800	1.300	1.025	0.788	36°-37°
9			.....	0.840	0.10	0.940	1.409	1.084 1.090	0.771	36°-37°
10			.....	1.213	0.10	1.313	1.665	1.273	0.764	36°-37°
11			.....	0.340	0.15	0.490	1.017	0.782	0.769	36°-37°
..	No flow		.....	— 0.16	0.16	0.000	0.000	0.000	.....	.....
12			.....	11.074	0.16	11.234	4.871	3.834	0.787	36°-37°
13			.....	9.628	0.15	9.778	4.545	3.555	0.782	36°-37°
..	No flow		.....	— 0.15	0.15	0.000	0.000	0.000	.....	.....
..	No flow, Dec. 19th, 1900.		.....	— 0.06	0.06	0.000	0.000	0.000	.....	.....
14			.....	0.127	0.06	0.187	0.627	0.2307	0.868*	36°-37°
15			.....	0.366	0.04	0.406	0.928	0.8107	0.874*	36°-37°
..	No flow		.....	— 0.04	0.04	0.000	0.000	0.000	.....	.....

Mean 0.729

Mean 0.779

* Experiments omitted in computing arithmetic means.

TABLE No. 3—(*Continued*).

Tube No. 6, 68.23 ft. from Inlet.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
	Dec. 18, 1900.			POINT AND PRESSURE.						
..	No flow			0.000	0.000		0.000	0.000		
1		20	279	20.678			6.609	4.592	0.695	
2		17	300	7.164			3.890	2.674	0.687	36°-37°
3		15	200	16.356			5.878	4.052	0.689	36°-37°
4		15	300	3.504			2.730	1.819	0.668	36°-37°
5		10	300	12.710			5.182	3.575	0.690	36°-37*
6		15	300	6.360			3.665	2.496	0.681	36°-37°
..	No flow			0.000			0.000	0.000		
7		9	300	4.636			3.129	2.162	0.691	36°-37°
8		11	240	2.692			2.385	1.616	0.678	36°-37°
9		11	300	0.947			1.414	0.931	0.659	36°-37°
10		10	300	1.080			1.510	1.001	0.668	36°-37°
11		10	405	4.674			3.142	2.171	0.691	36°-37°
12		10	300	3.724			2.805	1.905	0.679	36°-37°
13		10	300	2.900			2.475	1.674	0.676	36°-37°
14		13	300	0.287			0.778	0.490	0.630*	36°-37°
..	No flow			0.000			0.000	0.000		
				POINT AND CIRCUMFERENTIAL PIEZOMETER.						
..	No flow	Time, duration and weights same as for point and pressure.		— 0.020	+0.02	0.000	0.000	0.000		
1				15.790	0.02	15.810	5.779	4.592	0.794	36°-37°
2				5.280	0.02	5.300	3.346	2.674	0.799	36°-37°
3				12.422	0.02	12.442	5.127	4.052	0.790	36°-37°
4				2.474	0.02	2.494	2.205	1.819	0.792	36°-37°
5				9.668	0.02	9.688	4.524	3.575	0.790	36°-37°
6				4.792	0.02	4.812	3.188	2.496	0.783	36°-37°
..	No flow			— 0.020	+0.02	0.000	0.000	0.000		
7				3.690	0.02	3.710	2.799	2.162	0.772	36°-37°
8				1.943	0.02	1.963	2.037	1.616	0.793	36°-37°
9				0.657	0.02	0.677	1.195	0.931	0.779	36°-37°
10				0.847	0.02	0.867	1.353	1.001	0.746	36°-37°
11				3.554	0.01	3.564	2.744	2.171	0.791	36°-37°
12				2.784	0.01	2.794	2.489	1.905	0.784	36°-37°
13				2.188	0.01	2.198	2.155	1.674	0.777	36°-37°
14				0.220	0.008	0.228	0.694	0.490	0.706*	36°-37°
..	No flow			— 0.008	+0.008	0.000		0.000		

Mean 0.784

Tube *C*, 68.23 ft. from Inlet.

	Dec. 15, 1900.			POINT AND PRESSURE.						
..	No flow			0.000	0.000		0.000	0.000		
1		10	300	22.842			6.946	4.867	0.701	36°-37°
2		25	300	20.194			6.531			
3		15	300	12.356			5.109	3.549	0.694	36°-37°
4		20	300	3.756			2.817	1.992	0.707	36°-37°
5		9	300	1.018			1.466	1.005	0.685	36°-37°
..	No flow			0.000			0.000	0.000		
6		10	300	0.345			0.854	0.528	0.619*	36°-37°
7		10	300	2.702			2.389	1.667	0.698	36°-37°
8		13	300	7.384			3.949	2.750	0.696	36°-37°
9		19	300	16.502			5.915	4.119	0.696	36°-37°
10		17	300	1.174			1.575	1.065	0.676	36°-37°
11		16	280	22.284			6.862	4.805	0.700	36°-37°
12		18	230	12.384			5.115	3.554	0.695	36°-37°
13		8	200	2.360			2.233	1.544	0.692	36°-37°
14		10	300	0.347			0.856	0.539	0.630*	36°-37°
15		10	300	0.773			1.278	0.869	0.681	36°-37°
..	No flow			0.000			0.000	0.000		

Mean 0.688

* Experiments omitted in computing arithmetic means.

TABLE No. 3—(*Continued*).

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
				Point and Circumferential Piezometer.						
..	No flow.....	Time, duration and weights same as for point and pressure.	.....	— 0.020	+0.02	0.000	0.000	0.000	.....	.....
1	.....		.....	16.744	0.03	16.774	5.953	4.867	0.818	36°-37°
2	.....		.....	14.641	0.04	14.681	5.569	.....	.....	36°-37°
3	.....		.....	8.928	0.05	8.978	4.355	3.549	0.815	36°-37°
4	.....		.....	2.514	0.06	2.574	2.332	1.992	0.854	36°-37°
5	.....		.....	0.589	0.08	0.669	1.189	1.005	0.845	36°-37°
..	No flow.....		.....	— 0.100	0.10	0.000	0.000	0.000	.....	.....
				0.000		0.000			Mean 0.821	
6	.....		.....	0.165	0.00	0.165	0.590	0.528	0.895*	36°-37°
7	.....		.....	1.984	0.01	1.994	2.052	1.667	0.812	36°-37°
8	.....		.....	5.446	0.02	5.466	3.398	2.750	0.809	36°-37°
9	.....		.....	12.312	0.03	12.342	5.106	4.119	0.807	36°-37°
10	.....		.....	0.762	0.04	0.802	1.302	1.065	0.818	36°-37°
11	.....		.....	16.710	0.05	16.760	5.950	4.805	0.807	36°-37°
12	.....		.....	9.044	0.06	9.104	4.385	3.554	0.810	36°-37°
13	.....		.....	1.635	0.07	1.705	1.898	1.544	0.814	36°-37°
14	.....		.....	0.110	0.08	0.190	0.633	0.539	0.851*	36°-37°
15	.....		.....	0.418	0.08	0.498	1.025	0.869	0.848	36°-37°
..	No flow.....		.....	— 0.080	0.08	.....	.....	0.000	.....	.....

Tube No. 5, 68.23 ft. from Inlet.

Mar. 17, 1900.				Point and Pressure.						
..	No flow.....	.....	.....	0.000	.....	.....	0.000	0.000	.....	.....
1	.....	65	510	2.40	.....	.....	2.251	1.458	0.647	36°-38°
2	.....	6	300	17.40	.....	.....	6.063	4.165	0.687	36°-38°
3	.....	5	300	15.90	.....	.....	5.795	3.971	0.685	36°-38°
4	Mar. 23d, 1900.	7	300	23.48	.....	.....	7.059	4.941	0.700	36°-38°
5	.....	7	300	21.74	.....	.....	6.777	4.741	0.699	36°-38°
6	.....	7	300	21.42	.....	.....	6.727	4.714	0.701	36°-38°
7	.....	7	300	20.12	.....	.....	6.520	4.580	0.703	36°-38°
8	.....	7	300	19.10	.....	.....	6.352	4.486	0.706	36°-38°
..	No flow, Mar. 24th, 1900....	.....	.....	0.000	.....	.....	0.000	0.000	.....	.....
9	.....	9	300	19.00	.....	.....	6.335	4.413	0.697	36°-38°
10	.....	8	420	8.20	.....	.....	4.162	2.878	0.692	36°-38°
11	.....	8	420	8.70	.....	.....	4.287	2.975	0.694	36°-38°
12	.....	8	360	10.50	.....	.....	4.710	3.250	0.690	36°-38°
13	.....	8	360	11.56	.....	.....	4.942	3.590	0.727	36°-38°
..	No flow, Aug. 8th, 1900....	.....	.....	0.000	.....	.....	0.000	0.000	.....	.....
									Mean 0.694	
14	.....	22	300	0.80	.....	.....	1.300	0.988	0.722	75°-76°
15	.....	10	300	5.84	.....	.....	3.512	2.420	0.689	75°-76°
16	.....	17	300	5.44	.....	.....	3.389	2.299	0.678	75°-76°
17	.....	18	310	2.60	.....	.....	2.344	1.602	0.684	75°-76°
18	.....	12	300	3.16	.....	.....	2.584	1.776	0.688	75°-76°
19	.....	8	300	5.80	.....	.....	3.500	2.413	0.682	75°-76°
20	.....	14	300	4.34	.....	.....	3.028	1.978	0.653	75°-76°
21	.....	9	300	3.28	.....	.....	2.632	1.797	0.683	75°-76°
22	.....	13	360	4.86	.....	.....	3.204	2.219	0.692	75°-76°
23	.....	13	460	0.53	.....	.....	1.058	0.690	0.653	75°-76°
..	No flow, Aug. 9th, 1900....	.....	.....	0.000	.....	.....	0.000	0.000	.....	.....
24	.....	12	300	17.30	.....	.....	6.045	4.357	0.721	75°-76°
25	.....	16	300	1.20	.....	.....	1.592	1.153	0.724	75°-76°
26	.....	13	300	1.96	.....	.....	2.035	1.455	0.715	75°-76°
27	.....	7	300	16.50	.....	.....	5.904	4.270	0.723	75°-76°
28	.....	23	300	5.48	.....	.....	3.402	2.366	0.695	75°-76°
29	.....	7	300	13.20	.....	.....	5.280	3.673	0.696	75°-76°
..	No flow.....	.....	.....	0.000	.....	.....	0.000	0.000	.....	.....

* Experiments omitted in computing arithmetic means.

TABLE No. 3—(*Continued*).

Tube *D*, 68.30 ft. from Inlet.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
	Feb. 8, 1901.			POINT AND PRESSURE.—Double Centimeters.						
..	No flow			0.000	0.00	.....	0.000	0.000	.....	.....
1		15	720	1.510	.....	.....	2.52	1.741	0.691	36°-38°
2		15	600	1.492	.....	.....	2.51	1.736	0.692	36°-38°
..	No flow, Feb. 13th, 1901			0.000	.....	.....	0.000	0.000	.....	.....
3		14	300	11.16	.....	.....	6.87	4.803	0.699	36°-38°
4		13	300	6.59	.....	.....	5.38	3.725	0.706	36°-38°
5		15	230	6.12	.....	.....	5.09	3.617	0.711	36°-38°
6	Feb. 14th, 1901	23	300	3.77	.....	.....	3.98	2.802	0.704	36°-38°
7		22	420	2.07	.....	.....	2.96	2.046	0.691	36°-38°
8		17	300	1.12	.....	.....	2.18	1.502	0.689	36°-38°
9		14	300	0.63	.....	.....	1.63	1.070	0.656	36°-38°
10		13	300	0.75	.....	.....	1.78	1.239	0.696	36°-38°
11		14	450	0.38	.....	.....	1.266	0.764	0.603	36°-38°
12		23	625	0.19	.....	.....	0.895	0.492	0.549*	36°-38°
13		20	590	0.15	.....	.....	0.795	0.332	0.418*	36°-38°
..	No flow, Feb. 14th, 1901			0.000	0.00	.....	0.000	0.000	.....	.....
14		41	1 780	0.31	.....	.....	1.145	0.736	0.643	36°-38°
15		16	810	0.35	.....	.....	1.22	0.744	0.610	36°-38°
..	No flow, Mar. 19th, 1901			0.000	.....	.....	0.000	0.000	.....	.....
16		4	50	11.28	.....	.....	6.90	4.863	0.705	36°-38°
17		8	60	9.79	.....	.....	6.42	4.458	0.695	36°-38°
18		5	133	6.64	.....	.....	5.30	3.663	0.691	36°-38°
19		6	96	4.45	.....	.....	4.34	3.005	0.692	36°-38°
20		5	.....	3.40	.....	.....	3.78	.....	.....	.....
21		5	112	2.89	.....	.....	3.49	2.388	0.684	36°-38°
22		5	77	1.56	.....	.....	2.565	1.760	0.686	36°-38°
23		10	100	0.37	.....	.....	1.255	0.849	0.677	36°-38°
24		5	200	0.07	.....	.....	0.542	0.310	0.573*	36°-38°
25		5	40	0.02	.....	.....	0.292	0.184	0.630*	36°-38°
26		10	200	0.44	.....	.....	1.36	0.983	0.723	36°-38°
27		10	200	1.38	.....	.....	2.41	1.692	0.702	36°-38°
28		9	100	1.13	.....	.....	2.19	1.518	0.693	36°-38°
29		10	230	0.325	.....	.....	1.170	0.786	0.672	36°-38°
30		9	200	0.15	.....	.....	0.795	0.388	0.488*	36°-38°
31		11	200	8.07	.....	.....	5.84	4.083	0.699	36°-38°
..	No flow			0.01	.....	.....	0.000	0.000	.....	.....
	Feb. 8, 1901.			POINT AND CIRCUMFERENTIAL PIEZOMETER.						
..	No flow			— 0.10	+0.10	0.000	0.000	0.000	.....	.....
1				0.88	0.10	0.98	2.04	1.741	0.853	36°-38°
2				0.797	0.10	0.897	1.94	1.736	0.895	36°-38°
..	No flow			0.10	0.10	0.000	0.000	0.000	.....	.....
..	No flow, Feb. 13th, 1901			0.000	0.00	.....	0.000	0.000	.....	.....
3				7.17	.....	.....	5.51	4.803	0.872	36°-38°
4				4.39	.....	.....	4.31	3.725	0.864	36°-38°
5				4.01	.....	.....	4.11	3.617	0.880	36°-38°
6				2.52	.....	.....	3 26	2.802	0.859	36°-38°
7				1.37	.....	.....	2.41	2.046	0.849	36° 38°
8				0.69	.....	.....	1.705	1.502	0.881	36°-38°
9				0.36	.....	.....	1.232	1.070	0.868	36°-38°
10				0.48	.....	.....	1.425	1.239	0.869	36°-38°
11				0.19	.....	.....	0.895	0.764	0.853	36°-38°
12				0.07	.....	.....	0.543	0.492	0.905*	36°-38°
13				0.05	.....	.....	0.46	0.332	0.722*	36°-38°
..	No flow, Feb. 14th, 1901			0.000	0.00	.....	0.000	0.000	.....	.....
14				0.145	.....	.....	0.78	0.736	0.943	36°-38°
15				0.155	.....	.....	0.81	0.744	0.918	36°-38°
..	No flow, Mar. 19th, 1901			0.000	.....	.....	0.000	.....	.....	.....
16				7.23	.....	.....	5.525	4.863	0.880	36°-38°
17				6.23	.....	.....	5.13	4.458	0.869	36°-38°
18				4.22	.....	.....	4.22	3.663	0.868	36°-38°
19				2.85	.....	.....	3.46	3.005	0.868	36°-38°
20				2.16	.....	.....	3.02	.....	.....	.....
21				1.84	.....	.....	2.79	2.388	0.856	36°-38°
22				1.00	.....	.....	2.05	1.760	0.859	36°-38°
23				0.19	+0.04	0.23	0.975	0.849	0.871	36°-38°
24				0.01	0.04	0.05	0.460	0.310	0.675*	36°-38°
25				— 0.03	0.04	0.01	0.205	0.184	0.898*	36°-38°
26				0.30	.....	.....	1.125	0.983	0.874	36°-38°
27				0.89	.....	.....	1.940	1.692	0.872	36°-38°
28				0.70	.....	.....	1.720	1.518	0.883	36°-38°
29				0.15	0.04	0.19	0.895	0.786	0.878	36°-38°
30				0.05	0.04	0.09	0.615	0.388	0.630*	36°-38°
31				5.12	.....	.....	4.65	4.083	0.878	36°-38°
..	No flow			— 0.04	.....	.....	0.000	0.000	.....	.....

Time, duration and weights same as for point and pressure.

Mean 0.684

Mean 0.874

* Experiments omitted in computing arithmetic means.

TABLE No. 3—(*Concluded*).

Tube *E*, 68.30 ft. from Inlet.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
	Feb. 15, 1901.			Point and Circumferential Piezometer.						
..	No flow	.....	.....	0.000	0.00	.....	0.000	0.000	.....	.....
1	.....	40	730	0.62	.....	.....	1.62	1.272	0.785	36°-38°
2	.....			0.57	.....	.....	1.55	1.272	0.821	36°-38°
3	.....	100	300	7.68	.....	.....	5.70	4.575	0.803	36°-38°
4	.....		330	7.63	.....	.....	5.68	4.578	0.806	36°-38°
5	.....		320	7.62	.....	.....	5.67	4.578	0.807	36°-38°
..	No flow, Mar. 8th, 1901	.....	.....	0.000	0.00	.....	0.000	0.000	.....	.....
6	.....	80	360	4.69	.....	.....	4.46	3.615	0.811	36°-38°
7	.....		500	4.43	.....	.....	4.33	3.612	0.835	36°-38°
8	.....		.....	4.515	.....	.....	4.375	.....	.....	.....
9	.....		500	4.52	.....	.....	4.375	3.617	0.827	36°-38°
10	.....	54	300	1.32	.....	.....	2.36	1.986	0.841	36°-38°
11	.....		600	1.215	.....	.....	2.27	1.986	0.875	36°-38°
12	.....		300	1.275	.....	.....	2.32	1.979	0.853	36°-38°
..	No flow	.....	.....	.....	.....	.....	.....	.....	.....	.....

Mean 0.824

Tube *E*, 3.05 ft. from Inlet.

	Mar. 19, 1901.			Point and Circumferential Piezometer.						
..	No flow	.....	.....	0.000	0.00	.....	0.000	0.000	.....	.....
16	.....	Time, duration and weights same as for similar numbers of Tube D, observations being simultaneous.	.....	7.82	.....	.....	5.75	4.863	0.846	36°-38°
17	.....		.....	6.76	.....	.....	5.35	4.458	0.833	36°-38°
18	.....		.....	4.63	.....	.....	4.43	3.663	0.827	36°-38°
19	.....		.....	3.11	.....	.....	3.62	3.005	0.830	36°-38°
20	.....		.....	2.32	.....	.....	3.13	.....	.....	.....
21	.....		.....	2.02	.....	.....	2.92	2.388	0.818	36°-38°
22	.....		.....	1.04	.....	.....	2.09	1.760	0.842	36°-38°
23	.....		.....	0.26	.....	.....	1.045	0.849	0.813	36°-38°
24	.....		.....	0.032	.....	.....	0.367	0.310	0.846	36°-38°
25	.....		.....	0.018	.....	.....	0.278	0.184	0.662*	36°-38°
26	.....		.....	0.31	.....	.....	1.140	0.983	0.862*	36°-38°
27	.....		.....	0.95	.....	.....	2.005	1.692	0.844	36°-38°
28	.....		.....	0.75	.....	.....	1.780	1.518	0.853	36°-38°
29	.....		.....	0.22	.....	.....	0.955	0.786	0.823	36°-38°
30	.....		.....	0.085	.....	.....	0.595	0.388	0.652*	36°-38°
31	.....		.....	5.665	.....	.....	4.885	4.083	0.836	36°-38°
..	No flow	.....	.....	0.000	.....	.....	0.000	0.000	.....	.....

Mean 0.834

* Experiments omitted in computing arithmetic means.

Co-ordinates of Points *B*, *C* and *A*.

Tube condition.	*B*.		*C*.		*A*.	
	V.	*U*.	*V*.	*U*.	*V*.	*U*.
No. 3, point and pressure	2.1105	1.5422	3.1565	2.2956	4.8302	3.501
" ring	1.9935	1.5422	2.9392	2.2956	4.4524	3.501
No. 6, " pressure	2.2621	1.5300	3.2559	2.2256	5.0448	3.4778
" ring	1.9679	1.5300	2.8339	2.2256	4.3928	3.4778
C, " pressure	1.6835	1.1525	3.3832	2.3474	5.6494	3.9407
" ring	1.3778	1.1525	2.8692	2.3474	4.8578	3.9407
No. 5, " pressure	2.6763	1.8466	4.2336	2.9371	5.9272	4.1183
D, " "	1.6750	1.1119	2.9870	2.0560	5.3610	3.7410
" ring	1.2860	1.1119	2.3610	2.0560	4.2810	3.7410
E, " " at point 3.05 ft. from inlet	1.1390	0.9470	2.4810	2.0620	4.4980	3.7430

Equations of Rating Lines *B*, *C*, *A*.

Tube No. 3, point and pressure.....$U = 0.720\ V_c + 0.0229$
" " ring.....$U = 0.797\ V_c - 0.0469$
Tube No. 6, " pressure.....$U = 0.700\ V_c - 0.0535$
" " ring.....$U = 0.803\ V_c - 0.0500$
Tube *C*, " pressure.....$U = 0.708\ V_c - 0.0310$
" " ring.....$U = 0.801\ V_c + 0.0492$
Tube No. 5, " pressure.....$U = 0.6975\ V_c - 0.0158$
Tube *D*, " pressure.....$U = 0.713\ V_c - 0.0737$
" " ring.....$U = 0.878\ V_c - 0.0170$
Tube *E*, " ring.....$U = 0.834\ V_c - 0.0071$

TABLE No. 4.

Traverse.	Date.	Instrument.	Conditions.	Velocity, in Feet per Second.			Ratio: $\frac{V_m}{V_c}$	Ratios used in computations.
				Mean, by weight. U.	Mean, by tube. V_m.	Center, by tube. V_c.		
j.....	Feb. 15, 1901.	*E*	Hor. pt. and ring.	1.272	1.292	1.590	0.812	0.812
l.....	Mar. 8, "	"	Vert. " "	1.980	1.912	2.317	0.826	0.826
m.....	" " "	"	" " "	3.610	3.465	4.390	0.789	0.789
k.....	Feb. 15, "	"	Hor. " "	4.577	4.584	5.680	0.807	0.807
f.....	Feb. 14, "	*D*	" " "	0.729	0.544	0.780	0.698	.....
g.....	" " "	"	" " "	0.744	0.684	0.817	0.837	0.837
e.....	Feb. 8, "	"	" " "	1.738	1.614	1.990	0.811	0.811
h.....	Feb. 14, "	"	" " pres.	0.729	0.892	1.135	0.785	0.785
i.....	" " "	"	" " "	0.744	1.084	1.220	0.889	.....
d.....	Feb. 8, "	"	" " "	1.738	2.070	2.510	0.825	0.825
b.....	Dec. 18, 1899.	No. 3	" " ring.	2.007	2.046	2.510	0.815	0.815
a.....	" " "	"	" " pres.	2.007	2.281	2.740	0.832	0.832
c.....	Dec. 20, "	No. 5	" " "	3.569	3.798	5.281	0.729	.....

Mean ratio: 0.8139

Considering now the values of the ratios $\frac{U}{V_c}$, as shown in Table No. 3, and the plottings of the observations on the lower diagram on Plate III, it appears that there is some variation of this ratio with the velocity. This, however, is small; the observations falling approximately in straight lines which pass very nearly through the origin. It has been considered that such variations as appear are due rather to the shifting of the point of maximum velocity back and forth past the center, a condition which may or may not be common to all pipes, than to an actual variation of the ratio. Whether or not this assumption be correct, the traverses prove that in this pipe such a shifting exists. The conclusion has therefore been arrived at, that for practical purposes, within a range of velocity from 0.75 ft. to 6 ft., the foregoing ratio may be taken as a constant for each particular tube.

In determining the value of this ratio, the simplest process is to take the arithmetic means, and, restricting the computation to ratios where the true mean velocity was greater than 0.6 ft., the following values of such means are obtained for the several tubes:

TABLE No. 5.

Tubes.	Values of Mean Velocity, by Weight. / Velocity at Center, by Tube.	
	Point and pressure.	Point and ring.
Tube No. 3	0.729	0.779
" " 6	0.683	0.784
" C	0.693	0.821
" No. 5	0.694	
" D	0.684	0.874
" E	At 68.30 ft. from inlet.	0.824
" E	" 3.05 ft. " "	0.834

If the value for the ratio $\frac{V_m}{V_c} = 0.81$ be now applied to Tubes Nos. 3 and 6 the coefficients so obtained are: Tube No. 3, $\frac{0.729}{0.810} = 0.900$; and Tube No. 6, $\frac{0.683}{0.810} = 0.844$.

In order to study the results of the ratings more closely, the equations of the straight lines fitting the plotted experiments have been computed by the method* of successive centers of gravity *C*, *B* and *A* and the points *C*, *B* and *A* located on the lower diagram on Plate III, although the lines have not been drawn. The co-ordinates of *C*, *B*, and *A*, and the equations of the lines are given at the end of Table No. 3. Considering only the equations of point and pressure ratings of Tubes Nos. 3 and 6, for the present, which are:

$$U = 0.720\ V_c + 0.0229 \text{ for Tube No. 3, and}$$
$$U = 0.700\ V_c - 0.0535 \text{ for Tube No. 6,}$$

it appears that the coefficient in the former is less, and in the latter greater, than the arithmetic mean. Bearing in mind that the variation of the point of maximum velocity from the center of the pipe, increases the ratio $\frac{U}{V_c}$ and the coefficient of V_c in the equation, and that the traverses on the upper diagrams of Plate III show that a slight variation exists, it has been concluded that the arithmetic mean, which

*As described in *Transactions*, Am. Soc. C. E., Vol. xliv., p. 74, and in *Journal Association of Engineering Societies*, Vol. xxvi No. 3, March, 1901.

is the ratio of the co-ordinates of the point C, being greater than the tangent of the inclination of the mean line, indicates that the velocity was eccentric at the lower velocities, and, therefore, that the ratio of the co-ordinates of the point A would be a more correct value for the ratio $\frac{U}{V_c}$. This gives 0.725. Similarly, examining the locus of Tube No. 6, it appears that the ratio of the co-ordinates of B represents the condition of maximum velocity in the center, and this gives for $\frac{U}{V_c}$ the value 0.677. Applying to these values the ratio $\frac{V_m}{V_c} = 0.81$, as already deduced, the coefficients for the tubes become 0.896 for Tube No. 3, and 0.836 for Tube No. 6. The presence of the large constant in the equation of Tube No. 6, together with the fact that the experiments at low velocities have been generally less satisfactory than those at the higher ones lead the writers to view the coefficient obtained for Tube No. 6 as approximate only, and to depend for its rating on comparisons with Tube No. 3 made during the field work. The coefficient of Tube No. 3 is more satisfactory, the constant in the equation being only half as great, and the derivation being from the observations at the higher velocities, where the traverses show the velocity curve to be more normal. The results with Tube No. 5 and Tube C, however, showed, that the coefficients obtained for them in this pipe were too high, the former by more than 13%, and the latter by more than 6%; so that the conclusion was drawn that the coefficients of the other tubes were also probably high, somewhat in proportion to the areas at the pressure openings. The area of Tube No. 3 is about 20% of that of Tube No. 5 and about 21% of that of Tube C. Estimating the effect to be proportional to the area, this would indicate that the foregoing coefficient of Tube No. 3 might be from 1.2 to 2.6% too high. The writers consider these deductions to be based upon data too indefinite for rigid application, but still, as warranting a reduction of the coefficient of Tube No. 3, and the value 0.89 has been selected as being probably as near the correct one as the information will furnish.

In Closed Pipes, with Running Water: Comparisons with Field Data.—During the experiments on the 30-in. line made in 1899, Tube No. 6 was ordinarily read on the center at the gate-well at Hastings Street and Alexandrine Avenue, and a resistance gauge was read continuously during all those experiments, on the section known as the Brady tangent. From these observations, several hundred in number, the

ratio between H_f for the tangent and H_{vc} or V_c at the above location was established as $\frac{H_f \text{ Brady}}{4.0301} = H_v$ Tube No. 6 at Hastings. Then Tube No. 3 was substituted for Tube No. 6, and a series of observations made, whence the ratios between their coefficients were determined. Table No. 6 gives the comparison for these experiments.

TABLE No. 6.—TUBES NOS. 3 AND 6 ON CENTER IN 30-INCH PIPE.

No.	Number of observations.	Tube No. 6 by Brady Tan.		Tube No. 3 Observed.		Velocity.
		Head.	Velocity.	Head observed.	Head corrected.	
1....	18	1.144	2.478	1.028	1.013	2.334
2....	42	1.118	2.452	1.002	0.987	2.304
3....	24	1.633	2.963	1.516	1.501	2.842
4....	24	1.575	2.912	1.458	1.443	2.786
5....	18	1.666	2.993 reject.	1.455	1.440	2.784 reject.
	Mean, rejecting (5) =		2.701			2.5665.

Ratio $\frac{V \text{ by } T. \text{ No. } 6}{V \text{ by } T. \text{ No. } 3} = 1.052$, whence Coefficient of Tube No. 6 = $\frac{\text{Coefficient of Tube No. 3}}{1.052} = \frac{0.89}{1.052} = 0.846$, which value coincides closely with the arithmetic mean 0.844, and has been adopted.

Having established the coefficient of Tube No. 3, those of Tubes Nos. 4 and 5 are also obtained from comparisons of simultaneous observations with these tubes and Tube No. 3 made during the field work.

These comparisons are presented in Table No. 7, and include: First, a comparison on the 30-in. line between Tube No. 3 at Hastings Street with Tube No. 4 at Leland Street, both being on center; second, comparisons between Tube No. 3 and Tube No. 5 on the 30-in. line at Superior and Hastings Streets, and on the 16-in. line, two series at Military and Campbell Avenues on center; and third, a comparison between Tubes Nos. 4 and 5 on the 30-in. line at Leland and Hastings Streets, on center.

Rejecting from these comparisons those at low velocities, as indicated in the table, the ratios between the observed heads H_v are found from arithmetic means as follows:

$$\frac{H_v \text{ Tube No. 3}}{H_v \text{ Tube No. 4}} = 0.941; \quad \frac{H_v \text{ Tube No. 5}}{H_v \text{ Tube No. 3}} = 1.40, 1.37 \text{ and } 1.38; \text{ and}$$

$$\frac{H_v \text{ Tube No. 5}}{H_v \text{ Tube No. 4}} = 1.34.$$

TABLE No. 7.—Comparison of Tubes in Pipe Experiments in 1898.

Tube No. 3, Gauge No. 8, at Hastings.		Tube No. 4, Gauge No. 9, at Leland.	
30-in. pipe.			
Number in Table No. 22.	Mean reading. H_v	Mean reading. H_v.	H_v by *T.* No. 3. / H_v by *T.* No. 4.
113	0.451	0.487	0.92	
114	0.392	0.517	0.758	
115	0.378	0.485	0.779	
116	0.407	0.467	0.872	
117	0.386	0.479	0.806	
118	0.510	0.570	0.847	
119	0.531	0.568	0.935	
120	0.543	0.583	0.931	
121	0.750	0.817	0.918	
122	0.703	0.771	0.912	
123	1.029	1.093	0.941	
124	1.307	1.691	0.773	
125	1.468	1.764	0.832	
126	1.545	1.840	0.840	
127	1.600	1.825	0.871	
128	1.902	2.764	0.890	
133	2.296	2.287	1.004	
134	2.301	2.306	0.998	
135	2.380	2.432	0.978	
136	2.440	2.464	0.990	
137	2.545	2.550	0.998	
138	2.830	2.864	0.988	Mean, 0.941
139	3.144	3.216	0.978	
141	4.401	4.480	0.982	
142	3.961	4.039	0.981	
143	3.029	3.155	0.961	
144	2.832	2.932	0.965	
145	2.588	2.762	0.937	
146	2.669	2.777	0.962	
147	2.204	2.203	1.000	
148	2.147	2.248	0.954	
149	1.752	1.852	0.946	
150	1.582	1.695	0.933	
151	1.620	1.688	0.960	
152	1.525	1.648	0.919	
153	1.278	1.450	0.881	
154	1.296	1.391	0.932	
155	0.982	1.056	0.930	
156	1.030	1.080	0.954	
157	0.654	0.705	0.928	
158	0.346	0.420	0.824	
159	0.379	0.500	0.758	
160	0.325	0.372	0.874	
161	0.189	0.281	0.675	

Tube No. 3, Gauge No. 9, at Military and Army.		Tube No. 5, Gauge No. 5, at Campbell and Porter.	
16-in. pipe.			
Number in Table No. 31.	Mean reading. H_v.	Mean reading. H_v.	H_v by *T.* No. 5. / H_v by *T.* No. 3.
26	0.073	0.082	1.125	
27	0.189	0.211	1.116	
28	0.541	0.836	1.545	
29	0.845	1.145	1.355	
30	1.372	1.863	1.358	
31	1.935	2.623	1.356	
32	2.566	3.556	1.385	
33	3.171	4.500	1.419	
34	3.706	5.033	1.358	
35	3.722	5.135	1.379	
36	5.507	7.800	1.416	
37	2.460	3.420	1.390	
38	1.344	1.822	1.356	
39	0.673	0.870	1.293	
66	8.727	12.308	1.399	Mean, 1.373
67	10.356	14.388	1.373	
68	11.327	15.678	1.352	
69	11.030	15.691	1.402	
70	7.735	10.663	1.368	
71	3.933	5.236	1.331	

TABLE No. 7—(*Continued*).

Tube No. 3, Gauge No. 4, at Superior.	Tube No. 5, Gauge No. 5, at Hastings.	
30-in. pipe.		
Number in Table No. 22.		
192	0.752	0.840	1.117
193	0.721	0.869	1.211
194	0.696	0.792	1.138
195	0.996	1.288	1.293
196	1.457	2.189	1.502
197	1.819	2.671	1.468
198	1.875	2.706	1.449
199	1.861	2.731	1.468
200	2.162	3.241	1.499
201	2.630	3.381	1.284
202	2.627	3.327	1.263
203	3.134	4.139	1.321
204	1.698	2.420	1.429

Mean, 1.40

Tube No. 3, Gauge No. 9, at Military and Army.	Tube No. 5, Gauge No. 5, at Campbell and Porter.	
16-in. pipe.		
Number in Table No. 31.	Mean reading. H_v.	Mean reading. H_v.	$\frac{H_v \text{ by } T. \text{ No. } 5.}{H_v \text{ by } T. \text{ No. } 3.}$
42	0.017	0.125	7.353
43	0.146	0.069	0.472
44	0.165	0.080	0.485
45	0.573	0.682	1.190
46	0.570	0.710	1.246
47	0.604	0.582	0.963
48	1.170	1.559	1.333
49	1.161	1.578	1.359
50	1.551	2.081	1.342
51	1.506	2.063	1.370
52	1.505	2.047	1.360
53	1.517	2.089	1.377
54	1.934	2.579	1.333
55	2.893	3.996	1.381
56	2.870	3.960	1.379
57	3.843	5.327	1.387
58	4.835	6.546	1.354
59	4.869	6.788	1.394
60	5.895	8.070	1.390
61	5.847	8.080	1.382
62	6.682	9.278	1.382
63	7.435	10.411	1.391
64	8.944	12.346	1.373
65	8.770	12.555	1.420

Mean, 1.36

Tube No. 4, Gauge No. 9, at Leland.	Tube No. 5, Gauge No. 5, at Hastings.	
30-in. pipe.		
Number in Table No. 22.	Mean reading. H_v.	Mean reading. H_v.	$\frac{H_v \text{ by } T. \text{ No. } 5.}{H_v \text{ by } T. \text{ No. } 4.}$
166	0.811	0.964	1.188
172	2.9785	3.792	1.273
175	3.295	4.283	1.303
178	3.908	4.591	1.174
185	2.204	2.846	1.286
212	0.633	0.832	1.313
213	0.988	1.258	1.273
214	1.396	1.728	1.239
215	1.567	2.341	1.493
216	2.122	2.764	1.302
217	2.500	3.345	1.338
219	2.936	3.969	1.352
220	3.250	4.446	1.368
221	3.473	4.652	1.339
223	3.803	5.247	1.379
224	3.650	4.875	1.335
225	2.097	3.220	1.535
226	2.218	2.855	1.287

Mean, 1.32

The square roots of these ratios give the corresponding ratios between the velocities by these tubes. The ratios between the velocities are, therefore, $\frac{\text{Tube No. 3}}{\text{Tube No. 4}} = 0.97$; $\frac{\text{Tube No. 5}}{\text{Tube No. 3}} = 1.18$, 1.17 and 1.174; and $\frac{\text{Tube No. 5}}{\text{Tube No. 4}} = 1.16$.

Remembering that the ratio of the velocities is the reciprocal of the ratio of the coefficients, from the first ratio is obtained:

Coefficient of Tube No. 3 $= 0.89 = \frac{\text{Coefficient of Tube No. 4}}{0.97}$, or coefficient of Tube No. 4 $= 0.89 \times 0.97 = 0.863$.

From the second ratios are obtained:

Coefficient of Tube No. 5 $= \frac{\text{Coefficient of Tube No. 3} = 0.89}{1.18} =$

0.754; $\frac{0.89}{1.17} = 0.761$; and $\frac{0.89}{1.174} = 0.758$.

From the third ratio is obtained:

Coefficient of Tube No. 5 $= \frac{\text{Coefficient of Tube No. 4}}{1.16} = \frac{0.863}{1.16}$

$= 0.745$.

Considering that observations at Hastings, Leland and Superior Streets have shown that the maximum velocity in each place is very near the center, while similarly conclusive data were obtained for only one of the two locations on the 16-in. line; and considering also the relative number of experiments at the two places, it has been decided to give the value for the coefficient of Tube No. 5 obtained from Tube No. 4 double the weight that is given to each value obtained from Tube No. 3 directly, and to consider the latter as of equal weight among themselves, when the arithmetic mean is $\frac{1}{5}[0.754 + 0.761 + 0.758 + (2 \times 0.745)] = 0.752$, and the value 0.75 is adopted as the coefficient for Tube No. 5 and that of 0.86 for Tube No. 4.

As Tubes Nos. 1 and 2, and Tubes *A*, *B*, *C*, *D* and *E* have not been used in any experiments where a knowledge of their coefficients (beyond the fact that they are constant, which has already been demonstrated) would be of particular value to this investigation, the discussion of their ratings will be omitted at present.

In Closed Pipes, with Running Water: Adopted Coefficients.—To recapitulate, the coefficients adopted are:

	For V.	For H_v.	To reduce $\sqrt{H_{vc}}$ to V_c.
For Tube No. 3.....	0.89	0.7921	1.734
" " 4.....	0.86	0.7396	1.676
" " 5.....	0.75	0.5625	1.462
" " 6.....	0.846	0.7157	1.649

All are to be applied to the velocity computed from the observed head considered as that to which the velocity is due.

Comparison of Ratings in Still and Flowing Water: The Experimental Data.—There are now brought out for examination two series of coefficients for the same instruments, one obtained by a method of long standing for such apparatus, and a method involving very simple manipulations and computations; the other a series obtained by a more devious course of experiment, but one in which the conditions of actual service were much more closely approximated. As to the skill with which both ratings were performed, the writers believe them equally entitled to credence, and, if the values obtained had been but slightly different, they would probably have given the open trough rating the preference on account of the larger number of observations, and the greater simplicity of operation and reduction. Nevertheless, the results confirmed a long-lingering suspicion that the conditions under which an instrument operated when dragged through still water were not those existing when the water flowed past it.

For comparison, the coefficients obtained by the two methods are:

Tube No. 3, by tank rating,	0.926;	by pipe rating,	0.89.			
" 6 " "	0.950	" "	0.84.			
" 5 " "	0.859	" "	0.75.			

If the same method of rating in the 2-in. pipe were applied to Tube No. 5 and Tube *C* that was used for Tubes Nos. 3 and 6 the coefficients obtained would be 0.861 and 0.868, the former coinciding closely with the tank rating of the same tube. By this coincidence the writers were at first misled into believing that the coefficient of Tube No. 5 would be about 0.86, and that the ratios obtained in the original investigations were in error, supposedly on account of irregular distribution of velocities. The values obtained with Tubes Nos. 3 and 6 in the tank evidently contradict every other experiment, for this rating is the only case in which Tube No. 6 appears to have a higher

coefficient than Tube No. 3. If there is any advantage in the fine lines used in ocean shipbuilding over those of the bluff-bowed craft to be seen on the Great Lakes, it must be expected that Tube No. 3 would give a higher coefficient than Tube No. 6, from a mere inspection of its form. The writers, having made such investigations as to satisfy themselves that the velocities are not abnormally distributed, either at Hastings or Leland Streets, and but slightly so at Superior Street, have therefore arrived at the conclusion that the method of tank rating is not a proper one to apply to the instruments under consideration.

Distribution of Velocities in Circular Pipe.

Normal Conditions: Criteria of Accuracy.—The experiments made upon the 30, 16 and 12-in. pipes, in which the Pitot tubes were made to traverse, afford interesting information as to the conditions of the flowing water and the distribution of the axial components of the velocities of its particles. In addition to the experiments of this sort connected with the investigation of effects of curvature, some have been made upon one of the 42-in. force mains supplying the City of Detroit (Fig. 5), and, as already intimated, the first traverses were made in a 12-in. pipe which had no connection with the lines used in the general investigation, beyond the fact that it afforded an opportunity to ascertain whether the Pitot tube was capable of being applied to the measurement of water under the conditions which it was supposed would exist in the 30-in. line.

On the 30-in. pipe there have been more than fifty individual traverses made under the conditions of straight pipe; on 16-in., there have been seven; on 12-in., twenty-nine (besides some partial preliminary ones), and on 42-in., two. In the 2-in. pipe, eleven traverses have already been presented in the discussion of the rating of the tubes.

To convey some idea at once of the relative accuracy of this work, it may be stated that the conditions of the experiments on June 7th and 8th and on June 11th and 12th, 1898, were, as near as could be, the same, for the traverses made at the highest velocities, which are shown with others in Figs. 6 and 7. The pressure on the mains at the pumping station, 4 miles away, was probably the same, within ½ lb., on both occasions. The consumption from the whole distribution, both sets of experiments being made between midnight and 3 A. M.,

and only 4 days apart, must have been very nearly the same, and, as the two blow-offs operated for producing velocity were the same, and both were fully opened, it may be expected that the discharges through the pipe were practically identical. On June 7th and 8th two traverses were made at the highest velocity, from which a mean velocity of 3.967 ft. per second is obtained, and on June 11th and 12th three traverses were made, giving a mean velocity of 3.978 ft., from which it is seen that the tube shows the conditions duplicated within $\frac{3}{10}$ of 1 per cent. This was for the same tube in the same position, the traverses being made with Tube No. 3 at Hastings Street.

On August 13th and 14th a traverse was made at Leland Street with Tube No. 4, when the two blow-offs were fully open, the time being between 4 and 5 A. M., and the conditions as to pressure as nearly the same as those in the preceding cases as could be expected with the somewhat different consumption in August.

The mean velocity by this traverse was 3 988 ft. per second, which, when multiplied by the coefficient of Tube No. 4 = 0.86, gives 3.430 ft. per second as the true velocity.

The diameter of the pipe at Leland Street, as determined by the rod caliper, is $30\frac{1}{4}$ ins. = 2.521 ft.

Computing for Q, the discharge at Leland Street:

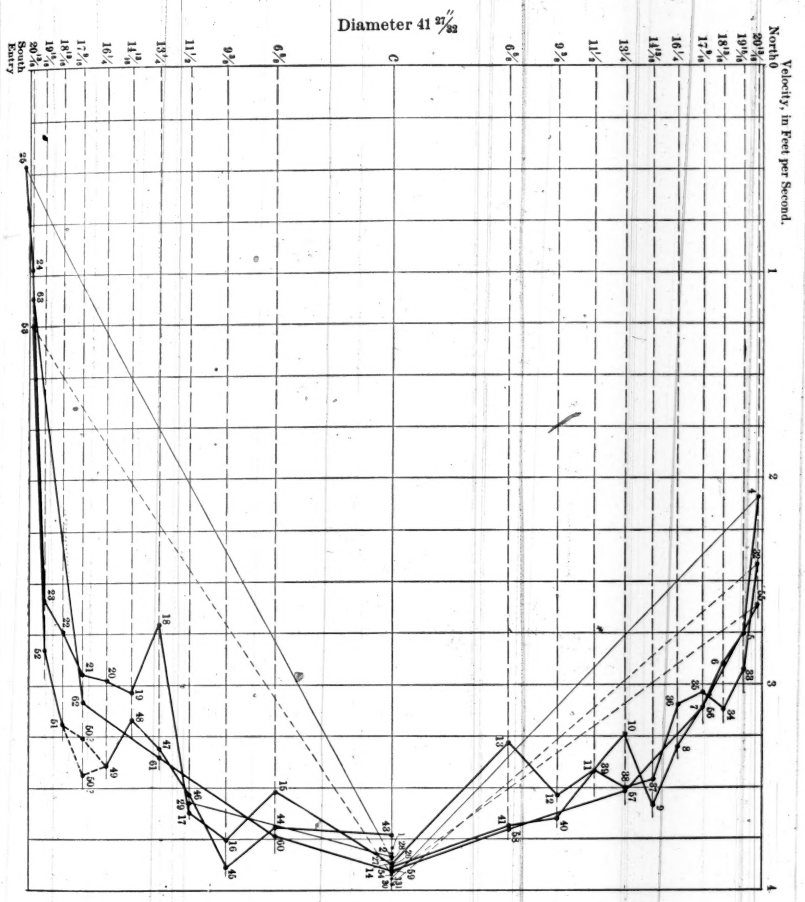
Diameter 2.521	log.	= 0.401573
d^2	"	= 0.803146 (2)
$\frac{\pi}{4} = 0.7854$	"	= 9.895091
Mean V = 3 439 ft.	"	= 0.535294
Q	"	= 1.233531 = 17.121 cu. ft.

Computing for Q at Hastings Street:

Mean of velocities observed in June $= \frac{3.967 + 3.978}{2} = 3.972$ ft., which, multiplied by the coefficient of Tube No. 3 = 0.89 = 3.972 × 0.89 = 3.535 ft.

Diameter, by rod caliper, is $30\frac{3}{64}$ ins. = 2.504 ft.

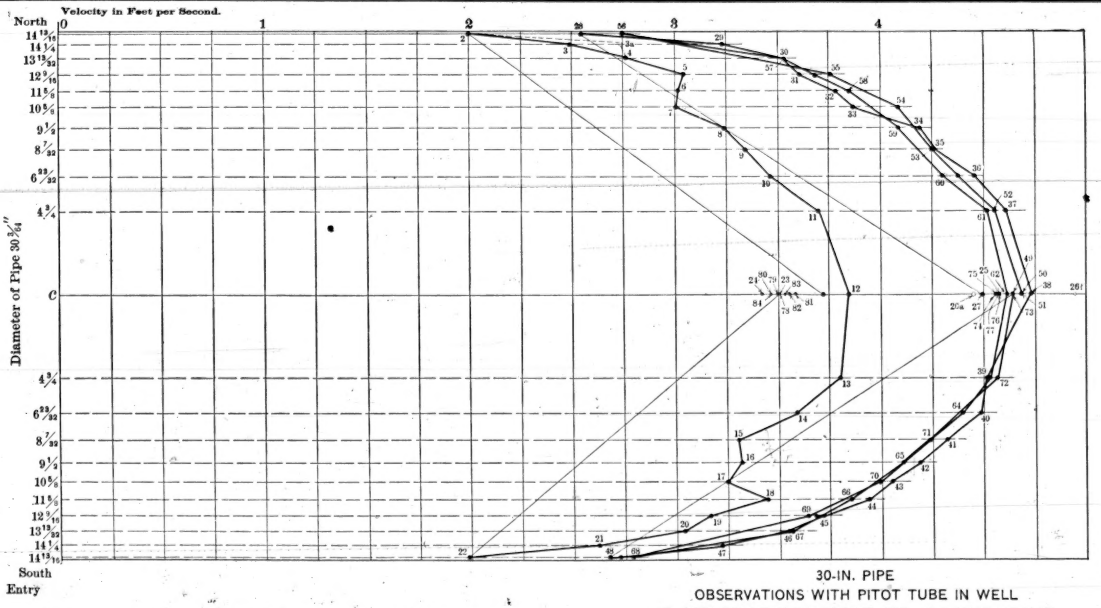
Diameter 2.504	log.	= 0.398634
d^2	"	= 0.797268 (2)
$\frac{\pi}{4} = 0.7854$	"	= 9.895091
Mean V = 3.535 ft.	"	= 0.548389
Q	"	= 1.240748 = 17.408 cu. ft.



42-IN. PIPE OBSERVATIONS WITH PITOT TUBE
AT ELMWOOD AND CHAMPLAIN,
DETROIT MICH.,
JUNE 20, 1898.
TUBE NO. 3, GAUGE NO. 8.
TIME 10:39 A. M. TO 4.32 P. M.
EACH POSITION IS THE MEAN OF 5 TO 7 OBSERVATIONS
EACH COMPRISING 4 READINGS OF THE GAUGE SCALE.

Traverse No. 1, Positions 3 to 26
" 2, " 31 " 54
" 3, " 54 " 64

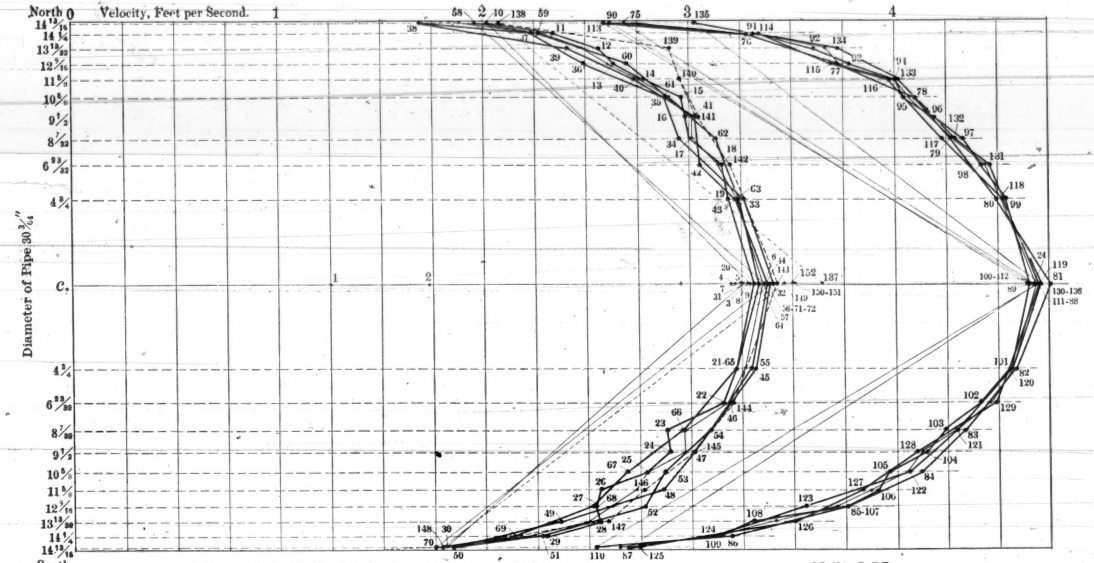
FIG. 5.



Traverse 4, Positions 1 to 24; 9:41:00 to 11:11:30 P.M. Canfield B.O. Full open.
" 5 and 6, " 25 " 84; 11:36:00 " 3:10:00 A.M. Canfield and Hastings B.O's Full open.
No. 6 is a double traverse.

30-IN. PIPE
OBSERVATIONS WITH PITOT TUBE IN WELL
AT CORNER OF ALEXANDRINE AVE AND HASTINGS ST.
DETROIT, MICH.
MADE JUNE 7TH AND 8TH, 1898.
WITH TUBE NO. 3 AND GAUGE NO. 8.
ENTRY HORIZONTAL ON CONCAVE SIDE OF PIPE LINE AND 757 ± DOWN-STREAM FROM P.T. OF 10 FT. RADIUS 90° CIRCULAR CURVE AND 14 FT. ± UP-STREAM FROM 10 IN. B.O. CONNECTION.
5 OBSERVATIONS 30 SECONDS APART TO EACH POSITION.
4 READINGS OF GAUGE SCALE FOR EACH OBSERVATION.

FIG. 6.



	Positions 1 and 2	9:19:00 to 9:27:15 P.M.	Canfield B.O. opening Full 9:26 to 9:30 P.M.
Traverse No. 7	10 to 31	10:16:00 " 11:22:15 "	
	32 " 38	11:36:00 " 11:56:15 "	
8	38 " 50	11:54:00 " 12:32:15 A.M.	
	50 " 56	12:30:00 " 12:50:15 "	
9	57 " 72	1:30:00 " 2:17:45 "	
			Hastings B.O. opening Full 2:17 to 2:21 A.M.
10	73 " 88	2:23:00 " 3:12:15 "	
11	89 " 111	3:15:00 " 4:23:15 "	
12	112 " 125	4:39:00 " 5:20:15 "	
13	125 " 136	5:18:00 " 5:48:15 "	
			Hastings B.O. closing Full 5:50 to 5:52 A.M.
14	137 " 149	5:53:00 " 6:14:15 "	
	150 " 102	6:16:00 " 6:22:15 "	
			Canfield B.O. closed at 6:52 A.M.

30-IN. PIPE

OBSERVATIONS WITH PITOT TUBE IN WELL AT CORNER OF ALEXANDRINE AVE. AND HASTINGS STREET, DETROIT MICH.,

MADE JUNE 11TH AND 12TH, 1898.

WITH TUBE NO. 3 AND GAUGE NO. 8.

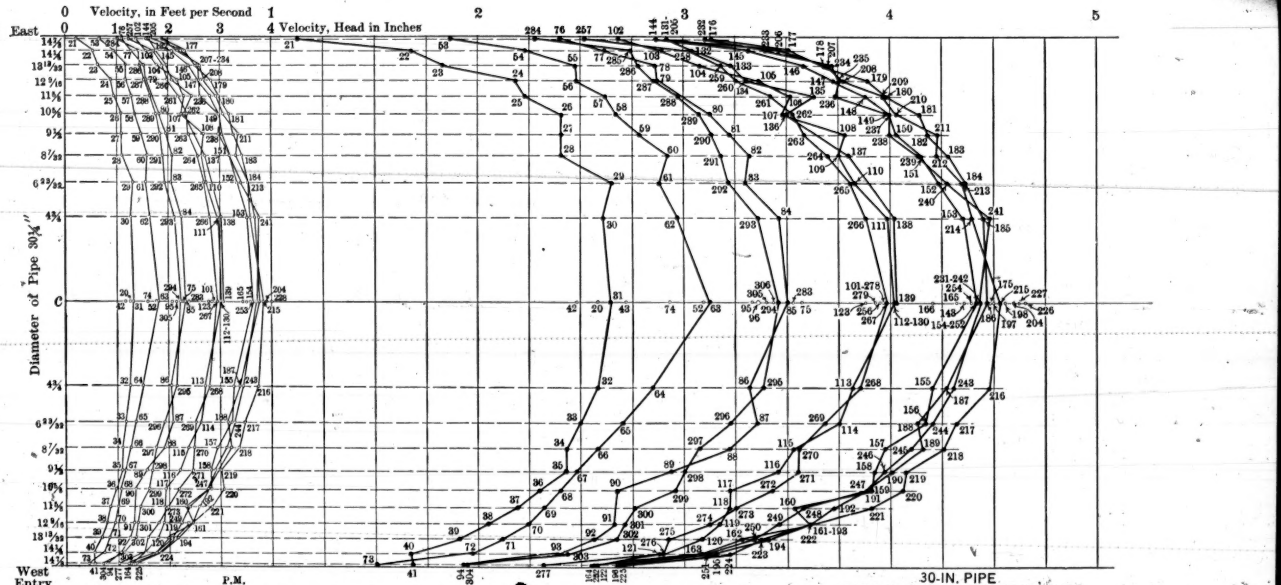
ENTRY HORIZONTAL ON CONCAVE SIDE OF PIPE LINE AND 737 FT. ± DOWN-STREAM FROM P.T. OF 10-FT. RADIUS, 90° CIRCULAR CURVE AND 14 FT. ± UP-STREAM FROM 10 IN. B.O. CONNECTION.

5 OBSERVATIONS 30 SECONDS APART FOR EACH POSITION.

4 READINGS OF GAUGE SCALE FOR EACH OBSERVATION.

Fig. 7.

Velocity in Feet per Second.

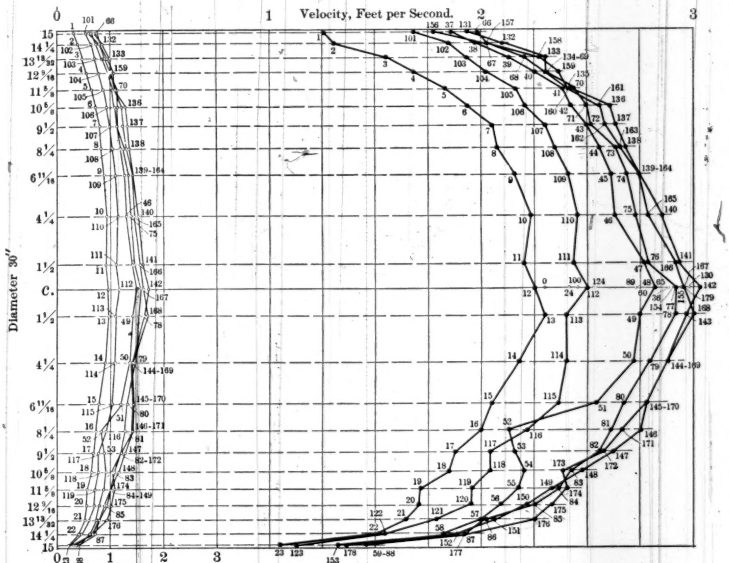


30-IN. PIPE
OBSERVATIONS WITH PITOT TUBE IN WELL AT CORNER OF RIVARD AND LELAND STS.
DETROIT, MICH.,
MADE AUG. 13TH AND 14TH, 1898.
WITH TUBE NO. 4 AND GAUGE NO. 9.
ENTRY INCLINED TOWARD POINT ABOUT 20° FROM HORIZONTAL ON CONVEX SIDE OF PIPE LINE AND 756 FT. ± FROM P.T. OF 15-FT. RADIUS 90° CIRCULAR CURVE.
3 OBSERVATIONS 15 SECONDS APART FOR EACH POSITION, 4 READINGS OF GAUGE SCALE FOR EACH OBSERVATION.

					P.M.
Traverse No. 31	Positions	1	to	20 on Center	11:17 to 12:33:45
32	"	21	" 41	traverse	12:35 " 12:56:45
33	"	58	" 73	"	1:36 " 1:56:45
34	"	76	" 94	"	2:09 " 2:27:45
35	"	102	" 122	"	2:35 " 2:56:45
36	"	131	" 138½	"	3:06 " 3:13:45
37	"	144	" 164	"	3:18 " 3:36:45
38	"	176	" 196	"	3:56 " 4:16:45
39	"	206	" 225	"	4:27 " 4:47:45
40	"	232	" 252	"	4:54 " 5:14:45
41	"	257	" 277	"	5:26 " 5:46:45
	"	284	" 304	"	5:53 " 6:13:45

Canfield B.O. opened to 5 T at 11:08:30
" " " 9 T during Pos. 5, and opened to 16 T between Pos. 17 and 18 at 12:30
" " " 24 T " " 49 at 1:33 Lunch 12:57 to 1:27 a.m.
" " " 38 T between " 74 and 75 at 2:00:40 Hose shaken out between these observations.
Hastings " " 7 T during " 97 at 2:30:30
" " " 18 T " " 141 " 3:15:30
" " " 21 T " " 166 " 3:40:30
" " " 34½ T between " 196 and 199 at 4:20:15
" " closed 21 T during " 227 " 4:50:30
" " " 11 T between " 254 and 255 " 5:22
" " " fully, during " 281 " 5:50:10
Canfield B.O. remaining open.
All times from Observers watch, which was with G.H.F. at 10:50 p.m. and 40 s. behind at 7:15 a.m.

FIG. 8.



Traverse 42	Positions	0 to 24
43		36 " 60
44		65 " 89
45		100 " 124
46		130 " 154
47		155 " 179

30-IN. PIPE
OBSERVATIONS WITH PITOT TUBE AT HASTINGS AND ALEXANDRINE,
DETROIT, MICH.

POSITIONS 0 TO 24, 36 TO 60 AND 65 TO 89 TAKEN WITH TUBE NO. 3, GAUGE NO. 5 ON DIAMETER 45° FROM VERTICAL. POSITIONS 100 TO 124, 130 TO 154, 155 TO 179 TAKEN WITH TUBE NO. 6, GAUGE NO. 5, AT SAME LOCATION.

POSITIONS 0 TO 89 FROM 9:50 TO 11:32 A.M.
POSITIONS 100 TO 179 FROM 1:23 TO 3:03 P.M.

JUNE 22D, 1899.

EACH POSITION IS THE MEAN OF 3 OBSERVATIONS COMPRISING 4 READINGS OF THE GAUGE SCALE.

FIG. 9.

The excess of discharge in June over that in August, therefore, appears to have been $\frac{17.408 - 17.121 \ (= 0.287 \text{ cu. ft.})}{17.121}$, or $1\frac{7}{10}$ per cent.

On the night of July 18th and 19th, 1898, three traverses were made at Hastings Street, with Tube No. 3, with the Canfield Street blow-off, only, open, the mean velocity of which gives 2.816 ft., and this, multiplied by the tube coefficient, 0.89, gives the true velocity as 2.506 ft. per second.

Log. 2.506 ft. $= 0.398981$
" Hastings Street area $= 0.692359$
" Q $= 1.091340$, whence $Q = 12.341$ cu. ft.

On the night of August 13th and 14th a traverse, made under similar conditions, with Tube No. 4, at Leland Street, gave a mean velocity of 2.847, which, multiplied by the coefficient, 0.86, gives a true velocity of 2.448 ft.

Log. 2.448 ft. $= 0.388811$
" Leland Street area $= 0.698237$
" Q $= 1.087048$, whence $Q = 12.219$ cu. ft.

$\frac{Q. \text{ Hastings}}{Q. \text{ Leland}} = \frac{12.341}{12.219} = 1.01$ nearly, or the two measurements agree within less than 1 per cent.

The several traverses are presented in Figs. 5 to 14 and on Plate IV, and are self-explanatory; and the velocities, reduced from the gauge readings, as already explained, are given in Table No. 9.

In examining these diagrams it will be noticed that nearly all the traverses show a region of peculiar disturbance about two-tenths of the diameter from the sides. The cause of this disturbance or how it occurs is not clear from these observations, but its presence in practically all the traverses precludes the possibility of its being an accident or an error of observation.

Normal Conditions: Observations with Low Velocities.—The traverses taken at the lower flows show much less regular variation in the velocities than the faster ones, and those in which the mean is less than 1 ft. per second are so irregular as to render them unsatisfactory for measuring purposes. In these traverses not infrequently a zero velocity would be recorded, and sometimes even a negative one for several seconds. At first, the writers were at a loss to account for the negative velocities, for, while it was possible to admit a condition of rest, it

EXPERIMENTS ON FLOW OF WATER IN PIPES.

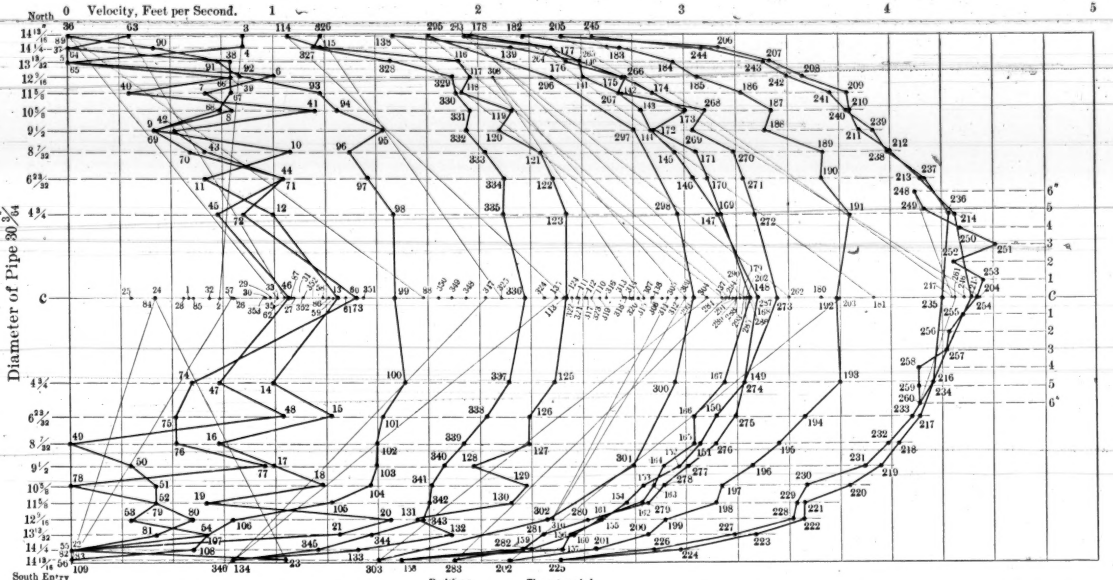


FIG. 10.

Traverse No.				Positions.		Time occupied.
15	12" B.O.	P.M. 10:26:00	Opened 4¼ Turns.	2 to 24	P.M. 10:00:30 to P.M.	10:25:00
16		11:17:30	" 6¼ "	33 " 57	" 10:46:30 "	11:15:00
17		A.M. 12:10:30	" 9¼ "	62 " 84	" 11:38:30 " A.M.	12:05:00
18		12:42:00	" 15¼ "	88 " 109	A.M. 12:18:00 "	12:41:50
19		1:46:00	" 34 "	113 " 135	1:20:00 "	1:44:40
20				137 " 158	1:47:00 "	2:09:40
21	10" B.O.	2:31:00	" 7 "	158 " 179	2:09:00 "	2:30:40
22		2:57:00	" 16 "	181 " 203	2:33:00 "	2:56:40
23				204 " 225	2:59:00 "	3:24:40
24				225 " 246	3:24:00 "	3:45:40
25		4:08:30	Closed Full	247 " 261	3:52:00 "	4:06:40
26	12" B.O.	5:14:00	" to 22 Turns	262 " 284	4:19:00 "	4:43:40
		5:29:00	" " 16 "	294 " 304	5:15:30 "	5:28:30
27		5:42:30	" " 12 "	307 " 311	5:33:00 "	5:41:00
		6:03:30	" " 9 "	325 " 347	6:14:00 "	6:38:40
		6:13:30	" " 7 "	Last reading No. 353 closed		6:53:30
		6:40:00	" " 4 "			
		6:46:00	" " Full			

30 IN. PIPE OBSERVATIONS WITH PITOT TUBE IN WELL

AT CORNER OF ALEXANDRINE AVE. AND HASTINGS ST.,

MADE JUNE 18TH AND 19TH 1898,

WITH TUBE NO. 3 AND GAUGE NO. 8

ENTRY HORIZONTAL ON CONCAVE SIDE OF PIPE LINE AND 737 FT. ± DOWN-STREAM FROM P.T. OF 10-FT. RADIUS, 90° CIRCULAR CURVE AND 14 FT. ± UP-STREAM FROM 10" B.O. CONNECTION.

3 OBSERVATIONS 15 SECONDS APART FOR EACH POSITION.

4 READINGS OF GAUGE SCALE FOR EACH OBSERVATION.

TABLE TO ACCOMPANY FIG. 11.

Traverse No.	Positions		Time. P. M.	
53.....	1 to 23	Traverse ..	10.55 to 11.18.45	Canfield B. O. opened 5 *T.*, 11.07.51 to 11.08.31, during Pos. 13.
54.....	20 to 38	Half traverse.	11.20 to 11.34.45	Canfield B. O. opened to 9 *T.*, 11.30.17 to 11.30.37, during Pos. 34.
55.....	39 to 53	Half traverse.	11.36 to 11.50.45	Gauge overhauled after Obs. 53, 11.51 to 12.00 M.
	54 to 69	On center..	12.00 to 12.15.45	
56.....	70 to 91	Traverse ..	12.17 to 12.38.45	Canfield B. O. opened to 16 *T.*, 12.29.35 to 12.30.05, during Pos. 82 and 83.
57.....	92 to 113	Traverse ..	12.39 to 1.00.45	Recess from 1.01 to 1.32 A. M.
	114 to 117	On center..	1.32 to 1.35.45	Canfield B. O. opened to 24 *T.*, 1.32.38 to 1.33.08, during Pos. 114 and 115.
58.....	118 to 138	Traverse ..	1.36 to 1.56.45	
	139 to 142	On center..	1.58 to 2.01.45	Canfield B. O. opened to 38 *T.*, 2.00.10 to 2.00.40, during Pos. 141.
59.....	143 to 165	Traverse ..	2.02 to 2.24.45	
	166 to 170	On center..	2.28 to 2.32.45	Hastings B. O. opened to 7 *T.*, 2.30.20 to 2.30 42, during Pos. 168.
60.....	171 to 192	Traverse ..	2.33 to 2.56.45	
	193 to 197	On center..	2.59 to 3.03.45	
61.....	198 to 203	Traverse ..	3.06 to 3.11.45	
	204 to 208	On center..	3.13 to 3.17.45	Hastings B. O. opened to 13 *T.*, 3.15.14 to 3.15.44, during Pos. 206.
62.....	209 to 230	Traverse ..	3.18 to 3.39.30	
	231	On center..	3.45 to 3.45.45	Hastings B. O. opened to 21 *T.*, 3.40.15 to 3.40.55.
63.....	232 to 256	Traverse ..	3.50 to 4.14.45	
	257 to 261	On center..	4.15 to 4.22.45	Hastings B. O. opened to 34½ *T.*, 4.20.12 to 4.20.48 during Pos. 259.
64.....	262 to 283	Traverse ..	4.23 to 4.44.45	
	284 to 288	On center..	4.49 to 4.53.45	Hastings B. O. closed to 21 *T.*, 4.50.13 to 4.51.03, during Pos. 285 and 286.
65.....	289 to 310	Traverse ..	4.54 to 5.15.45	
	311 to 315	On center..	5.19 to 5.23.45	Hastings P. O. closed to 11 *T.*, 5.21.35 to 5.22.35, during Pos. 313 and 314.
66.....	316 to 337	Traverse ..	5.24 to 5.46.45	
	338 to 341	On center..	5.49 to 5.52.45	Hastings B. O. closed completely 5.50.16 to 5.50.41 during Pos. 339.
67.....	342 to 365	Traverse ..	5.53 to 6.16.45	Canfield B. O. remaining open.

All times from observer's watch.

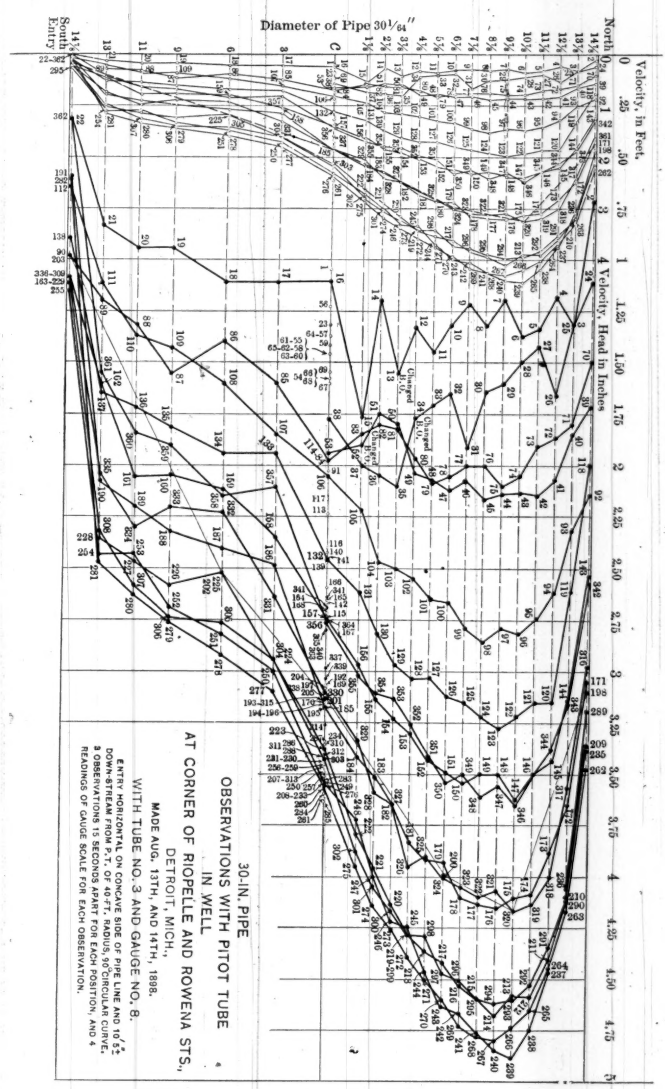


Fig. 11.

For table accompanying Fig. 11, see opposite page.

TABLE TO ACCOMPANY FIG. 12.

Traverse No.	Positions		Time. P. M.	
68.....	1 to 23	Traverse...	9.55 to 10.17.45	
	24 to 45	On center..	10.18 to 10.39.45	Canfield B. O. opened 3 *T.*, 10.30.35 to 10.30.45, during Pos. 36.
69.....	46 to 66	Traverse...	10.40 to 11.01.45	Velocities not plotted.
	67 to 73	On center..	11.02 to 11.08.45	Canfield B. O. opened to 5½ *T.*, 11.05.00 to 11.05.20, during Pos. 70.*
70.....	74 to 95	Traverse...	11.09 to 11.30.45	Canfield B. O. opened to 8 *T.*, 11.30.00 to 11.30.12, during Pos. 95.
71.....	96 to 117	Traverse...	11.31 to 11.52.45	
	118 to 121	On center..	11.53 to 11.56.45	Canfield B. O. opened to 12 *T.*, 11.55.00 to 11.55.15, during Pos. 120.
72.....	122 to 142	Traverse...	11.57 to 12.17.45	
	143 to 146	On center..	12.18 to 12.21.45	Canfield B. O. opened to 17 *T.*, 12.20.02 to 12.20.15, during Pos. 145.
73.....	147 to 167	Traverse...	12.22 to 12.42.45	Recess from 12.44 to 1.00.
	168 to 171	On center..	12.43 to 1.02.45	Canfield B. O. opened to 25 *T.*, 1.01.00 to 1.01.20, during Pos. 170.
74.....	172 to 192	Traverse...	1.03 to 1.23.45	
	193 to 196	On center..	1.24 to 1.27.45	Canfield B. O. opened to 38¼ *T.*, 1.25.30 to 1.26.10, during Pos. 194 and 195.
75.....	197 to 217	Traverse...	1.28 to 1.48.45	
	218 to 220	On center .	1.49 to 1.53.45	Hastings B. O. opened to 5 *T.*, 1.52.00 to 1.52.15, during Pos. 219.
76.....	221 to 241	Traverse...	1.54 to 2.14.45	
	242 to 246	On center..	2.15 to 2.18.45	Hastings B. O. opened 9 *T.*, 2.16.30 to 2.16.45, during Pos. 243.†
77.....	247 to 267	Traverse...	2.19 to 2.39.45	
	268 to 271	On center..	2.40 to 2.43.45	Hastings B. O. opened to 15 *T.*, 2.42.00 to 2.42.30, during Pos. 270.
78.....	272 to 292	Traverse...	2.44 to 3.04.45	
	293 to 295	On center..	3.05 to 3.07.45	Hastings B. O. opened to 22 (24) *T.*, 3.05.50 to 3.06.20, during Pos. 293 and 294.
79.....	296 to 316	Traverse...	3.08 to 3.28.45	
	317 to 319	On center..	3.29 to 3.31.45	Hastings B. O. opened to 34½ *T.*, 3.30.10 to 3.30.50, during Pos. 318.‡
80.....	320 to 340	Traverse...	3.32 to 3.52.45	
	341 to 343	On center..	3.53 to 4.13.45	Shook out connections.
	344	At east side	4.14 to 4.14.45	Hastings B. O. closed to 18 *T.*, 4.15, during Pos. 344.
81.....	345 to 365	Traverse...	4.15 to 4.35.45	
	366 to 368	On center..	4.36 to 4.38.35	Hastings B. O. closed to 12 *T.*, 4.37, during Pos. 367.
82.....	369 to 387	Traverse...	4.39 to 5.00.45	4.53 to 4.56, excessive vibrations which could not be read.
83.....	388 to 409	Traverse...	5.01 to 5.22.45	Hastings B. O. closed to 6 *T.*, 5.01.00 to 5.01.25, during Pos. 388.
	410 to 413	On center..	5.23 to 5.36.45	Hastings B. O. closed completely, 5.35.10 to 5.35.30, between Pos. 412 and 413.
84.....	414 to 434	Traverse...	5.37 to 5.57.45	
	435 to 438	On center..	5.58 to 6.01.45	Canfield B. O. closed to 20 *T.*, 6.00.00 to 6.00.45, during Pos. 437.
85.....	439 to 460	Traverse...	6.02 to 6.23.45	Canfield B. O. closed to 13 *T.*, 6.25.00 to 6.25.20, between Pos. 460 and 461.
86.....	461 to 484	Traverse...	6.26 to 6.49.45	Canfield B. O. closed to 8 *T.*, 6 49.00 to 6.49.10, during Pos. 484.

NOTE.—Times of observation and of closing B. O's. by observer's watch. Times of opening B. O's. by G. S. W.'s watch. Watches set together at 8.30 P. M.

* Velocities plotted in full black lines from 11.09 to 3.53.

† Position 245 omitted.

‡ Took only 10½ turns.

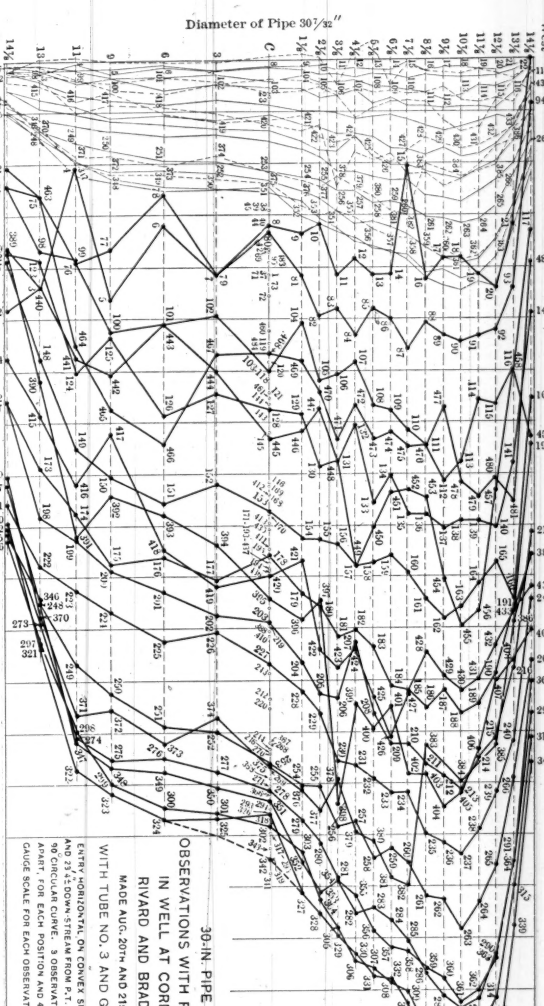
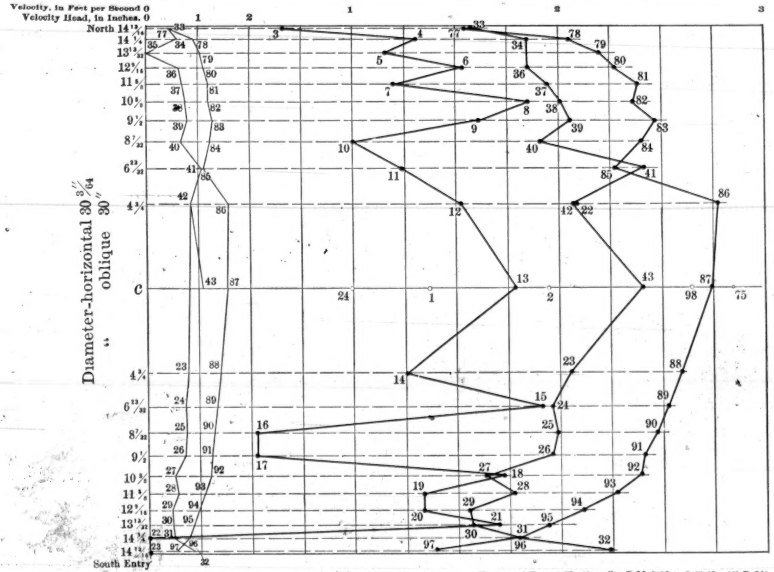


FIG. 12.

For table accompanying Fig. 12, see opposite page.



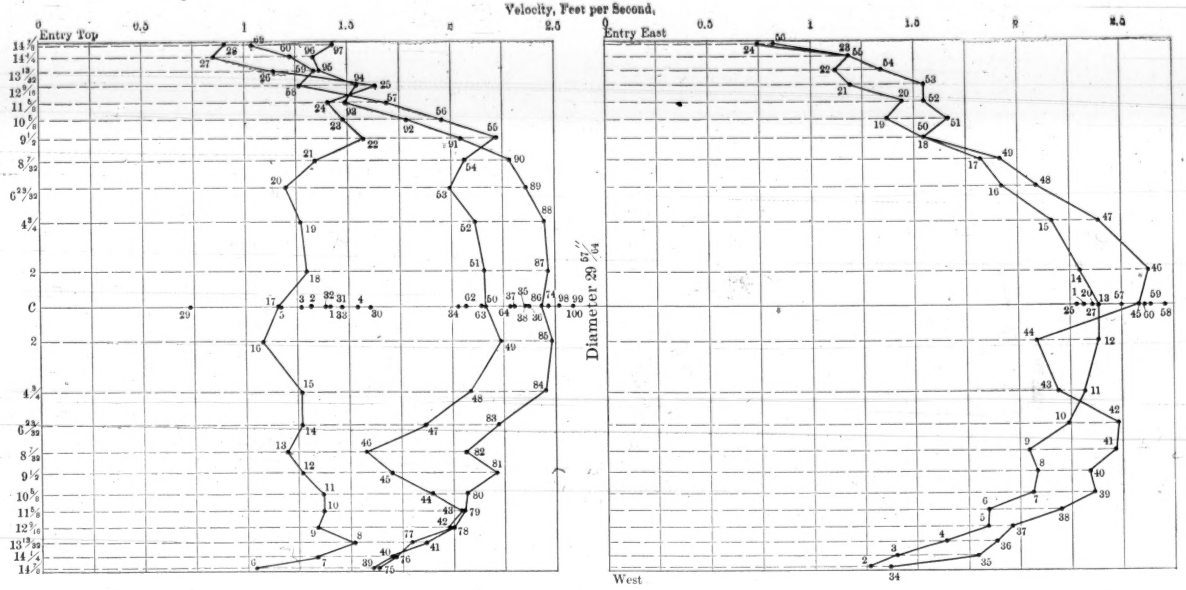
Traverse 28 June 24, 1898 Positions 1 to 24 traverse with Tube 3 and Gauge 8 on Horizontal Tap at Hastings St. P.M. 8:20 to 8:47:45. All B.O's. closed.
29 July 17, 1898 " 22 to 43 " " " 4 " " 9 " Oblique " " " " A.M. 1:13 " 1:38:45 Hastings B.O. 7 T. open
30 " " 75 to 98 " " " " " " " " " " " " 2:16 3:50:45 " " Full "

Intermission between Pos. 82 and 83 from 2:28:45 to 3:34. Velocities of this Traverse are corrected for apparent error due to leak in Gauge No. 9.
Positions 1 to 24 were taken with Pressure Column of Gauge connected to ¼ in. tap in top of Pipe.

30-IN. PIPE OBSERVATIONS WITH PITOT TUBE.

EACH POSITION IS THE MEAN OF 8 OBSERVATIONS COMPRISING
4 READINGS OF THE GAUGE SCALE

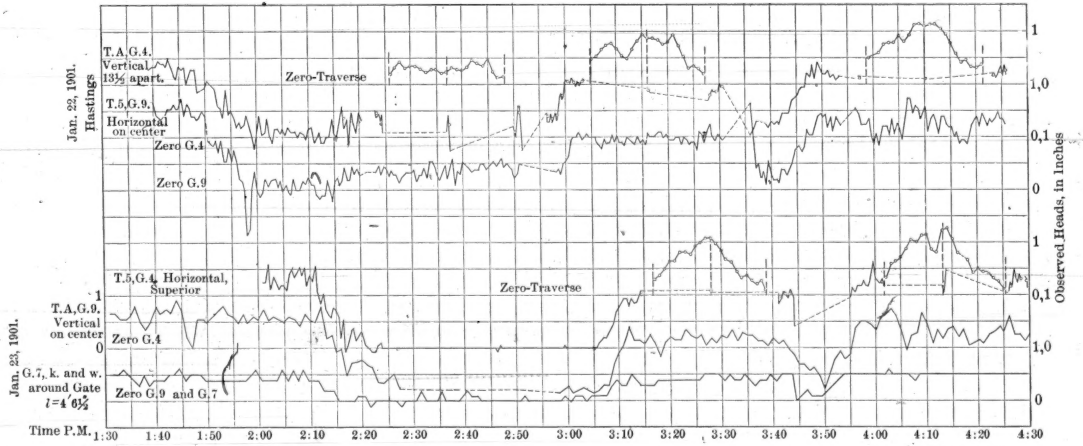
FIG. 18.



TUBE A, GAUGE No. 4, VERTICAL
AT HASTINGS,
2:23 TO 4:25 P.M. JAN. 22, 1901.

TUBE 5, GAUGE No. 4, HORIZONTAL.
AT SUPERIOR,
3:14 TO 4:30 P.M. JAN. 23, 1901.
LOCATION 2 FT. DOWN-STREAM FROM A 30 IN. GATE WITH BONNET TO THE EAST.

FIG. 14.



30-IN. PIPE

OBSERVATIONS WITH PITOT TUBES

POSITIONS ON TRAVERSES ARE MEANS OF 3 OBSERVATIONS COMPRISING 4 READINGS OF THE GAUGE SCALE EACH.

OTHER OBSERVATIONS:
JAN. 22, 1901, 30 SECONDS APART.
" 23, " 1 MINUTE "

Traverse 48	Positions 5 to 29	Hastings
49	38 " 63	"
50	74 " 98	"
51	1 " 25	Superior
52	33 " 57	"

FIG. 15.

TABLE No. 8.—Comparison of Traverses in Straight Pipe.

Size, in inches.	Number.	Date.	Location.	Tube.	Gauge.	Diameter, in inches.	Velocity at center.	Mean velocity.	Ratio $V_m \div V_c$.	Ratio Mean ÷ Max.
42	1	June 2, 1898.	Elmwood and Champlain.	3	8	$41\frac{37}{64}$	3.884	3.160	0.813	.....
42	2	"	"	3	8	$41\frac{37}{64}$	3.914	3.319	0.848	.....
42	3	"	"	....	....	$41\frac{37}{64}$	Partial.		.....	.....
30	4	June 7, 8, 1898.	Hastings.	3	8	$30\frac{51}{64}$	3.72	3.191	0.858	.....
30	5	"	"	3	8	$30\frac{51}{64}$	4.65	3.972	0.854	.....
30	6	"	"	3	8	$30\frac{51}{64}$	4.64	3.963	0.854	.....
30	7	June 11, 12, 1898.	"	3	8	$30\frac{51}{64}$	3.29	2.808	0.853	.....
30	8	"	"	3	8	$30\frac{51}{64}$	3.38	2.822	0.835	.....
30	9	"	"	3	8	$30\frac{51}{64}$	Partial.		.....	.....
30	10	"	"	3	8	$30\frac{51}{64}$	.....	.....	.....	.....
30	11	"	"	3	8	$30\frac{51}{64}$	4.67	3.978	0.854	.....
30	12	"	"	3	8	$30\frac{51}{64}$	Partial.		.....	.....
30	13	"	"	3	8	$30\frac{51}{64}$	Partial.		.....	.....
30	14	"	"	3	8	$30\frac{51}{64}$	Partial.		.....	.....
30	15	June 18, 19, 1898.	"	3	8	$30\frac{51}{64}$	0.940	0.859	0.914	.....
30	16	"	"	3	8	$30\frac{51}{64}$	0.986	0.567	0.575	.....
30	17	"	"	3	8	$30\frac{51}{64}$	1.070	0.609	0.569	.....
30	18	"	"	3	8	$30\frac{51}{64}$	1.630	1.179	0.723	.....
30	19	"	"	3	8	$30\frac{51}{64}$	2.405	2.014	0.837	.....
30	20	"	"	3	8	$30\frac{51}{64}$	3.292	2.767	0.840	.....
30	21	"	"	3	8	$30\frac{51}{64}$	3.307	2.806	0.848	.....
30	22	"	"	3	8	$30\frac{51}{64}$	3.779	3.247	0.859	.....
30	23	"	"	3	8	$30\frac{51}{64}$	4.403	3.633	0.825	.....
30	24	"	"	3	8	$30\frac{51}{64}$	4.284	3.696	0.863	.....
30	25	"	"	3	8	$30\frac{51}{64}$	3.396	2.874	0.846	.....
30	26	"	"	3	8	$30\frac{51}{64}$	3.038	2.546	0.822	.....
30	27	"	"	3	8	$30\frac{51}{64}$	2.147	1.894	0.882	.....
30	28	June 24, 25, 1898.	"	3	8	$30\frac{51}{64}$	1.621	1.299	*0.801	.....
30	29	July 16, 17, 1898.	Hastings, oblique.	4	9	30	2.40	1.862	0.776	.....
30	30	"	"	4	9	30	2.750	2.317	0.843	.....
30	31	Aug. 13, 14, 1898.	Leland.	4	9	$30\frac{1}{4}$	2.585	2.240	0.866	.....
30	32	"	"	4	9	$30\frac{1}{4}$	3.070	2.551	0.831	.....
30	33	"	"	4	9	$30\frac{1}{4}$	3.473	2.847	0.820	.....
30	34	"	"	4	9	$30\frac{1}{4}$	3.972	3.448	0.868	.....
30	35	"	"	4	9	$30\frac{1}{4}$	Partial.		.....	.....
30	36	"	"	4	9	$30\frac{1}{4}$	4.380	3.828	0.874	.....
30	37	"	"	4	9	$30\frac{1}{4}$	4.475	3.933	0.879	.....
30	38	"	"	4	9	$30\frac{1}{4}$	4.578	3.988	0.871	.....
30	39	"	"	4	9	$30\frac{1}{4}$	4.428	3.862	0.872	.....
30	40	"	"	4	9	$30\frac{1}{4}$	3.970	3.428	0.863	.....
30	41	"	"	4	9	$30\frac{1}{4}$	3.460	3.075	0.889	.....
30	42	June 22, 1899.	Hastings, oblique.	3	5	30	2.210	1.864	0.843	.....
30	43	"	"	3	5	30	2.820	2.317	0.822	.....
30	44	"	"	3	5	30	2.857	2.437	0.853	.....
30	45	"	"	3	5	30	2.502	2.114	0.845	.....
30	46	"	"	6	5	30	2.985	2.490	0.834	.....
30	47	"	"	6	5	30	2.950	2.479	0.840	.....
30	48	Jan. 22, 1901.	Hastings, vertical.	*A*	4	30±	1.322	1.314	0.994	.....
30	49	"	"	*A*	4	30±	2.209	1.825	0.826	.....
30	50	"	"	*A*	4	30±	2.462	1.987	0.807	.....
30	51	Jan. 23, 1901.	Superior, horizontal.	5	5	$29\frac{21}{64}$	2.353	1.762	0.749	.....
30	52	"	"	5	5	$29\frac{21}{64}$	2.568	1.961	0.764	.....
16	87	Sept. 13 and 18, 1898.	Military and Army Avenue.	3	9	$16\frac{1}{16}$	4.485	3.766	0.840	.....
16	88	"	"	3	9	$16\frac{1}{16}$	Partial.		.....	.....
16	89	"	"	3	9	$16\frac{1}{16}$	2.900	2.372	0.818	.....
16	90	"	"	3	9	$16\frac{1}{16}$	3.940	3.291	0.835	.....
16	91	"	"	3	9	$16\frac{1}{16}$	5.590	4.705	0.842	.....
16	92	"	"	3	9	$16\frac{1}{16}$	6.830	5.781	0.846	.....
16	93	"	"	3	9	$16\frac{1}{16}$	7.700	6.648	0.863	.....
12	94	Nov. 4, 1898.	Kercheval and South.	3	8	11.992	4.240	3.666	0.865	0.841
12	95	"	"	3	8	11.992	5.075	4.505	0.888	0.844
12	96	Nov. 16, 1898.	Reid and South.	3	8	12.003	2.685	2.238	0.834	0.826
12	97	"	"	3	8	12.003	3.733	3.302	0.885	0.847
12	98	"	"	3	8	12.003	4.563	4.067	0.891	0.877
12	99	"	"	3	8	12.003	5.455	4.991	0.915	0.880
12	100	"	"	3	8	12.003	6.273	5.695	0.908	0.878
12	101	June 6, 1899.	Kercheval and South.	3	9	11.992	0.779	0.585	0.751	0.747
12	102	June 7, 1899.	"	3	9	11.992	1.509	1.240	0.841	.....

* This traverse made with point of Tube No. 3 and orifice in pipe wall.

TABLE No. 8—(*Continued*).

Size, in inches.	Number.	Date.	Location.	Tube.	Gauge.	Diameter, in inches.	Velocity at center.	Mean velocity.	Ratio $V_m \div V_c$.	Ratio Mean ÷ Max.
12	*A*	Dec. 9, 1897.	Woodward and City Limits.	2	4	11⅞	1.42	1.063	0.748	.....
12	*B*	"	Mean of 3	2	4	11⅞	3.18	2.565	0.807	.....
12	*C*	"	" 5	2	4	11⅞	5.04	4.148	0.823	.....
12	*D*	"	" 6	2	4	11⅞	5.97	4.781	0.801	.....
			2-IN. PIPE.							
2	*a*	Dec. 18, 1899.	Cornell University.	Hor.	3	2.096	2.51	2.046	0.815	.....
2	*b*	"	"	"	3	2.096	2.74	2.281	0.832	.....
2	*c*	Dec. 20, 1899.	"	"	5	2.096	5.281	3.798	0.729	.....
2	*d*	Feb. 8, 1901.	"	"	*D*	2.096	2.51	2.07	0.825	.....
2	*e*	"	"	"	*D*	2.096	1.99	1.618	0.813	.....
2	*f*	Feb. 14, 1901.	"	"	*D*	2.096	0.78	0.5444	0.698	.....
2	*g*	"	"	"	*D*	2.096	0.817	0.6836	0.837	.....
2	*h*	"	"	"	*D*	2.096	1.135	0.892	0.785	.....
2	*i*	"	"	"	*D*	2.096	1.22	1.084	0.889	.....
2	*j*	Feb. 15, 1901.	"	"	*E*	2.096	1.59	1.292	0.812	.....
2	*k*	"	"	"	*E*	2.096	5.68	4.584	0.809	.....
2	*l*	March 8, 1901.	"	Vert.	*E*	2.096	2.317	1.912	0.826	.....
2	*m*	"	"	"	*E*	2.096	4.39	3.465	0.789	.....

was difficult to imagine a reversed flow, when it was known positively that the only outlet was down stream. On further experiment, it was found that if, when Tube No. 3 was reading a high velocity in the positive direction, the point was turned away from the direction of the axis of the pipe, the reading would be decreased and that, soon after the point passed an angle of 45° with the axis, negative readings occurred. Making a complete revolution of the tube, it was found that the maximum negative reading occurred when the point was about at right angles to the axis, the readings approaching zero again after this point was passed, and being considerably nearer to zero when the tube was reversed than when it was only half reversed. From these observations, it was concluded that the zero and negative readings were due to oblique currents, and, as an obliquity of a little more than 45° was evidently sufficient to produce them, it was determined to treat all negative readings as zero. This, undoubtedly, has the effect of giving too low a discharge to the section, but it did not seem possible to determine the true axial velocity when such readings occurred, and the writers preferred to limit the use of the instrument to those cases where such readings do not occur. Traverses containing them are nevertheless presented, as it has been the purpose of the writers to present the results as obtained, be they good, bad or indifferent.

TABLE No. 9.

42-In. Pipe. June 2d, 1898. Elmwood and Champlain Streets. Tube No. 3. Horizontal.				30-In. Pipe. Normal Conditions. June 7th, 8th, 1898. Hastings Street and Alexandrine Avenue. Tube No. 3. Horizontal.							30-In. Pipe. Normal Conditions. June 11th, 12th, 1898. Hastings Street and Alexandrine Avenue. Tube No. 3. Horizontal.									
	(1)	(2)	(3)		(4)		(5)	(6)			(7)	(8)			(9)	(10)	(11)	(12)	(13)	(14)
	Vel.	Vel.	Vel.		H_v	Vel.	Vel.	Vel.	Vel.	Vel.	Vel.	Vel.	Vel.	Vel.	Vel.	Vel.	Vel.	Vel.	Vel.	Vel.
Center ...	3.944	3.965	.....	Center ...	2.57	3.72	4.49	.....	.....	.....	3.29	.....	.....	.....	3.36	4.67	4.64	4.65	4.70	3.64
$20\frac{13}{16}$	2.088	2.422	2.620	$14\frac{13}{16}$	0.74	2.00	2.55	2.75	.....	.....	2.02	1.69	.....	.....	1.96	2.69	2.62	2.59	3.03	2.07
$19\frac{15}{16}$	2.763	2.926	.....	$14\frac{1}{4}$	1.15	2.49	3.23	.....	3.53	.....	2.34	2.24	.....	.....	2.27	3.28	3.31	3.34	.....	.....
$18\frac{7}{8}$	2.898	3.120	.....	$13\frac{23}{32}$	1.42	2.77	3.53	.....	.....	.....	2.56	.....	2.41	.....	.....	.....	3.61	.....	3.73	2.91
$17\frac{7}{8}$	3.113	3.035	3.109	$12\frac{7}{16}$	1.72	3.04	3.61	3.76	.....	.....	2.64	2.49	.....	.....	2.70	3.72	3.78	3.69	.....	.....
$16\frac{7}{8}$	3.300	3.095	.....	$11\frac{7}{32}$	1.69	3.02	3.78	.....	3.85	.....	2.78	.....	2.74	.....	.....	.....	4.00	3.98	.....	2.96
$14\frac{3}{8}$	3.587	3.464	.....	$10\frac{3}{8}$	1.68	3.01	3.87	4.09	.....	.....	2.96	2.88	.....	.....	2.89	4.10	4.05	.....	4.02	.....
$13\frac{1}{4}$	3.239	3.496	3.519	$9\frac{1}{4}$	1.95	3.24	4.19	.....	4.09	.....	2.98	.....	3.03	.....	.....	.....	4.19	.....	.....	3.04
$11\frac{1}{4}$	3.424	3.424	.....	$8\frac{7}{32}$	2.07	3.34	4.26	4.25	.....	.....	3.00	2.95	.....	.....	3.13	4.27	4.33	4.23	4.29	.....
$9\frac{3}{8}$	3.540	3.654	.....	$6\frac{23}{32}$	2.23	3.46	4.46	.....	4.30	.....	3.16	.....	3.05	.....	.....	.....	4.42	.....	4.46	3.20
$6\frac{5}{8}$	3.280	3.681	3.698	$4\frac{3}{8}$	2.54	3.70	4.61	4.55	4.52	.....	3.19	3.26	3.23	.....	3.23	4.49	4.53	4.52	.....	.....
Center ...	3.867	3.888	3.895	Center ...	2.74	3.84	4.73	4.68	4.61	4.64	3.31	3.40	3.37	3.38	3.33	4.75	4.65	4.71	4.70	3.42
$6\frac{5}{8}$	3.519	3.691	3.733	$4\frac{3}{8}$	2.69	3.80	4.51	.....	4.52	4.56	3.22	.....	3.29	3.31	3.22	4.58	4.55	4.56	.....	.....
$9\frac{3}{8}$	3.749	3.881	.....	$6\frac{23}{32}$	2.40	3.59	4.48	.....	4.39	.....	3.16	.....	3.18	.....	.....	.....	4.41	.....	4.48	3.20
$11\frac{1}{4}$	3.624	3.523	.....	$8\frac{7}{32}$	2.02	3.30	4.32	.....	.....	4.24	2.88	.....	.....	3.09	2.95	4.33	4.24	4.29	4.10	.....
$13\frac{1}{4}$	2.714	3.313	3.345	$9\frac{1}{4}$	2.04	3.32	4.18	.....	4.10	.....	2.90	.....	3.01	.....	.....	.....	4.14	.....	.....	3.01
$14\frac{3}{8}$	3.042	3.171	.....	$10\frac{3}{8}$	1.96	3.25	4.05	.....	.....	3.99	2.78	.....	.....	2.87	2.69	4.12	3.96	4.06	.....	.....
$16\frac{1}{4}$	2.981	3.394	.....	$11\frac{5}{8}$	2.20	3.44	3.94	.....	3.85	.....	2.56	.....	2.86	.....	.....	.....	3.90	.....	3.83	2.77
$17\frac{9}{16}$	2.953	3.264	3.079	$12\frac{1}{16}$	1.86	3.16	3.72	.....	.....	3.65	2.53	.....	.....	2.77	2.52	3.76	3.76	3.56	.....	.....
$18\frac{3}{8}$	2.735	3.190	.....	$13\frac{1}{8}$	1.71	3.03	3.54	.....	3.56	.....	2.55	.....	2.36	.....	.....	.....	3.30	.....	3.50	2.59
$19\frac{5}{16}$	2.594	2.833	.....	$14\frac{1}{4}$	1.28	2.62	3.22	.....	.....	.....	2.29	.....	.....	2.27	2.13	3.19	3.13	3.11	.....	.....
$20\frac{13}{16}$	0.984	0.998	0.868	$14\frac{3}{8}$	0.73	1.98	2.67	.....	2.78	.....	1.80	.....	1.83	.....	1.75	2.69	2.53	2.74	.....	1.77
Center ...	3.860	3.916	.....	Center ...	2.27	3.50	4.64	.....	.....	.....	3.25	.....	.....	.....	3.38	4.69	4.69	.....	.....	3.46

TABLE No. 9—(*Continued*).

30-IN. PIPE. NORMAL CONDITIONS.

August 13th, 14th, 1898. Rivard and Leland Streets. Tube No. 4.—Oblique.											June 22d, 23d, 1899, Hastings St. and Alexandrine Ave. Tube No. 3.—Oblique.						January 22d, 1901. Hastings St. and Alexandrine Ave. Tube *A*.—Vertical.			January 23d, 1901. Alley east of St. Antoine and Superior Streets. Tube No. 5.—Horizontal.	
(31) Vel.	(32) Vel.	(33) Vel.	(34) Vel.	(35) Vel.	(36) Vel.	(37) Vel.	(38) Vel.	(39) Vel.	(40) Vel.	(41) Vel.	(42) Vel.	(43) Vel.	(44) Vel.	(45) Vel.	(46) Vel.	(47) Vel.	(48) Vel.	(49) Vel.	(50) Vel.	(51) Vel.	(52) Vel.
2.58	3.10	3.56	3.97	4.02	4.26	4.48	4.60	4.43	3.95	3.52	2.30	2.88	.....	2.46	2.98	.....	1.80	2.14	2.52	.....	2.59
1.12	1.89	2.40	2.68	2.92	2.87	3.13	2.92	3.10	2.52	2.28	1.26	1.81	1.98	1.68	1.93	1.77	0.91	1.04	1.43	1.27	1.37
1.68	2.23	2.62	2.89	3.05	3.32	3.51	3.47	3.40	3.03	2.74	1.30	1.97	2.02	1.84	2.09	2.02	0.86	1.23	1.34	1.40	1.80
1.83	2.48	2.86	3.08	3.23	3.58	3.68	3.70	3.72	3.18	2.78	1.55	2.13	2.20	1.93	2.30	2.27	1.15	1.34	1.37	1.64	1.89
2.18	2.48	2.87	3.36	3.30	3.73	3.89	3.77	3.75	3.26	2.86	1.68	2.25	2.30	2.02	2.30	2.36	1.64	1.27	1.55	1.85	1.96
2.23	2.62	.....	3.52	3.63	3.88	3.98	3.97	3.74	3.42	2.98	1.83	2.38	2.43	2.16	2.42	2.41	1.41	1.69	1.49	1.85	2.20
2.41	2.67	3.12	3.48	3.50	4.00	4.14	4.03	4.00	3.53	3.07	1.93	2.42	2.49	2.20	2.61	2.56	1.48	1.96	1.78	2.07	2.37
2.41	2.78	3.22	3.78	.....	4.03	4.18	4.23	4.00	3.58	3.13	2.05	2.49	2.51	2.30	2.63	2.58	1.58	2.22	2.05	2.09	2.34
2.41	2.92	3.32	3.75	3.80	4.14	4.28	4.22	4.13	3.70	3.18	2.07	2.56	2.63	2.34	2.68	2.64	1.34	2.07	2.28	2.05	2.47
2.65	2.88	3.30	3.82	....	4.23	4.36	4.35	4.27	3.81	3.22	2.16	2.61	2.68	2.41	2.74	2.74	1.20	2.00	2.36	2.24	2.48
2.61	2.97	3.46	3.98	4.02	4.35	4.45	4.39	4.48	3.88	3.36	2.23	2.65	2.73	2.45	2.85	2.78	1.27	2.12	2.45	2.32	2.19
2.64	3.12	3.50	4.02	4.03	4.42	4.47	4.53	4.43	3.98	3.46	2.26	2.83	2.92	2.51	3.03	2.95	1.16	2.16	2.43	2.39	2.59
2.58	2.85	3.32	3.82	.....	4.20	4.27	4.48	4.30	3.86	3.38	2.18	2.73	2.80	2.40	2.88	2.88	1.27	2.09	2.45	2.16	2.39
2.50	2.71	3.35	3.75	.....	4.17	4.13	4.32	4.20	3.68	3.23	2.05	2.54	2.67	2.37	2.78	2.78	1.27	1.87	2.22	1.92	2.09
2.43	2.58	3.22	3.58	.....	3.97	4.15	4.26	4.10	3.55	3.07	2.00	2.13	2.62	2.22	2.76	2.67	1.20	1.58	2.07	1.82	1.92
2.43	2.48	2.94	3.46	.....	3.92	4.00	4.07	3.97	3.55	3.00	1.88	2.16	2.57	2.04	2.63	2.58	1.27	1.70	2.21	1.55	1.55
2.30	2.40	2.67	3.22	.....	3.90	3.88	4.06	3.88	3.42	2.96	1.86	2.21	2.43	2.04	2.48	2.39	1.37	1.90	2.07	1.37	1.66
2.19	2.32	.....	3.22	.....	3.53	3.72	3.90	3.58	3.25	2.76	1.72	2.18	2.37	1.96	2.34	2.41	1.37	2.04	2.05	1.44	1.55
2.05	2.24	2.66	3.17	.....	3.61	3.61	3.60	3.46	3.12	2.71	1.71	2.09	2.26	1.96	2.22	2.34	1.34	1.98	2.00	1.19	1.55
1.91	2.12	2.56	3.08	.....	3.28	3.40	3.36	3.33	2.92	2.67	1.65	2.00	2.03	1.79	2.07	2.26	1.52	1.87	1.80	1.12	1.34
1.67	1.99	2.38	2.88	.....	3.08	3.13	3.32	3.10	2.89	2.43	1.52	1.82	1.93	1.55	1.89	1.92	1.34	1.70	1.72	1.19	1.19
1.68	1.50	1.91	2.57	.....	2.54	2.62	2.64	2.55	2.32	1.97	1.06	1.46	1.46	1.13	1.33	1.37	1.04	1.61	1.64	0.74	0.82
2.48	2.94	3.33	3.88	.....	4.32	4.48	4.65	4.42	3.97	3.40	2.42	2.74	2.73	2.58	2.90	2.95	1.17	2.375	2.47	2.28	2.50

TABLE No. 9—(*Continued*).

30-In. Pipe, Normal Conditions.

June 18th, 19th, 1898.

Hastings Street and Alexandrine Avenue.

Tube No. 3. Horizontal.

(15) Vel.	(16) Vel.	(17) Vel.	(18) Vel.	(19) Vel.	(20) Vel.	(21) Vel.	(22) Vel.	(23) Vel.	(24) Vel.	(25) Vel.	(26) Vel.	(27) Vel.	(28) Vel. June 24th, 1898. Hastings and Alexandrine. Tube No. 3. Horizontal. Point of tube and wall of pipe.	(29) Vel.	(30) Vel. July 16th, 17th, 1898. Hastings and Alexandrine, Tube No. 3. Oblique.
0.728	1.011	1.0500	1.735	2.411	3.182	3.279	3.915	4.426	4.360	3.503	3.216	2.113	1.945	.....	.....
0.846	0.000	0.3025	0.000	1.122	1.584	1.964	2.221	2.414	2.554	1.942	1.762	1.234	0.669	1.57	1.55
0.846	0.000	0.0000	0.4213	1.234	2.158	2.357	2.694	3.168	3.088	2.355	.....	1.198	1.310	1.85	2.04
0.000	0.7940	0.0000	0.7334	1.895	2.486	2.432	2.950	3.415	3.390	2.488	.....	1.584	1.160	0.00	2.20
0.9922	0.7940	0.794	0.8460	1.964	2.506	2.695	3.070	3.580	3.504	2.712	2.357	1.870	1.530	1.85	2.37
0.6682	0.3025	0.794	1.234	1.917	2.696	2.827	3.280	3.790	3.705	2.678	.....	1.894	1.198	1.94	2.38
0.7940	1.1980	0.735	1.306	2.158	2.792	3.008	3.415	3.800	3.778	3.097	.....	1.964	1.855	2.01	2.36
0.4213	0.5200	0.4213	1.526	2.097	2.840	2.841	3.387	3.847	3.824	3.040	2.775	1.942	1.615	2.05	2.47
1.0810	0.6682	0.6003	1.372	2.299	2.949	3.048	3.670	3.998	3.985	3.225	.....	2.032	1.000	1.91	2.40
0.6683	1.0370	1.037	1.467	2.357	3.040	3.112	3.658	4.150	4.170	3.280	.....	2.118	1.233	2.41	2.27
0.9920	0.733	0.846	1.584	2.415	3.155	3.169	3.800	4.310	4.277	3.337	2.965	2.118	1.524	2.12	2.77
1.3060	1.070	1.404	1.578	2.415	3.347	3.322	3.729	4.302	4.246	3.442	3.040	2.221	1.772	2.40	2.74
0.9920	0.733	0.6003	1.639	2.357	3.280	3.182	3.751	4.202	4.204	3.280	2.949	2.138	1.250	2.05	2.59
1.2710	1.037	0.5195	1.526	2.240	3.140	3.040	3.579	4.127	4.095	3.240	.....	2.032	1.916	1.96	2.52
0.734	0.000	0.5195	1.497	2.240	3.070	3.040	3.451	4.028	3.976	3.141	.....	1.917	0.5197	1.98	2.47
0.992	0.302	0.9480	1.497	1.965	2.965	2.889	3.322	3.904	3.870	3.009	2.738	1.820	0.5197	1.96	2.40
1.234	0.421	0.0000	1.467	2.221	2.778	2.841	3.168	3.789	3.593	2.889	.....	1.761	1.720	1.63	2.38
0.668	0.421	0.4213	1.270	2.139	2.712	2.776	3.140	3.568	3.532	2.810	.....	1.747	1.332	1.77	2.27
1.556	0.3025	0.6003	0.794	1.694	2.578	2.568	2.888	3.568	3.519	2.524	2.322	1.719	1.332	1.55	2.10
1.306	0.6684	0.4213	0.667	1.845	2.434	2.450	2.810	3.332	3.240	2.303	.....	1.467	1.690	1.57	1.94
0.000	0.000	0.0000	0.6003	1.404	2.392	2.240	2.559	2.965	2.841	2.200	.....	1.199	0.000	1.80	1.80
1.037	0.000	0.0000	0.0000	0.794	1.612	.....	1.876	2.378	.....	1.869	1.497	0.792	0.000	2.23	1.38
0.4215	0.794	0.4213	.....	2.379	.....	.....	3.741	.....	.....	3.197	3.097	2.032	0.904	.....	2.67

TABLE No. 9—(*Continued*).

30-IN. PIPE.—ABNORMAL CONDITIONS.

Aug. 20th, 21st, 1898.

Brady and Rivard Streets.

Tube No. 3—Horizontal.

(68) Vel.	(69) Vel.	(70) Vel.	(71) Vel.	(72) Vel.	(73) Vel.	(74) Vel.	(75) Vel.	(76) Vel.	(77) Vel.	(78) Vel.	(79) Vel.	(80) Vel.	(81) Vel.	(82) Vel.	(83) Vel.	(84) Vel.	(85) Vel.	(86) Vel.
0.17	0.52	0.98	1.50	1.73	2 10	2.43	2.64	3.09	3.31	3.65	3.74	3.93	3.54	2.76	2.78	2.27	1.31	1.37
0.00	0.92	0.22	0.81	1.23	1.64	1.87	2.29	2.62	2.92	3.17	3.28	3.41	3.01	2.40	2.77	1.56	1.83	0.98
0.80	1.37	1.11	1.50.	1.90	2.62	2.64	2.94	3.31	3.48	3.89	4.00	4.20	3.89	2.72	2.92	2.66	1.50	2.14
1.11	1.23	1.31	1.67	2.27	2.43	2.97	3.28	3.55	3.90	4.34	4.54	4.86	4.35	3.32	3.00	2.84	2.08	2.03
1.04	1.04	1.33	1.64	2.23	2.54	3.14	3.38	3.73	4 08	4.55	4.72	4.80	4.51	.....	3.34	3.00	2.67	2.12
0.96	1.23	1 37	1 94	2.23	2.65	3.17	3.53	3.85	4.21	4.60	4.77	4.90	4.57	3.50	3.54	3.06	2.74	2.05
0.96	1.17	1.33	2.04	2.27	2.71	3.05	3.47	3.79	4.07	4.53	4.67	4.85	4.45	.....	3.62	2.98	2.52	1.68
1.07	1.08	1 27	1.87	2.20	2.61	3.14	3.41	3.75	4.05	4.42	4.70	4.77	4.40	3.32	3.46	2.87	2.12	1.87
0.52	0.65	1.40	1.80	2.20	2.48	3.06	3.31	3.55	3.89	4.27	4.51	4.57	4.31	4.00	3.46	3.07	2.08	1.87
1.04	1.55	1.25	1.69	2.01	2.40	3.03	3.31	3.55	3.82	4.17	4.46	4.45	4.30	3.96	3.13	3.28	2.10	1.80
1.04	0.00	1.20	1.67	2.14	2.43	2.84	3.17	3.48	3.87	4.10	4.40	4.43	4.22	3.78	3.24	3.08	2.26	1.83
0.97	0.00	1.33	1.46	1.77	2.45	2.76	2.82	3.42	3.69	4.13	4.36	4.33	4.05	3.73	3.12	2.85	2.45	1.67
1.04	1.58	1.20	1.52	1.94	2.34	2.79	3.09	3.40	3.71	3.96	4.20	4.24	3.97	3.48	3.60	2.94	1.94	1.83
0.80	0.00	1.23	1.55	1.97	2.32	2.64	3.07	3.17	3.52	3.79	4.07	4.13	4.03	3.63	2.62	2.85	1.68	1.61
0.84	0.00	1.04	1.27	1.72	2.32	2.58	2.92	3.06	3.51	3.68	3.83	4.03	3.88	3.53	2.71	2.43	1.80	1.46
0.80	0.00	0.83	1.50	1.69	2.18	2.40	2.72	2.92	3.41	3.53	3.71	3.65	3.58	3.42	2.61	2.50	1.83	1.42
1.04	0.37	1.04	1.23	1.61	2.05	2.52	2.68	2.77	3.24	3.44	3.65	3.67	3.52	3.18	2.34	2.54	1.50	1.58
0.80	0.37	0.65	1.27	1.72	2.14	2.40	2.57	2.81	3.22	3.37	3.62	3.67	3.44	3.30	2.20	2.37	1.27	1.85
1.15	0.00	0.91	1.31	1.33	2.01	2.43	2.47	2.67	3.06	3.38	3.50	3.54	3.42	3.13	2.10	1.80	1.52	1.68
0.52	0 00	0.98	0.96	1.50	1.87	2.18	2.32	2.56	2.92	3.26	3.24	3.43	3.28	3.16	2.30	2.05	1.43	1.37
1.09	0.00	0.65	0.92	0.96	1.43	1.97	2.20	2.45	2.67	2.72	2.82	2.84	2.61	2.69	1.55	1.72	1.08	0.65
1.20	0.92	0.60	0.60	0.92	0.96	1.67	1.64	2.14	2.05	2.26	2 32	2.23	2.46	2.01	0.90	1.43	0.91	0 52
1.04	0.73	1.08	0.60	1.61	2.05	2.43	2.64	3.13	3.35	3.51	3.74	3.87	3.81	3.48	.....	2.24	0.80	1.64

TABLE No. 9—(*Continued*).

30-In. Pipe. Abnormal Conditions.

August 13th, 14th, 1898.

Riopelle and Rowena Streets.

Tube No. 3.—Horizontal.

	(53) Vel.	(54) Vel.	(55) Vel.	(56) Vel.	(57) Vel.	(58) Vel.	(59) Vel.	(60) Vel.	(61) Vel.	(62) Vel.	(63) Vel.	(64) Vel.	(65) Vel.	(66) Vel.	(67) Vel.
Center	1.05	.....	.....	1.55	.....	2.17	2.66	3.15	3.13	3.54	3.36	3.62	3.43	3.13	2.64
$14\frac{7}{8}$	0.72	1.11	1.72	1.50	2.14	2.00	2.54	3.05	3.10	3.37	3.38	3.48	3.20	2.98	2.57
$13\frac{7}{8}$	1.32	1.32	1.85	1.80	2.32	2.61	3.14	3.65	.....	4.10	4.06	4.17	4.12	3.57	3.17
$12\frac{7}{8}$	1.18	1.68	2.07	1.87	2.62	3.15	3.43	3.88	.....	4.40	4.47	4.45	4.34	4.00	3.38
$11\frac{7}{8}$	1.33	1.42	2.14	1.91	2.74	3.15	3.50	4.08	.....	4.58	4.75	4.64	4.53	4.13	3.50
$10\frac{7}{8}$	1.37	1.50	2.13	2.05	2.82	3.22	3.60	4.10	4.17	4.58	4.88	4.73	4.59	4.17	3.65
$9\frac{7}{8}$	1.20	1.60	2.13	2.10	2.79	3.27	3.58	4.15	.....	4.68	4.85	4.80	4.62	4.07	3.56
$8\frac{7}{8}$	1.32	1.64	2.17	2.00	2.86	3.22	3.50	4.13	.....	4.59	4.77	4.78	4.58	4.10	3.61
$7\frac{7}{8}$	1.22	1.91	2.07	2.00	2.79	3.15	3.54	4.10	.....	4.53	4.67	4.73	4.51	4.05	3.50
$6\frac{7}{8}$	1.32	1.40	2.12	2.05	2.67	3.12	3.50	3.98	3.98	4.42	4.63	4.60	4.47	4.00	3.54
$5\frac{7}{8}$	1.45	1.46	2.07	2.08	2.65	3.03	3.43	3.91	.....	4.36	4.47	4.50	4.29	3.90	3.42
$4\frac{7}{8}$	1.33	1.52	2.03	1.97	2.55	3.03	3.30	3.83	.....	4.28	4.25	4.39	4.28	3.95	3.26
$3\frac{7}{8}$	1.55	2.10	1.80	1.82	2.50	2.97	3.23	3.65	.....	4.14	4.22	4.26	4.17	3.65	3.13
$2\frac{7}{8}$	1.20	2.04	1.75	1.80	2.47	2.82	3.13	3.52	.....	3.96	4.00	4.14	4.04	3.59	3.11
$1\frac{7}{8}$	1.76	2.00	1.90	1.85	2.22	2.62	2.97	3.42	.....	3.80	3.72	3.95	3.88	3.34	3.02
Center	1.11	1.77	1.94	1.97	2.10	2.46	2.75	3.17	3.16	3.38	3.54	3.56	3.43	3.10	2.76
3	1.11	.....	.....	1.60	1.85	1.94	2.35	2.48	.....	2.94	3.00	3.10	2.94	2.64	2.10
6	1.11	.....	.....	1.40	1.60	1.94	2.12	2.40	2.52	2.52	2.82	2.92	2.76	2.23	2.14
9	0.95	.....	.....	1.55	1.43	1.82	2.04	2.32	0.98	2.58	2.69	2.76	2.74	2.20	1.90
11	0.95	.....	.....	1.32	1.37	1.72	2.05	2.20	.....	2.47	2.43	2.66	2.58	2.30	1.84
13	0.83	.....	.....	1.20	1.12	1.64	1.60	2.07	.....	2.35	2.43	2.47	2.32	2.03	1.55
$14\frac{7}{8}$	0.32	.....	.....	0.98	0.65	0.90	1.09	0.60	.....	1.11	1.15	0.61	1.12	1.09	0.31
Center	1.32	.....	.....	2.03	2.24	2.47	2.71	3.07	3.08	3.44	3.46	3.53	3.36	2.98	2.79

TABLE No. 9—(*Concluded*).

16-In. Pip .

September 13th and 16th, 1898.
Military and Army Avenues.
Tube No. 3—Vertical.

	(87) Vel.	(88) Vel.	(89) Vel.	(90) Vel.	(91) Vel.	(92) Vel.	(93) Vel.
Center	4.45	5.54	2.90	3.95	5.68	6.91	7.54
7/8	3.08	.....	1.69	2.49	3.79	4.55	5.41
7 9/32	3.24	4.06	1.98	2.77	4.05	5.11	5.88
7 7/32	3.42	.....	2.20	3.00	4.45	5.39	6.28
6 31/32	3.65	4.37	2.35	3.18	4.46	5.66	6.49
6 25/32	3.71	.....	2.40	3.34	4.76	5.86	6.75
5 27/32	3.97	4.74	2.45	3.42	4.86	6.00	6.82
5 7/16	3.95	.....	2.52	3.50	4.96	6.26	6.94
4 3/8	4.00	5.14	2.57	3.48	4.95	6.39	7.16
3 9/16	4.43	.....	2.62	3.67	5.57	6.53	7.35
2 13/32	4.36	5.26	2.82	3.88	5.26	6.43	7.59
1" below	4.53	.....	2.81	3.92	5.52	6.73	7.75
Center	4.52	5.41	2.89	3.94	5.62	6.78	7.70
1"	4.57	.....	2.87	3.90	5.55	6.82	7.64
2 17/32	4.36	5.26	2.79	3.86	5.43	6.61	7.55
3 9/16	4.19	.....	2.69	3.69	5.26	6.42	7.40
4 3/8	4.04	5.24	2.61	3.59	5.14	6.21	7.28
5 1/16	4.12	.....	2.61	3 50	5.03	6.11	7.09
5 21/32	3.86	4.62	2.48	3.46	4.78	5.96	6.88
6 3/4	3.78	.....	2.40	3.31	4.66	5.59	6.77
6 11/16	3.68	4.39	2.32	3.20	4.58	5.56	6.47
7 9/32	3.38	.....	2.05	3.01	4.30	5.28	6.22
7 19/32	3.11	3.88	1.77	2.67	3.93	5.03	5.72
7 7/8	2.59	.....	1.64	2.35	3.38	4.16	4.69
8 1/4	0.52	.....	0.30	.....	.....	.....	.....
Center	4.45	5.38	2.92	3.94	5.43	6.86	7.87

12-In. Pipe.

November 4th, 1898. Kercheval and South St.
Tube No. 3—Vertical.

November 16th, 1898.
Reid and South Streets.
Tube No. 3—Vertical.

June 6th, 7th, 1899. Kercheval and South St.
Tube No. 3—Vertical.

December 9th, 1897.
Woodward Avenue and City Limits.
Tube No. 2—Vertical.

	(94) Vel.	(95) Vel.	(96) Vel.	(97) Vel.	(98) Vel.	(99) Vel.	(100) Vel.	(101) Vel.	(102) Vel.
Center	4.25	4.99	2.59	3.76	4.49	5.59	6.34	0.700	1.600
5 21/32	2.82	3.35	1.43	2.73	3.78	4.43	5.10	0.300	0.883
5 5/32	2.87	3.76	2.05	3.34	3.78	4.71	5.41	0.533	0.983
5 5/32	3.36	3.87	2.40	3.36	4.06	4.96	5.77	0.583	1.267
4 23/32	3.50	4.33	2.35	3.35	4.32	5.32	6.05	0.483	1.217
3 27/32	3.63	4.49	2.61	3.61	4.52	5.44	6.25	0.400	1.267
3 3/32	3.93	4.65	2.64	3.83	4.41	5.49	6.37	0.633	1.333
2 25/32	4.15	4.61	2.54	3.83	4.61	5.63	6.47	0.800	1.417
2 5/32	4.10	4.99	2.59	3.90	4.62	5.67	6.49	0.783	1.467
1" below	4.23	5.11	2.57	3.76	4.64	5.63	6.20	.....	.....
Center	4.23	5.07	2.71	3.69	4.57	5.43	6.20	0.750	1.467
1"	4.36	5.34	2.59	3.65	4.45	5.41	6.04	.....	.....
2 3/32	4.23	5.12	2.64	3.48	4.33	5.25	5.92	0.750	1.533
2 23/32	3.99	5.00	2.11	3.34	4.05	4.95	5.77	0.567	1.333
3 13/32	4.13	4.90	2.14	3.14	4.03	4.96	5.58	0.533	1.367
4 3/32	3.44	4.70	2.10	3.05	3.75	4.74	5.38	0.700	1.183
4 23/32	3.81	4.51	1.43	2.77	3.68	4.62	5.21	0.683	1.117
5 5/32	3.28	4.23	1.91	2.64	3.50	4.36	4.81	0.383	0.850
5 15/32	2.79	3.94	1.61	2.67	3.07	3.86	4.37	0.267	0.900
5 25/32	2.67	3.32	1 69	1.98	2.79	3.50	3.73	0.283	0.633
Center	4.25	5.17	2.73	3.79	4.62	5.37	6.35	0.917	1.500

	(A) Vel.	(B) Vel.	(C) Vel.	(D) Vel.
Top	3.51	3.31	2.00	0.77
5 1/4	4.05	3.67	2.20	0.84
5	4.39	3.94	2.32	0.96
4	4.89	4.25	2.56	1.04
2 3/4	5.16	4.62	2.76	1.20
1 1/2	5.70	4.83	3.01	1.29
Center	5.97	5.06	3.18	1.43
1 1/2	5.77	4.93	3.14	1.29
2 3/4	5.34	4.52	2.87	1.20
4	4.80	4.18	2.61	1.13
5	4.40	3.83	2.39	1.09
5 1/2	4.29	3.59	2.31	0.87
Bottom	3.97	3.33	2.20	0.74

Normal Conditions: Relation of Central to Mean Velocity.—As already stated, it was discovered early in these investigations that for the 30-in. pipe a nearly constant ratio existed between the mean velocity as determined by the traverse and the velocity at the center, and this fact was at once taken advantage of to measure the discharge without the labor of traversing.

Table No. 8 shows the elements of seventy-six traverses, giving the ratios of $\frac{V_m}{V_c}$. From this table, selecting those traverses which when plotted show the nearest approach to normal conditions, a mean value of this ratio of 0.84 has been arrived at for the 30-in. pipe, and this value seems also to hold for the 16, 12, and 42-in. pipes. The value 0.81 for this ratio has been already deduced for the 2-in. brass pipe, from which it may appear that the ratio decreases slightly with the diameter; but traverses at other points in this pipe have given a ratio of 0.84, so that, for such diameters as are met in ordinary practice, it seems entirely safe to use a factor near to 0.84. There appears, on the face of Table No. 8, some evidence that the ratio changes with the velocity, but if such a change occurs it is quite small within the limits of the experiments. The irregularity of the ratio seems more likely to be due to the occurrence of disturbances as the tube passed the center, rather than to an actual variation of the ratio. Observations made with a tube on the center continuously showed the velocity there to be continually varying through small limits, but enough to change the value of this ratio 1 or 2% between the maximum and minimum.

Normal Conditions: The Equation of the Velocity Curves.—The constancy of the ratio $\frac{V_m}{V_c}$ points to a definite form of the surface represented by the plottings of the axial components of the velocities, and ordinary judgment would indicate the curve of the right section to be one that might become tangent or very nearly so to the pipe wall. Of the common curves, that which most naturally comes to mind as fulfilling this condition is the ellipse. An inspection of the plotted traverses where normal conditions existed shows that the velocity at the pipe wall was probably about half of the maximum. As already pointed out, the velocity at this position has never been measured on account of the impossibility of doing so with tubes of finite dimensions, but, from the inclination of the line between the two positions nearest the edge, it

seems fair to assume the foregoing value. If this be so, then the velocity prism is made up of a cylinder of length $\frac{1}{2} V_c$ and a dome of some form, the volume of which is $(0.84 - 0.50)\ V_c\ A = 0.34\ V_c\ A$.

Recalling, now, that the volume of the ellipsoid of revolution is two-thirds of the circumscribing cylinder of equal length, and the volume of half such a figure is $\frac{1}{3}$ or 0.333 of such a cylinder, it is apparent at once that the establishing of the ratio $\frac{V_m}{V_c} = 0.84$ stamps the velocity curve as a very close approximation to the ellipse in general form, in spite of its many irregularities. The writers are aware that previous investigators have assumed this curve to be a parabola, possibly modified by some other curve near the sides, but this assumption calls for a ratio $\frac{V_m}{V_c} = 0.75$, which is altogether too far from the foregoing experimental values to be admitted, notwithstanding that a distinguished predecessor in this line of experiment has deduced a parabolic equation for this curve from a comprehensive mathematical investigation of the supposed internal relations of the particles and their motions. It is, perhaps, fair to themselves to state that the elaborate investigation above referred to, was unknown to the writers until after the completion of the work of 1898.

The experiments show that even in straight pipe the value of the ratio $\frac{V_m}{V_c}$ may vary somewhat as obtained by experiment. Whether an actual variation occurs after normal flow is once established they have not regarded as proven except where special disturbing causes intervene.

Abnormal Conditions: Distortions of the Velocity Curve.—While, under normal conditions of flow in straight pipe, the velocity curve assumes a form approximating closely to the ellipse, it is very susceptible to small distortions. The effect of curvature is probably one of the most potent in producing such distortions, and Figs. 11 and 12 show a series of traverses taken on horizontal diameters of 30-in. pipe at points a few feet beyond the P. T.'s of 90° circular curves. Fig. 11 shows traverses taken 10 ft. 5 ins. down stream from a 40-ft. radius curve, and Fig. 12 those taken 23 ft. 4 ins. from a 15-ft. radius curve, for mean velocities in both cases ranging from less than 1 ft. to about $3\frac{1}{2}$ ft. per second.

These traverses, particularly those of Fig. 11, show the gradual displacement of the point of maximum velocity from a position near the center at slow velocities to one at about the location of the peculiar

disturbances already referred to in the consideration of normal flow, when the maximum velocity reaches a little above 3½ ft. Beyond this position an increase of 30% in maximum velocity fails to shift the point of the locus. This, perhaps, explains why the losses observed when water flows around a single curve of 180°, are not twice as great as when it passes around one of 90°, and of the same radius. The velocity curve probably receives its maximum possible distortion at velocities above 3 or 4 ft., before the water has passed the first quadrant, so that there is no more energy absorbed in this operation until a tangent is reached and the velocity curve begins to be brought back to the normal form. It is interesting to notice that the point of maximum velocity is at the same place on the diameter in both figures, thus showing that a variation of radius from 40 to 15 ft. has very little effect in changing the distribution in this respect.

But it will also be observed that there is a marked difference in the remaining characteristics of the velocity curves in the two figures, those following the short-radius curve, showing less total distortion than those following the longer ones. This may be in part, or perhaps wholly, accounted for by the excess of distance existing between the curve and the diameter of traverse in the former, thus permitting the velocities to begin their readjustment, but the fact that the point of maximum velocity is in the same position in both cases seems to show that the readjustment has at least not progressed unequally in the two cases, and so appears to controvert the above suggestion.

If the distortion of the velocity curve accounts for the excess of loss of head of the first half of a 180° curve over that of the second, it must be a leading factor in curve resistance, and it might be suggested that the curve producing the least distortion would offer the least resistance to the passage of water around it, when it follows that the loss in the short-radius curve would be the less.

Passing by this question for the present, the effect of the distortion of the velocities upon the loss of head to be observed in the tangent following the curve becomes of interest, for it is readily seen that if energy is absorbed in producing the original distortion, energy must also be absorbed in bringing about a readjustment to the normal arrangement, and a question at once arises as to the limits within which the curve resistance is to be measured, for it must evidently extend beyond the curve. The logical conclusion is that the curve

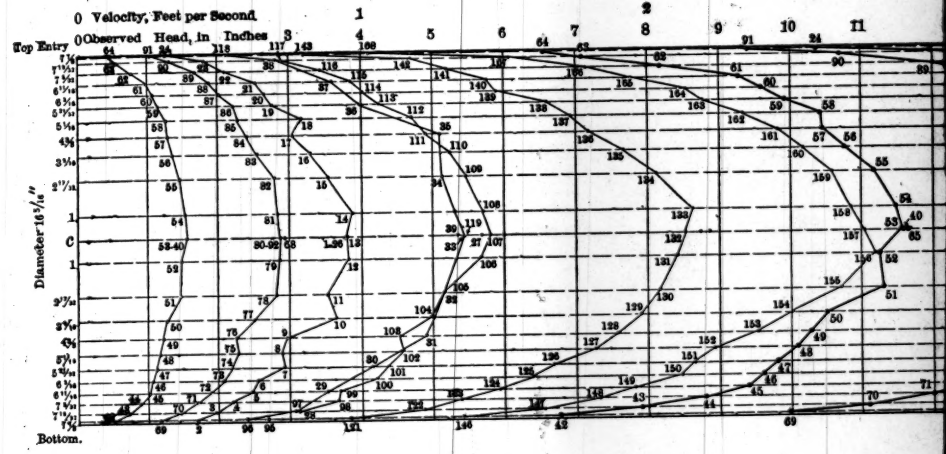
resistance must be assumed to extend to the point where the distribution of velocities again becomes normal, and the curve should be charged with the excess loss of head in the section between this point and its beginning, over the loss in an equal length of similar straight pipe. It now becomes of interest to determine how far away from the P. T. of the curve this point of normal rearrangement is, and on this question the data obtained are not conclusive.

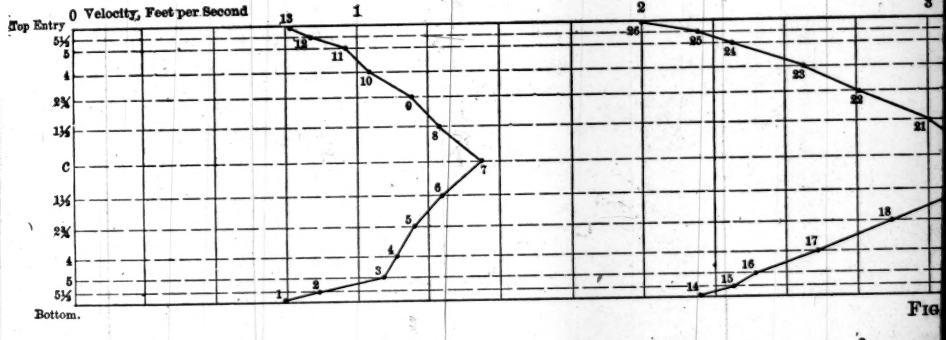
Abnormal Conditions: Readjustment of the Velocity Curve.—It has been considered that the distribution at Hastings Street was normal, although, even there, occasional evidences of eccentricity in the point of maximum velocity have been observed. This location is 737 ft. down stream from a 10-ft. radius circular curve, and the intervening pipe contains no known disturbing elements, and on the whole it seems safe to conclude that the distortion caused by a 10-ft. radius curve does not extend 737 ft. or 295 diameters down stream in 30-in. pipe, except at very low velocities.

Looking now at the traverses at Superior Street (Fig. 14), it appears that while the maximum velocity is in the center, there, nevertheless, is a distortion, which, however, may be due to the presence of a gate with the bonnet on the slow side of the traverse, only 2 ft. up stream. This location is 238 ft. down stream from a 6-ft. radius curve, and the conclusion must be considered as doubtful that distortion is apparent 238 ft. or 95 diameters down stream from a 6-ft. radius curve in 30-in. pipe.

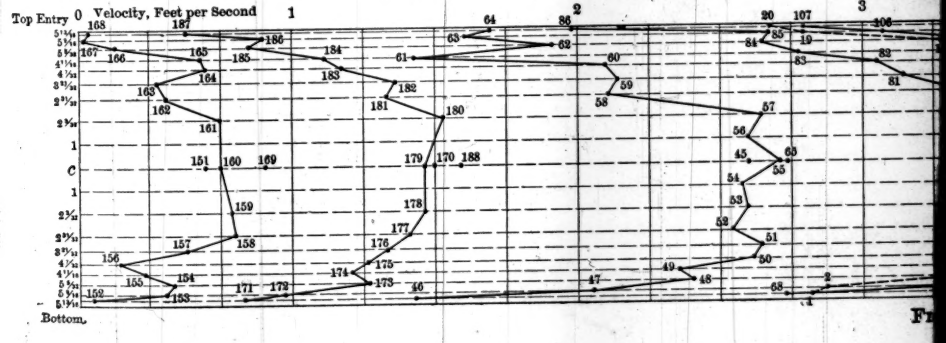
The traverses made at Leland Street (Fig. 8), show a slight distortion, which is, with one exception, toward the concave side of the curve, and it may, perhaps, be justly questioned whether this distortion may not be due to preceding branches in the pipe, of which there are three on the slow and one on the fast side of the traverses, the nearest being one of the former, a 30 x 10-in. Y-branch distant about 270 ft. up stream. A length of 756 ft. of pipe intervenes between this traverse and the preceding 15-ft. radius curve. This location, therefore, cannot be considered as furnishing conclusive evidence either one way or the other.

The 16-in. traverses, Plate IV, Fig. 1, which were taken on a vertical diameter, show a slight vibration of the point of maximum velocity up and down past the center as the velocity changes. These traverses were taken 328 ft. from a compound curve having a mean radius of about 3½ ft., there being no intervening specials. It seems fairly safe


0 Velocity, Feet per Second
Top Entry 0 Observed Head, in Inches
Diameter 16ths"
Bottom.


0 Velocity, Feet per Second
Top Entry
Bottom.

Fig


Top Entry 0 Velocity, Feet per Second
Bottom.

Fi

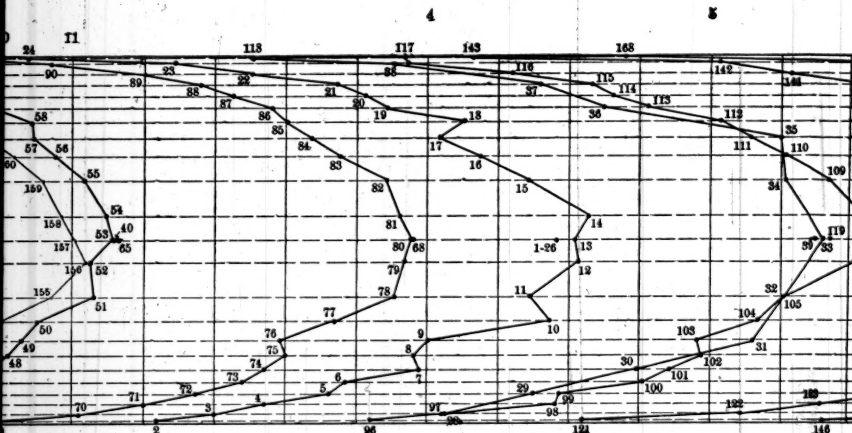


FIG. 1

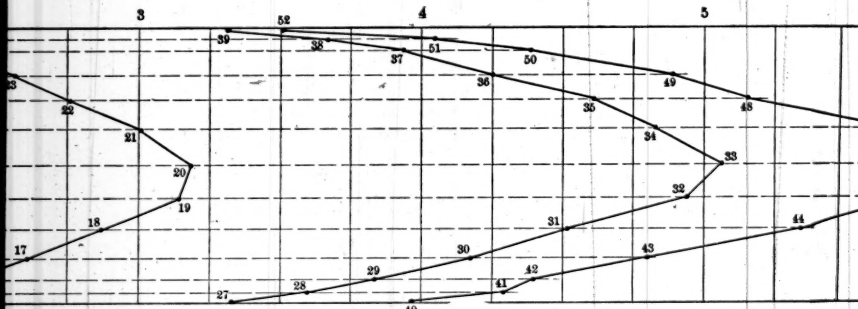


FIG. 2

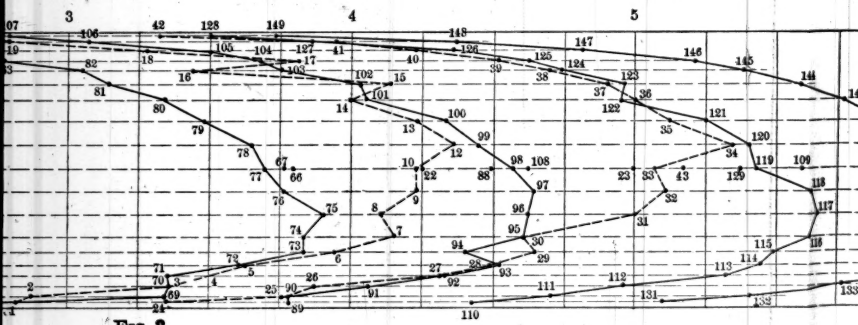


FIG. 3

PLATE IV.
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ON FLOW OF WATER IN PIPES.

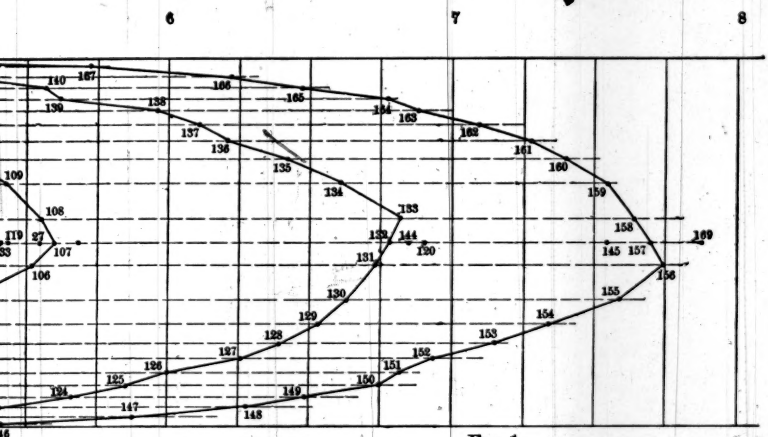

FIG. 1

Traverse No.	87	Position	1 to 26	Military and Army Aves.,				Sept. 13,	1898;	Diameter	16"
" "	88		27 " 39	"	"	"	"	"	"	"	"
" "	89		40 " 65	"	"	"	"	Sept. 18,	1898;	"	"
" "	90		68 " 92	"	"	"	"	"	"	"	"
" "	91		96 " 119	"	"	"	"	"	"	"	"
" "	92		120 " 144	"	"	"	"	"	"	"	"
" "	93		145 " 169	"	"	"	"	"	"	"	"

Tube No. 3.

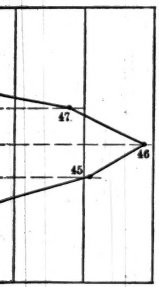

FIG. 2

Traverse No.	A	Positions	1 to 13	Woodward and City Limits,				Dec. 9,	1897;	Diameter	11¼"
" "	B	"	14 " 26	"	"	"	"	"	"	"	"
" "	C	"	27 " 39	"	"	"	"	"	"	"	"
" "	D	"	40 " 52	"	"	"	"	"	"	"	"

Tube No. 2.

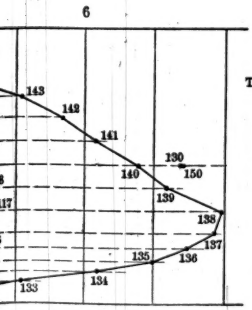

FIG. 3

Traverse No.	94	Positions	1 to 22	Kercheval and South,				Nov. 4,	1898;	Diameter	12.003"*
" "	95	"	23 " 43	"	"	"	"	"	"	"	"
" "	96	"	45 " 65	Reid and South				Nov. 16,	1898;	"	" †
" "	97	"	66 " 86	"	"	"	"	"	"	"	"
" "	98	"	88 " 108	"	"	"	"	"	"	"	"
" "	99	"	109 " 129	"	"	"	"	"	"	"	"
" "	100	"	130 " 150	"	"	"	"	"	"	"	"
" "	101	"	151 " 169	Kercheval and South,				June 6,	1899;	"	11.992"
" "	102	"	170 " 188	"	"	"	"	June 7,	1899;	"	"

* Location 2026 Ft. from nearest up-stream disturbance.
† Location 26 Ft. down-stream from a wide open 12-in. Gate, Bonnet on top.
Tube No. 3

to conclude that the effect of the distortion due to a 3½-ft. radius horizontal curve is not apparent in a vertical traverse 328 ft. or 252 diameters down stream in 16-in. pipe.

The 12-in. traverses on South Street, Plate IV, Fig. 3, which were also taken vertically, show a decided distortion at Reid Street, where the traverse was taken 25.75 ft. below a gate which was examined and found to be wide open, and of which no part projected into the waterway.

As the point of maximum velocity is below the center, it would seem that the effect of the opening into the bonnet of the gate was to retard the flow at the top of the pipe, and corresponds to the observed condition at Superior Street. The traverses at Kercheval Avenue were made on a diameter 2 025.75 ft. from the nearest up-stream obstruction of any kind, which was the same gate, and the preceding 2 000 ft. was probably as carefully laid as any section of pipe ever put into the ground, it being as nearly in a straight line as it was possible for levels and alignment to make it. In spite of these excellent preceding conditions the traverses Nos. 94, 95, 101 and 102 show a distortion of the velocities, the maximum being persistently a little above the centers. As the gate at Reid is the only disturbing cause, it seems that one of two conclusions is to be drawn; (*a*) that the maximum velocity will not remain exactly in the center under even the most favorable conditions; or (*b*) that disturbances of velocities may extend more than 2 000 diameters in 12-in. pipe. The writers consider it very improbable that an unknown obstruction existed in the preceding pipe.

Traverses *A*, *B*, *C* and *D*, Plate IV, Fig. 2, are means of series of traverses taken on 12-in. pipe, the individual observations being three to a position, all taken within a period of 20 seconds, after which the instrument was moved, thus giving a position every 30 seconds. The location was in straight pipe, about 30 ft. down stream from a 16 to 12-in. reducer, which was 10 ft. down stream from a 16 x 6-in. cross. The pipe preceding this cross was ordinarily straight, and 16 ins. in diameter for several thousand feet, the next special being a 16 x 6-in. tee about 250 ft. up stream. These traverses would seem to indicate that the effect of the reducer, if any, was to straighten out or make normal the velocity distribution, if the cross and tee mentioned had distorted it.

When these traverses were made, the water was passed through a

12-in. Venturi meter about 40 ft. down stream from the diameter of the traverse. The discharge was measured through this meter simultaneously with the tube work, by attaching one of the mercury gauges, described under "Differential Gauges," to the up-stream and throat connections, and computing the discharge from the observed difference in head, which was read every 30 seconds.

The results were as follows, Tube No. 2 being used:

Traverse.	Mean V by tube, feet per second.	Mean V by meter, feet per second.	$\frac{V \text{ meter.}}{V \text{ tube.}}$
A=Mean of 3	1.063	1.045	0.983
B= " " 6	2.565	2.408	0.935
C= " " 5	4.148	3.733	0.900
D= " " 6	4.781	4.487	0.938

Tube No. 2 was never rated, except by this comparison.

Abnormal Conditions: Effect of Distortion of the Velocity Curve upon the Discharge, as Measured by Pitot Tube.—Small distortions of the velocity curve do not appear to seriously affect the accuracy of a discharge measurement by traversing with the Pitot tube, although they will, of course, reduce the apparent discharge when it is based on the observation of the velocity at the center. It follows, therefore, that when the latter method is used, the position of the maximum velocity should be established. To establish this, examinations on two diameters approximately at right angles to each other are necessary, at least, if the maximum velocity is eccentric. So far as the present investigation goes it indicates that when the velocity is found to be a maximum at the center on one diameter it may be considered so on all, but until traverses are made on the second diameter where the distortions are great, this cannot be positively affirmed, and the writers would not feel warranted in advancing it as a proposition.

Computations of discharges from the traverses taken under the excessive distortion observed near the curves do not correspond with those obtained by simultaneous traverses at the Leland Street location, nor does the ratio $\frac{V_m}{V_c}$, or even $\frac{V_m}{V_{max.}}$ correspond with their values in the case of normal distribution.

Effect of Form of Tube: The Rating Means and Equations.—At the end of Table No. 3 are given the co-ordinates of the points B, C and A of the straight line deduced from the rating experiments by the

method of centers of gravity, and also the equations of these lines. The arithmetic means of the ratios $\frac{U}{V_c}$ are also given, the experiments marked with an asterisk being rejected in this computation.

It will be observed that all the equations involve a constant probably produced by the variation of the maximum velocity from the center of the pipe. The plottings of the rating experiments show eccentricities in the loci of all the tubes at the low velocities, and, by omitting from the computation one or more of the experiments included, the constants could be reduced. These constants are not, in the opinion of the writers, to any great extent, due to errors of observation, but rather, as indicated above, to minor changes from the general law of distribution within the pipe. How much they are influenced by the form of the tubes is an unsolved problem.

The differences in the coefficients, or in the values of the ratios $\frac{U}{V_c}$, are evidently due in the main to the form of the tubes, and, whether the effects of form be investigated by the arithmetic means or by the line equations, similar differences appear in most cases.

Effect of Form of Tube: Effect of the Point.—When the tubes have been experimented upon with the triple gauge and readings on the circumferential or ring piezometer observed, it is possible to study the effect of the changes in the point separately from those of the tube as a whole. This is possible with Tubes Nos. 3 and 6, and Tubes *C*, *D* and *E*. Table No. 10 gives the experimental results:

TABLE No. 10.—Point and Ring Ratios and Coefficients.

Tube.	$\frac{U}{V_c}$		Coef. $= \frac{U}{V_c} \times \frac{V_m}{V_c} = \frac{U}{0.81\ V_c}$	
	By means.	By line.	By means.	By line.
No. 3	0.799	0.797	0.962	0.983
No. 6	0.784	0.803	0.968	0.992
C	0.821	0.801	1.014	0.989
D	0.874	0.878	1.079	1.084
E	0.834	0.834	1.029	1.029

This shows a range of more than 12% between the point of Tube No. 3 and that of Tube *D*. The point of the former is quite similar to that used by Darcy, who considered its coefficient as ranging from

0.875 to 1.057. The differences in the foregoing values show that an assumption of unity may not be, in all cases, within the limits of allowable error, as Tube *D* is seen to give a result 8% too small, on this assumption. The forms of the points are well shown on Plate I and Fig. 4. The effect of similarity of points is shown by a comparison of Tube No. 3 with Tube No. 6, and Tube *C* with Tube *E*.

Effect of Form of Tube: Effect of the Pressure Orifices.—Having considered the effect of the point, it is now possible to arrive at some conclusion regarding the remainder of the tube, by an examination of the experiments on point and pressure openings which have been made on Tubes Nos. 3, 5 and 6, and Tubes *C* and *D*.

TABLE No. 11.—Point and Pressure Ratios and Coefficients.

Tube.	$\frac{U}{V_c}$		Coef. $= \frac{U}{V_c} \times \frac{V_m}{V_c} = \frac{U}{0.81\ V_c}$	
	By means.	By line.	By means.	By line.
No. 3	0.729	0.720	0.900	0.889
No. 6	0.683	0.700	0.843	0.864
C	0.693	0.703	0.857	0.868
No. 5	0.694	0.6975	0.857	0.861
D	0.684	0.713	0.843	0.881

Here, it is seen that the effect of the remainder of the tube is in some cases to reverse the tendency of the point. Thus, Tube No. 3, which for "Point and Ring" had the lowest coefficient, has the highest for the "Point and Pressure," and Tube *D*, which had a coefficient for "Point and Ring" 12% higher than Tube No. 3, now has one nearly 1% lower. It appears, therefore, that side openings located as in Tube *D* will increase the tube reading considerably, and thus enable it to be used more accurately at low velocities. Comparisons of the forms of Tubes Nos. 6 and 5 and Tube *C* are less reliable because of the area these tubes occupied in the pipe, which is shown in Fig. 4, although the effect of similarity of form between Tube *C* and Tube No. 5 is easily apparent.

Effect of Form of Tube: Effect of the Presence of the Tube on the Distribution of Velocities.—It might easily be supposed that the insertion of a large tube in a small pipe would cause the water to rush past it at a higher velocity and, therefore, with decreased pressure, so that, while the point was reading the normal velocity, the pressure openings

would read low and the apparent velocity would be increased. From the position of the ring openings, more than $\frac{3}{4}$ in. up stream from the point, it seems safe to conclude that this effect would only be apparent on the side openings in the tube itself, and the first conclusions drawn from the rating experiments, when a coefficient of 0.861 was obtained for Tube No. 5, were in conformity with this supposition, for, assuming the rating in still water, which gave 0.859, to be reliable, the evidence seemed to be strongly in its favor. An examination of the distribution of the velocities shown by the traverse with Tube No. 5, in comparison with those made with other tubes, showed clearly that there was a large effect due to the contracted area, and when the pipe comparisons on 30-in. and 16-in. pipe, made under widely varying conditions, showed the ratio of Tube No. 5 to Tube No. 3 to be as $\frac{0.89}{0.75}$ and this was checked by a rating of Tube *C* in a 5-in. pipe which gave for it a coefficient for V of 0.80 and for H_v of 0.64, and a comparison between Tube No. 5 and Tube *C* by *A*, its duplicate, in the 30-in. pipe, it appeared to be conclusive that the effect of the contraction of area was the opposite of that supposed above, and it may be conjectured that the presence of the tube caused a division of the stream with the maximum velocity away from the sides of the instrument.

Effect of Form of Tube: The Effect of Form in the Tank Ratings.—In the tank ratings the effect of form shows itself somewhat differently than in the pipe. The finer lines of Tube No. 3 permit it to pass through the water without much disturbance, while the more "bluff" outlines of Tube No. 6 seem to cause a body of water to be swept ahead of it and perhaps act as a cushion or fender to the effects of the impact against the water beyond. This does not mean a generation of velocity in the rating trough, necessarily, but simply a carrying along of a small mass of water in front of the point.

The sinuous line of Tube No. 5 in the tank rating seems to be accounted for by the possible generation of a wave, of amplitude varying with the velocity, such that at certain speeds its direction past the pressure openings causes them to read high and at others to read low.

Differential Gauges.

Construction.

Mercury Differential Gauges.—In 1894 two differential mercury gauges, Nos. 1 and 2, were constructed for the Engineering Department

of the Detroit Water-Works, the general design of each consisting of a glass U-tube of $\frac{1}{8}$ in. internal diameter, each end of which was connected to a separate $\frac{1}{8}$-in. drawn-brass tube leading down the back of the gauge and terminating at the bottom in shut-off cocks and coupling for hose. The U-tubes were partly filled with mercury, and the pressure was communicated to the tops of these columns by the water in the hose, thence up through the tubes on the back of the gauge, through the top connections and down in the glass tube. As the U-tubes finally were broken in handling, and were found expensive and difficult to replace, the gauges* were rebuilt in 1896, as shown in Plate V, and used in various tests and experiments during 1896–98. The glass tubes have an inside diameter of $\frac{1}{8}$ in., and a standard with a funnel top is provided at one side to enable the gauge to be filled with mercury without disturbing the connections at the top of the glass.

A by-pass is also provided near and below the main shut-off cocks, and by opening this equalizer the mercury should return to a zero reading. The tripods for the various gauges are not shown on any of the illustrations.

Oil Differential Gauges.—For measuring the losses of head in the experimental lengths—except on the 12-in. 2 000-ft. tangent—two special differential gauges were used, designated as Nos. 6 and 7, each consisting of two glass tubes $\frac{5}{32}$ in. internal diameter, and 36 ins. long, mounted upon a board in parallel positions about 3 ins. apart, as shown on Plate V. The tops were connected by a brass casting having a vent or filling opening closed by a cap. At the bottom a brass casting connected each tube to a cock, which terminated in a coupling for the attachment of hose. A scale, graduated to inches and tenths, was mounted between the two tubes.

When these gauges were partly filled with water, the upper part of the tubes containing air, the length of that part of the scale between the tops of the two water columns would give the difference of head at the up and down-stream sides of the instrument, directly, which could be read, under the conditions of the experiments, to $\frac{1}{20}$ in.

This might be considered highly satisfactory by those who are accustomed to the use of Bourdon gauges or mercury columns, but could

* The general design was similar to the one built and used by E. Kuichling, M. Am. Soc. C. E., and described by him in his valuable paper, "On the Loss of Head Resulting from the Passage of Water Through a 24-In. Stop Valve," *Transactions*, Am. Soc. C. E., Vol. xxvi., after which these gauges were in part modeled.

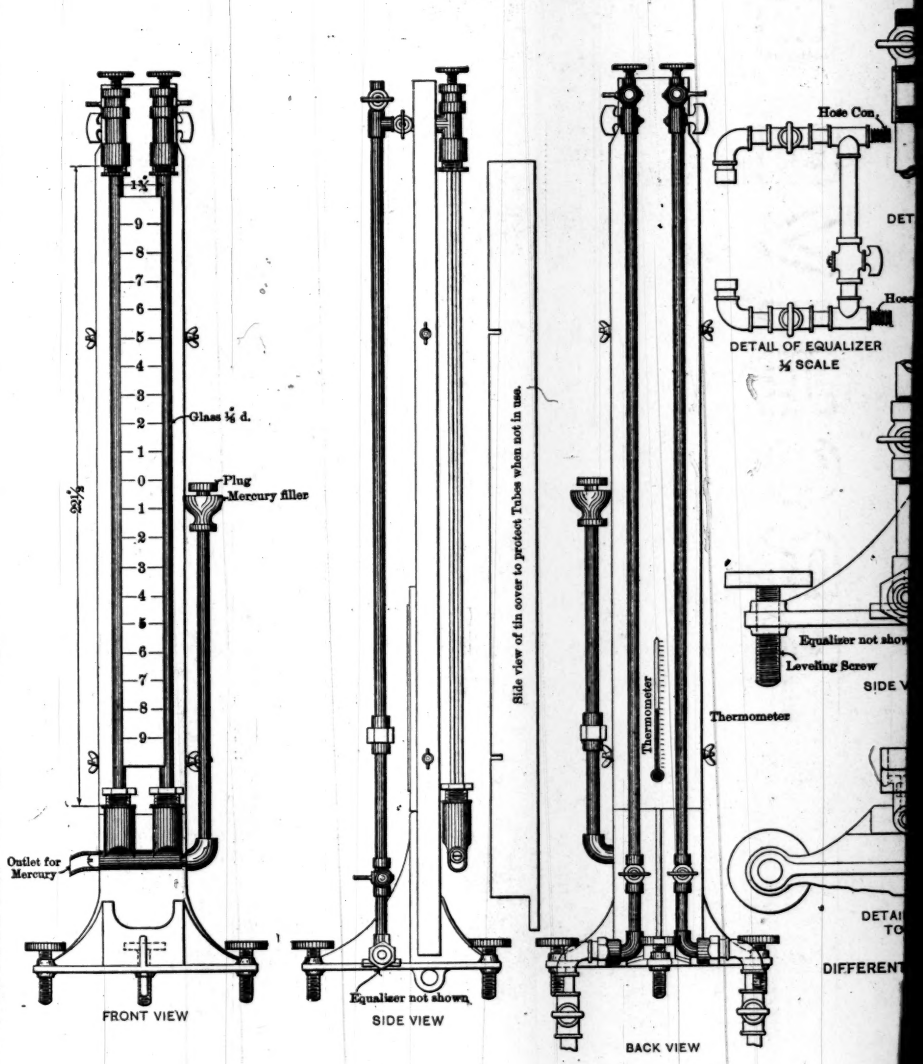
Glass ⅛ d.
Plug
Mercury filler
Outlet for Mercury
FRONT VIEW
Side view of tin cover to protect Tubes when not in use.
Equaliser not shown
SIDE VIEW
Thermometer
BACK VIEW
Hose Con
DETAIL OF EQUALIZER
¼ SCALE
Equaliser not shown
Leveling Screw
SIDE V
Thermometer
DETAI
TO
DIFFERENT

PLATE V
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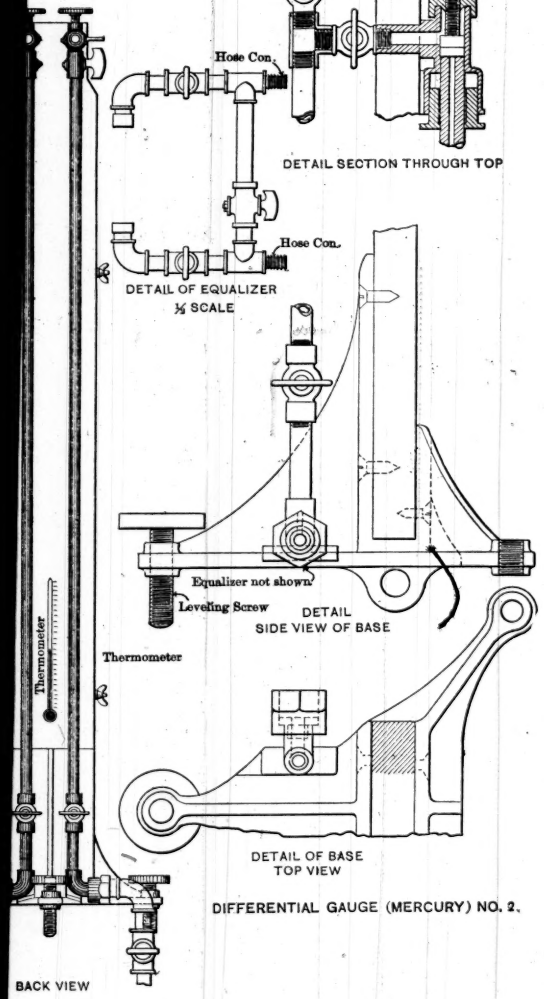
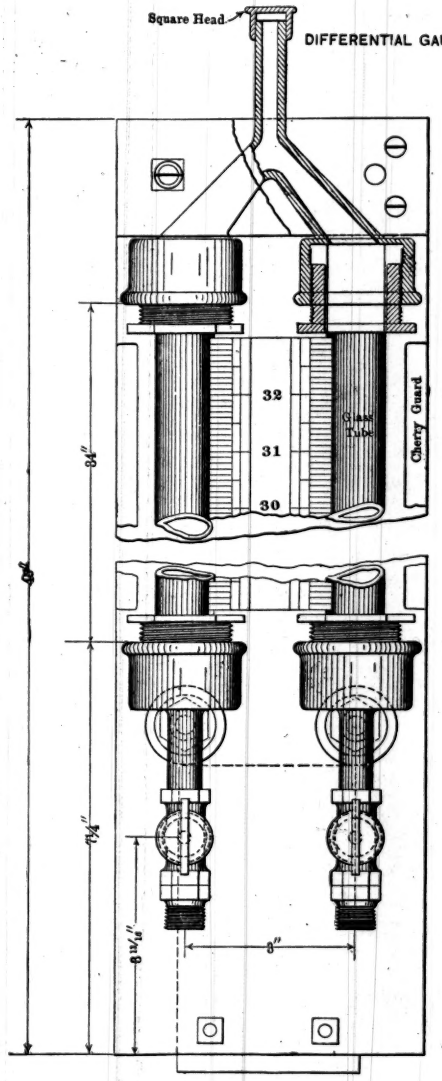
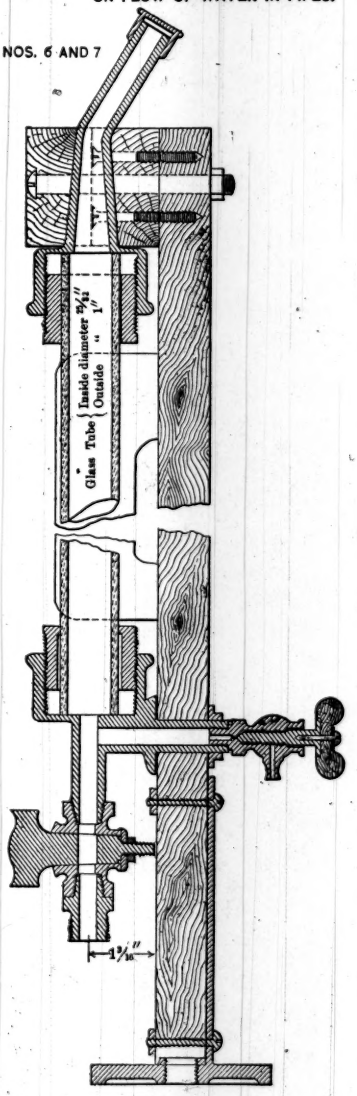
WILLIAMS, HUBBELL AND FENKELL
ON FLOW OF WATER IN PIPES.



DIFFERENTIAL GAUGE (MERCURY) NO. 2.



DIFFERENTIAL GAUGES NOS. 6 AND 7
FRONT VIEW



SECTION

hardly be efficient in measuring the loss of head in 25 ft. of 30-in. pipe at velocities of less than 1 ft. per second.

To increase the fineness of the reading of the instrument, the air in the upper portion of the tube was replaced by oil, whereby the value of one division of the scale in head of water became greatly decreased. With ordinary kerosene the reading of the scale was about five times, and with sperm oil about nine times, the head of water, as will be shown later.

Kerosene, gasoline and sperm oil were tried at various times in these experiments,* and although all gave good results, kerosene was found to be the most satisfactory, for the following reasons: (1) It evaporates very slowly; (2) it does not destroy rubber hose very quickly; (3) it remains perfectly fluid under all ordinary conditions and responds instantly to changes in pressure; (4) it has a convenient specific gravity; (5) it will not explode if used at night near lanterns; and (6) it can be obtained anywhere.

An opening with a cap is provided at the top for filling with oil and allowing the escape of air, and two small cocks near the bottom are used for blowing out the air in the hose, or part of the oil, if found necessary. Much trouble was experienced for two years with oil leaking through the stuffing-boxes, but since using common yellow soap in the packing, in place of tallow, no leak has ever been observed. It may be remarked here that these gauges have given excellent satisfaction under various conditions, and the very fact that they are easy to observe, there being ordinarily no vibration, adds greatly to the value of the results obtained. They can also be so constructed as to indicate with great accuracy H_v as observed with Pitot tubes in gas or air flues, and, by properly graduating the scale, V_m, in gas or air, can be read directly at any time. The form of stuffing-boxes shown is not the best, and on these or any other gauges no valves should be used which are not of approved design and carefully inspected. More trouble has been experienced with poorly constructed plug cocks in the various gauges used than all other leaks combined, although this form of valve is generally preferable to any other style, as it contains no chamber or bonnet which may retain air.

Cornell Triple Gauge.—This gauge is similar to Nos. 6 and 7, with the exception that three glass tubes 24 ins. long, are used instead of

* So far as known to the writers, the use of oil in this manner is original with them.

two 36 ins. long; thus permitting pressure differences to be observed between three points instead of two.

Method of Conducting Experiments.

Apparatus: Piezometers.—With the exception of the four special piezometer castings set in the 12-in. line, and a few taps horizontally in the side of the 16-in. pipe and some of the 30-in. curves, all connections to the various pipes were made with corporation cocks screwed into the top of the main as nearly at right angles to the center line as possible. On the 30-in. line these cocks were $\frac{1}{2}$ in. in diameter, the hole first being drilled from the inside, normal to the axis of the pipe, threaded from the outside, and painted with graphite paint or red lead and oil. On

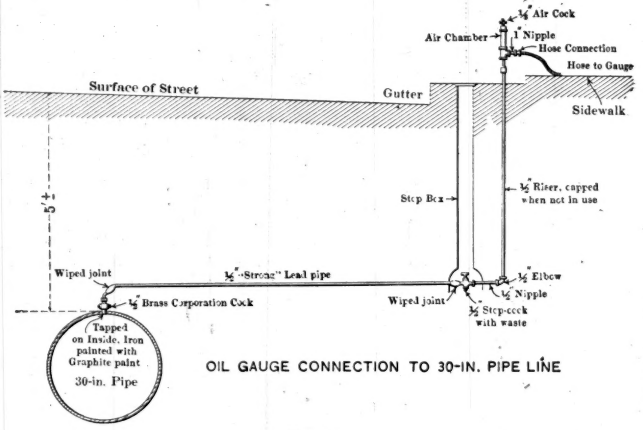


FIG. 16.

the 12 and 16-in. lines, $\frac{5}{8}$-in. cocks were used, the inner end or bottom being plugged, and a $\frac{1}{8}$-in. hole tapped through the plug. They were inserted with a tapping machine. No difference could be detected in the results obtained by the two methods, but it may be observed that if pressures are very low, too small an opening will not allow sufficient flow through the lead pipe and hose to remove the air.

Apparatus: Lead Connections.—Nearly all piezometers, except those at the ends of the 2 000-ft. tangents on the 12-in. line, were connected to the side of the road, as shown in Fig. 16, with $\frac{1}{2}$-in. lead pipe, and provided with a stop and waste cock and a riser to the surface. This

method has proved very satisfactory where pipe is laid in streets subject to more or less travel. In a few cases, where the cock was located in a gate-well, no lead pipe was required.

Apparatus: Air Chambers.—The first operation in connecting the oil gauges consisted in removing the cap from the riser at the side of the street and screwing on an air chamber made of 2-in pipe. The air valve at the top was opened and the water turned on by the stop cock on the lead connection. As soon as all air seemed to have been removed, the necessary amount of hose was attached near the bottom of the air chamber and water turned into it and allowed to run for some minutes, the hose being violently shaken in the meantime and the air valve finally closed. It is impossible to say just how much value these air chambers have. It is obvious that they should be placed at the street main, but as this could not be done, they were placed as close to it as the street would permit. Air chambers were used in all experiments where not otherwise stated.

Apparatus: Hose.—Two sizes of hose were generally used, $\frac{1}{4}$-in. and $\frac{3}{8}$-in., in all, several hundred feet. This hose was divided into 50-ft. lengths, and all couplings were threaded alike and provided with rubber washers to allow of any arrangement of lengths and obtain tight joints. It was thought, perhaps, that different sizes or unsymmetrical distribution of lengths might affect the results, but a careful investigation of readings obtained in the field and experiments made while calibrating the gauges, gave no evidence that such a condition existed. This was true even in the extreme case on South Street, 12-in. line, where the gauge was connected to 1 000 ft. of $\frac{3}{8}$-in. lead pipe on one side and to only 25 ft. of $\frac{1}{2}$-in. lead pipe on the other. In connecting the gauges, however, it was the intention to have the same length of each size on either side of the gauge, and in but few instances, was this rule disregarded. Experience seems to show that $\frac{3}{8}$-in. hose is preferable to $\frac{1}{4}$-in. hose for general work, because it is easier to remove all air, as a higher velocity through it can be obtained if the inlet openings are sufficiently large, and, while small air bubbles will fill the entire cross-section of the $\frac{1}{4}$-in. size, the same bubble would give a clear waterway in the $\frac{3}{8}$-in. hose. In a few cases $\frac{1}{2}$-in. hose was used, and was found to work as well, if not better, than the $\frac{3}{8}$-in. size, but it is not as easily carried from place to place, and is, therefore, not as well adapted to field use.

Manipulations: Connecting Gauges.—The gauge was usually set up about mid way between piezometers, either in the shade of a tree, a building, or an umbrella, except when work was done at night. When satisfied that the hose was clear of air, it was connected to the gauge and the lower valves opened. If the oil had been used for some time, and its line of contact with the water was not sharp and well defined, the cap at the top of the gauge was removed and all oil blown out. The bottom cocks were then closed, and oil fed in at the top of the gauge through a funnel, while water was slowly drawn out of one of the waste cocks near the bottom. When a sufficient quantity of oil had been introduced into the gauge, the top cap was screwed on and the small blow-off cocks closed; the bottom valves were then opened, and, if there was no flow through the main, the dividing surface between oil and water in both tubes should stand at the same level; but such a result was seldom obtained, although the deviation was always very slight.

There appear to be three principal reasons for the gauges not registering zero:

(1) Small leaks anywhere, or even "sweating" joints, are sufficient to give a slight + or — reading. In two instances a minute puncture was found in the hose, but in all cases, all joints above ground were made tight and every precaution taken to eliminate this error. Leaks at the gauge affect the readings more than if near one of the piezometers, and are of more importance when small hose, long lengths of hose, or small piezometer openings are used.

(2) A second reason was air in connections, which was, however, removed as thoroughly as possible. If the gauge gave readings apparently in error, all joints were carefully examined for leaks, and the hose disconnected and shaken for several minutes with full flow through it before being re-connected. Air bubbles in a horizontal pipe apparently affect the results much less than if the same is vertical.

(3) A third reason is a different temperature of water in the connections on either side of the gauge, as when one hose is shaded and the other exposed to the sun; especially in vertical portions of the connection. This error could not be entirely eliminated in the field, especially in work done during the day time.

Before taking any readings with the mercury gauges, they were

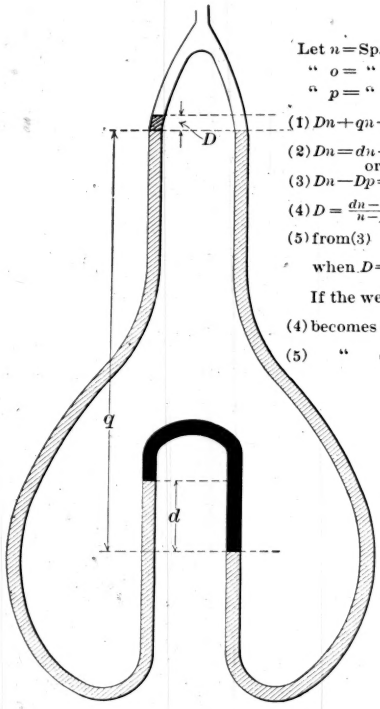
tried with the equalizer open to ascertain if the gauge balanced properly. It may be stated, however, that it is doubtful how much value an arrangement has that permits equalizing a gauge to test its returning to zero; and such an arrangement is not attached to the oil gauges. As a matter of fact, the instrument is only a part of the apparatus, for the tubes, piezometers and other connections should also be included in the test, and, in order to try the gauge properly, after being connected, it should be read with a condition of "no flow," when, if it returns to a level position, it is known that the entire apparatus is working satisfactorily. Equalizing the instrument alone is sometimes detrimental to the results obtained, as readings are used as taken, when corrections should be applied, or they should be rejected altogether.

Manipulations: Observing and Recording.—All differential gauges were read once every 30 seconds. Those containing kerosene were easily read to the nearest 0.05 in.—the equivalent of about 0.00085 ft. of water—and all calibrations and some other observations were taken to the nearest 0.01 in., the scales being divided into inches and tenths and numbered up from 0 at the bottom. The observer recorded his own readings, which were timed by a well-regulated watch, and took temperature readings from a thermometer attached to the gauge.

Two men were required with the mercury gauge, an observer and a recorder, and as there was frequently considerable vibration in the columns of mercury, and as the direction of movement of the tops of the mercury columns must necessarily, at the same moment, be in opposite directions, the readings of the two columns were observed simultaneously at one extreme and then at the other, and an average used as an observation. The scale on Gauge No. 1 is divided into inches and tenths, 0 being at the center, the numbers reading up and down from this point; all below the 0 marked —, and above +. This manner of dividing gauge scales is not satisfactory, as the sign of the readings occasionally confuses the observer, or is omitted from the notes. A thermometer was attached to each gauge, and it was intended that readings should be taken every 10 minutes. This rule was strictly adhered to in all later work, with both oil and mercury differential gauges. The effect of temperature upon the observed results was not at first fully appreciated, and in some of the earlier work the thermometer was not read as frequently as it should have been.

The calibrations show that temperature effects must not be neglected, and in making reductions the temperature has been interpolated where not actually observed. In no case, however, have such interpolated figures been published in the log of experiments given, without calling special attention to the fact.

CALIBRATION OF OIL



Let n = Sp. gr. of water.
" o = " " " oil.
" p = " " " air.

(1) $Dn+qn-dn=qn-do+Dp$

(2) $Dn=dn-do+Dp$

or

(3) $Dn-Dp=dn-do$

(4) $D=\frac{dn-do}{n-p}$ or $D=\frac{n-o}{n-p}$ when $d=1$

(5) from (3) $=\frac{Dn-Dp}{n-o}$ or $d=\frac{n-p}{n-o}$

when $D=1$

If the weight of air Dp is neglected

(4) becomes (6) $D=\frac{n-o}{n}$ when $d=1$

(5) " (7) $d=\frac{n}{n-o}$ " $D=1$

FIG. 17.

Calibrations.

Importance.—Few of the operations connected with experimental hydraulics are of more importance, or are more difficult to perform, than the careful calibrations of the instruments used in the observations, and few have been neglected as much in the past, even by those

whose experiments will be the foundation of important practical formulas in their respective lines for many years to come.

Fully realizing this, some of the first work carried on by the writers was an attempted calibration of part of the gauges, and from that time down to the present much attention has been given this subject, it being their endeavor to apply such coefficients to the field notes that the final results obtained might be as near the true ones as the instruments described and time and resources at their disposal would permit.

Many of the first investigations were unsatisfactory, but the ones given are those applied to most of the experiments; in a few cases, however, where it seemed possible that a different rating should obtain, on account of oil of different specific gravity being used, other values were adopted.

Kerosene.—Attention is called to Fig. 17 for a proper understanding of the principle upon which the oil differential gauge works, and the method adopted for its calibration with kerosene.

Gauges Nos. 6 and 7 were used, one suspended above the other, the lower one partly filled with oil and the upper one connected to it and having its top cap removed. The remainder of the lower gauge, the hose and the top gauge were filled with water, and great care was taken to have no air in either of the gauges or connections. In this condition the columns of oil should stand level. Part of the water in one tube of the upper gauge was then drawn out through the small cock at its bottom until $D = 1$ in. $\pm$, and D and d were observed. Another 1 in. $\pm$ of water was then drawn out, and D and d again observed, and so on until about one-half of the water in the upper gauge had been removed, when, in a like manner, it would be lowered in the other tube until the water near the bottom of the upper gauge read level, and the oil in the lower gauge should have returned to zero.

Eleven observations were usually made, and the temperature of the water was taken as drawn from the upper gauge. Many series of observations have been taken in this manner with oils of various specific gravities, and in Table No. 12 are shown the results obtained from one such calibration, every reading being taken by two observers, each without reference to the other. Column 13 gives the value of $\frac{d}{D}$ as finally corrected, and Column 14 its variations from this one calibration, the air Dp not being considered.

TABLE No. 12.—KEROSENE CALIBRATION, No. 11, IN DETAIL.

Date and time. July 17th, 1899. (1)	Observer. (2)	Gauge No. 6—Water. Left. (3)	Right. (4)	D. (5)	Gauge No. 7—Kerosene. Left. (6)	Right. (7)	d. (8)	Temp. Fahr. G. 6. (9)	G. 7. (10)	$\frac{d}{D} = \frac{N-P}{N-O}$ For each observer. (11)	Average. (12)	$\frac{N}{N-O}$ From final results. (13)	Diff. (14)
12.40 P. M.	A	Gauge full of water to top			18.06	18.06	0.00	.....	.....	....	.....	.....	.....
	B				18.06	18.06	0.00	.....	.....	....	.....	.....	.....
12.50	A	30.35	30.35	0.00	18.06	18.06	0.00	77°	76°	....	.....	.....	.....
	B	30.34	30.34	0.00	18.06	18.06	0.00			....	.....	.....	.....
12.55	A	27.20	28.13	0.98	15.71	20.34	4.63	78°	76°	4.724	4.696	4.7903	+0.057
	B	27.19	28.18	0.99	15.73	20.35	4.62			4.667			
1.00	A	23.62	25.70	2.08	13.05	22.94	9.89	77½°	77°	4.755	4.753	4.7842	+0.024
	B	23.60	25.68	2.08	13.09	22.93	9.84			4.731			
1.04	A	20.59	23.58	2.99	10.81	25.16	14.35	78°	77°	4.799	4.793	4.7842	+0.013
	B	20 55	23.54	2.99	10.82	25.13	14.31			4.786			
1.10	A	17.21	21.26	4.05	8.30	27.60	19·30	78°	77°	4.765	4.773	4.7842	−0.016
1.15	B	17.20	21.23	4.03	8.33	27.60	19.27			4.781			
1.20	A	13.90	18.97	5.07	5.88	30.02	24.14	78°	77°	4.761	4.764	4.7842	−0.006
	B	13.89	18.95	5.06	5.87	30.00	24.13			4.767			
1.25	A	11.46	15.40	3.94	8.49	27.44	18.95	78°	78°	4.809	4.793	4.7782	+0.033
	B	11.44	15.41	3.97	8.47	27.43	18.96			4.776			
1.30	A	9.18	12.13	2.95	10.90	25.09	14.19	78½	78°	4.810	4.793	4.7782	+0.052
	B	9.13	12.11	2.98	10.89	25.07	14.18			4.758			
1.34	A	6.90	8.95	2.05	13.16	22.83	9.67	78½	78½°	4.717	4.747	4.7753	+0.059
1.35½	B	6.90	8.91	2.01	13.20	22.80	9.60			4.776			
1.39	A	4.53	5.55	1.02	15.70	20.42	4.72	79½°	78½°	4.627	4.660	4.7753	−0.066
1.40	B	4.52	5.53	1.01	15.68	20.42	4.74			4.693			
1.45	A	2.12	2.12	0.00	18.08	18.09	0.01	79°	79°	....	.....	.....	.....
	B	2.12	2.12	0.00	18.06	18.06	0.00			....	.....	.....	.....
.....	A	0.72	0.10	−0.62	19.52	16.64	−2.88	79°	79°	4.645	4.731	4.7723	−0.172
.....	B	0.70	0.10	−0.60	19.52	16.63	−2.89			4.817			

In Table No. 13 are given the results obtained in 11 series of calibrations (No. 11 being the one given in detail in Table No. 12), the ones marked "diverging" being the readings obtained when D was increased and "converging" when D was diminished.

It was thought that the effect of D getting larger and then smaller might be apparent in the results—which are shown in Fig. 18, where

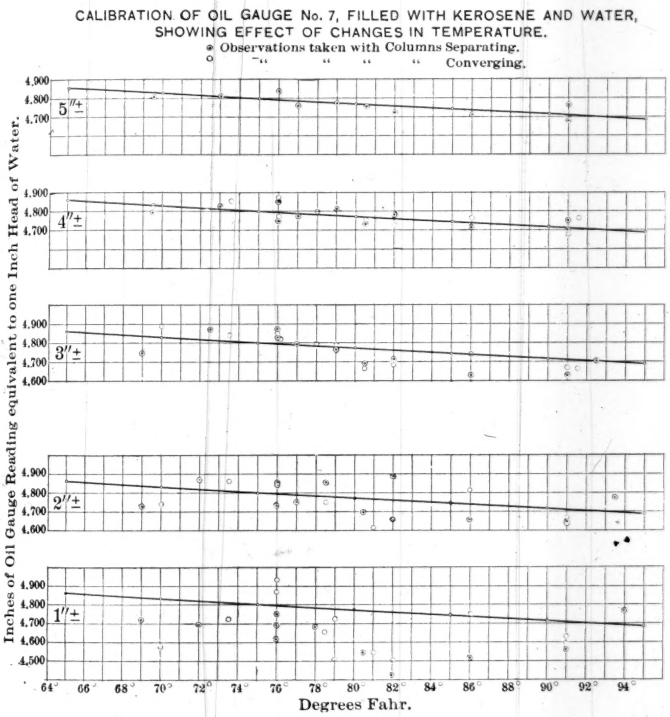


FIG. 18.

all readings are plotted,—and for this reason readings were taken at 1 in. ±, 2 ins. ±, 3 ins. ±, etc., instead of only at 5 ins. ±. Column 42 gives the average results of $\frac{d}{D}$ derived from the readings with D at 3 ins. ±, 4 ins. ±, and 5 ins. ±. It is to be borne in mind that the results presented in this table and obtained by the foregoing calibra-

TABLE No. 13.—KEROSENE CALIBRATIONS.

DIVERGING.

No.	D = 0 ins. ±		D = 1 in. ±				D = 2 ins. ±				D = 3 ins. ±				D = 4 ins. ±				D = 5 ins. ±			
	(1) D.	(2) d.	(3) T. Fahr.	(4) D.	(5) d.	(6) $\frac{d}{D}$	(7) T. Fahr.	(8) D.	(9) d.	(10) $\frac{d}{D}$	(11) T. Fahr.	(12) D.	(13) d.	(14) $\frac{d}{D}$	(15) T. Fahr.	(16) D.	(17) d.	(18) $\frac{d}{D}$	(19) T. Fahr.	(20) D.	(21) d.	(22) $\frac{d}{D}$
1..	0.00	0.00	69°	1.08	5.10	4.722	69°	2.01	9.50	4.722	69°	3.32	15.75	4.744	69½°	3.98	19.10	4.799	69½°	5.00	24.04	4.808
2..	0.00	−0.03	72°	−0.95	4.46	4.695	72°	2.00	9.54	4.870	72½°	3.12	15.22	4.878	73°	3.96	19.10	4.823	73°	5.02	24.16	4.812
3..	0.00	+0.02	76°	1.05	4.85	4.619	76°	1.90	8.99	4.731	76°	2.80	13.73	4.868	76°	3.95	18.76	4.749	76°	5.00	24.18	4.836
4..	0.00	0.00	76°	0.94	4.47	4.755	76°	1.97	9.55	4.848	76°	2.97	14.35	4.831	76°	3.98	19.33	4.857	76°	5.00	24.18	4.836
5..	0.00	0.00	78°	1.00	4.69	4.690	78½°	2.00	9.70	4.850	79°	3.00	14.30	4.767	79°	4.00	19.22	4.805	79°	5.00	23.87	4.776
6..	0.00	0.00	80½°	0.98	4.46	4.551	80½°	2.02	9.48	4.696	80½°	2.99	14.04	4.695	80½°	4.02	19.03	4.734	80½°	5.10	24.27	4.759
7..	0.00	0.00	82°	0.95	4.20	4.421	82°	2.03	9.94	4.891	82°	3.08	14.50	4.708	82°	4.00	19.10	4.775	82°	4.98	23.52	4.723
8..	0.00	0.00	86°	1.00	4.52	4.520	86°	1.96	9.13	4.658	86°	2.96	13.70	4.628	86°	4.20	19.80	4.714	86°	5.20	24.50	4.711
9..	0.00	0.00	94°	1.02	4.87	4.774	93½°	1.98	9.46	4.778	92½°	(3 20)	(15.05)	4.709	91°	4.13	19.50	4.746	91°	5.00	23.73	4.765
10..	0.00	0.00	91°	0.98	4.48	4.571	91°	1.97	9.16	4.644	91°	3.00	13.88	4.627	91°	4.17	19.59	4.698	91°	5.00	23.40	4.680
11..	.....	.....	76°	.....	.....	4.696	77°	.....	.....	4.753	77°	.....	.....	4.792	77°	.....	.....	4.773	77°	.....	.....	4.764

CONVERGING.

No.	D = 4 ins. ±				D = 3 ins. ±				D = 2 ins. ±				D = 1 in. ±				D = 0 ins. ±		AVERAGE OF READING, AT 3 ins. ±, 4 ins. ±, 5 ins. ±. Cols. 11–30.	
	(23) T. Fahr.	(24) D.	(25) d	(26) $\frac{d}{D}$	(27) T. Fahr.	(28) D.	(29) d.	(30) $\frac{d}{D}$	(31) T. Fahr.	(32) D.	(33) d.	(34) $\frac{d}{D}$	(35) T. Fahr.	(36) D.	(37) d.	(38) $\frac{d}{D}$	(39) D.	(40) d.	(41) T. Fahr.	(42) 1 in. Water $D = \frac{d}{D}$
1..	69½°	3.98	19.21	4.827	70°	3.06	14.65	4.788	70°	1.96	9.28	4.734	70°	1.01	4.62	4.574	0.00	0.00	69½°	4.796
2..	73½°	4.05	19.65	4.852	73½°	3.07	14.86	4.840	73½°	2.07	10.06	4.860	73½°	0.98	4.63	4.724	0.00	0.00	73°	4.838
3..	76°	4.18	20.00	4.760	76°	3.03	14.61	4.822	76°	2.05	9.93	4.844	76°	1.07	5.28	4.934	0.00	0.00	76°	4.831
4..	76°	4.03	19.60	4.863	76°	3.18	15.42	4.849	76°	2.06	9.97	4.840	76°	1.00	4.87	4.870				
5..	79°	4.00	19.22	4.805	79°	3.00	14.34	4.780	79°	2.00	9.45	4.725	79°	1.00	4.51	4.510	0.00	0.00	79°	4.787
6..	80½°	4.07	19.31	4.744	80½°	3.12	14.60	4.679	81°	1.96	9.05	4.612	81°	1.00	4.55	4.550	0.00	0.00	80½°	4.728
7..	82°	3.90	18.42	4.723	82°	3.00	14.06	4.687	82°	1.94	9.05	4.660	82°	1.00	4.50	4.500	0.00	0.00	82°	4.726
8..	86°	4.00	19.04	4.760	86°	3.10	14.68	4.735	86°	2.00	9.62	4.810	86°	1.03	4.90	4.757	0.00	0.00	86°	4.713
9..	90½°	4.10	19.50	4.756	91½°	3.04	14.17	4.661	91°	2.03	9.42	4.640	91°	1.03	4.73	4.529	0.00	0.00		
10..	91°	3.87	18.16	4.692	91°	2.99	13.96	4.668	91°	2.05	9.58	4.673	91°	0.97	4.48	4.626	0.00	0.00	91°	4.669
11..	78°	.....	.....	4.793	78°	.....	.....	4.793	78½°	.....	.....	4.747	79°	.....	.....	4.731	.....	.....	77½°	4.783

tions, are affected by the presence of the air Dp which does not exist in the field observations. A correction, therefore, should be applied to eliminate this factor before using them with such data. It was not deemed advisable to include the readings taken with D at 1 in. $\pm$ and 2 ins. $\pm$, owing to the effect of observation errors.

As it was impossible, with the apparatus used, to get the necessary range in temperatures to cover all the experiments, the specific gravity of the kerosene was observed with an oil hydrometer, and, near the bottom of Fig. 19, the readings thus taken are shown with their respective temperatures, and the average computed line.

TABLE No. 14.—Temperature Reduction Curve.

(1) Number.	(2) $T.$ Fahr.	(3) $\frac{N-p}{N-O}$ By hydrometer.	(4) $\frac{N-p}{N-O}$ Actual observed value.	(5) Excess of actual over theoretical.	(1) Number.	(2) $T.$ Fahr.	(3) $\frac{N-p}{N-O}$ By hydrometer.	(4) $\frac{N-p}{N-O}$ Actual observed value.	(5) Excess of actual over theoretical.
1.....	69½°	4.646	4.796	0.150	7.....	82°	4.570	4.726	0.156
2.....	73°	4.624	4.838	0.214	8.....	86°	4.549	4.713	0.164
3 and 4..	76°	4.605	4.831	0.226	9 and 10.	91°	4.523	4.699	0.176
11.....	77½°	4.596	4.783	0.187					9)1.6230
5.....	79°	4.587	4.787	0.200					
6.....	80½°	4.578	4.728	0.150			Mean excess.		0.1803

It was very difficult, with the hydrometer used, to obtain the actual specific gravity, as the angle that the oil makes with the glass is very obtuse, but by always holding an incandescent light at the same angle outside of the glass vessel containing the liquid, and below its surface, and always observing from the same point above this surface, very close relative readings could be taken. A comparison is made, in Table No. 14, between the average line obtained from the hydrometer readings and the results of the average calibrations from Table No. 13, and the latter are found to average 0.1803 in excess of the former. In Table No. 15, from the specific gravity of water, and that of the oil, as found by the hydrometer, the value of D,—Column 5,—is given when the air Dp is taken into consideration, and Column 6 when it is not. As this

air does not exist in field observations, 0.1803 is added to $\frac{d}{D}$ in Column 6, the result, Column 7, giving $\frac{d}{D}$ for 1 in. of water at 60°, which is the coefficient applied to most of the field readings. Fig. 19 shows graphically the operation just described, the dotted lines taking into account the air $D\,p$, and the full lines omitting this.

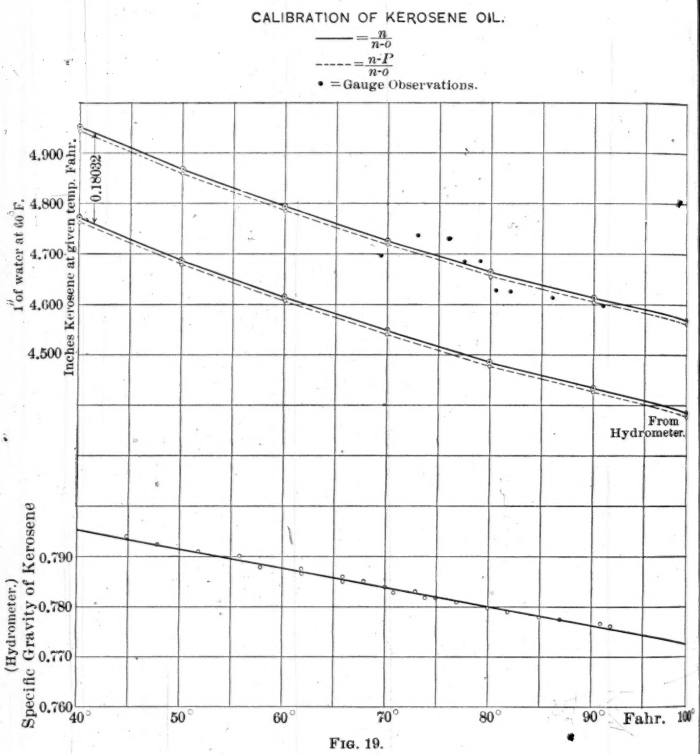


FIG. 19.

Gasoline.—In a manner similar to that described under the kerosene calibrations, the specific gravity of gasoline was observed with a hydrometer and plotted in Fig. 20. Previous to this—September 19th, 1898—four calibrations were made with the differential gauges on the gasoline used on the 12-in. and 16-in. lines, and are as follows :

T. 68° Fahr..... 3.440
" 3.514
T. 67° Fahr..... 3.506
" 3.457
4)13.917
T. 67½° Fahr..... 3.4792 = $\frac{n-p}{n-o}$

Using the rate of change per degree, as found by the hydrometer and the actual calibrations as given above, Table No. 16 was constructed, with no allowance made for the air $D\,p$.

TABLE No. 15.—KEROSENE CALIBRATIONS.

Temp. Fahr.	Sp. Gr. Ker. Hydrometer	Sp. Gr. Water = N.	Sp. Gr. Air.	From Columns (2), (3) and (4).		Corrected curve = Column (6) + 0.1808.	Multiplier to reduce kerosene to 60° Fahr.
				$\frac{N-P}{N-O}$	$\frac{N}{N-O}$		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
40°.....	0.7955	1.00092	0.00127	4.8654	4.8725	5.0528	0.96875
50°.....	0.7917	1.00068	0.00125	4.7824	4.7884	4.9687	0.98517
60°.....	0.7879	1.00000	0.00123	4.7090	4.7147	4.8950	1.00000
61°.....						4.8879	1.00143
62°.....						4.8809	1.00285
63°.....						4.8740	1.00427
64°.....						4.8671	1.00568
65°.....						4.8604	1.00708
66°.....						4.8537	1.00848
67°.....						4.8471	1.00986
68°.....						4.8406	1.01123
69°.....						4.8341	1.01259
70°.....	0.7840	0.99895	0.00120	4.6413	4.6474	4.8277	1.01394
71°.....						4.8213	1.01529
72°.....						4.8150	1.01664
73°.....						4.8088	1.01798
74°.....						4.8026	1.01931
75°.....						4.7964	1.02063
76°.....						4.7903	1.02194
77°.....						4.7842	1.02323
78°.....						4.7782	1.02450
79°.....						4.7723	1.02574
80°.....	0.7801	0.99762	0.00118	4.5809	4.5862	4.7665	1.02696
81°.....						4.7608	1.02818
82°.....						4.7552	1.02939
83°.....						4.7497	1.03059
84°.....						4.7443	1.03177
85°.....						4.7390	1.03294
86°.....						4.7338	1.03409
87°.....						4.7285	1.03523
88°.....						4.7233	1.03636
89°.....						4.7182	1.03748
90°.....	0.7763	0.99603	0.00116	4.5277	4.5329	4.7131	1.03859
91°.....						4.7080	1.03969
92°.....						4.7031	1.04078
93°.....						4.6982	1.04186
94°.....						4.6934	1.04292
95°.....						4.6887	1.04397
96°.....						4.6841	1.04500
97°.....						4.6795	1.04602
98°.....						4.6750	1.04702
99°.....						4.6706	1.04802
100°.....	0.7725	0.99410	0.00114	4.4809	4.4860	4.6663	1.04901

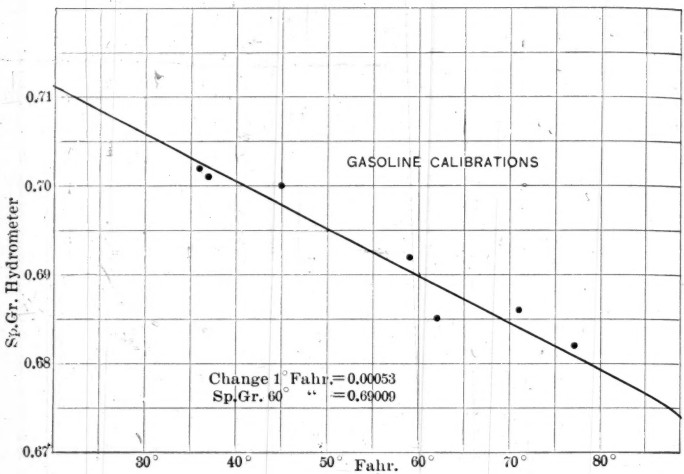


FIG. 20.

TABLE No. 16.—GASOLINE CALIBRATIONS.

$T.$, Fahr.	Specific Gravity.		1 in. Water. $D = \frac{N-P}{N-O}$	$T.$, Fahr.	Specific Gravity.		1 in. Water. $D = \frac{N-P}{N-O}$	$T.$, Fahr.	Specific Gravity.		1 in. Water. $D = \frac{N-P}{N-O}$
	Water $N.$	Gasoline $O.$			Water $N.$	Gasoline $O.$			Water $N.$	Gasoline $O.$	
(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
40°	1.00089	0.72647	3.6470	54°			3.5560	68°			3.4760
41°			3.6405	55°			3.5500	69°			3.4705
42°			3.6336	56°			3.5440	70°	0.99893	0.71057	3.4650
43°			3.6259	57°			3.5380	71°			3.4598
44°			3.6202	58°			3.5320	72°			3.4546
45°			3.6135	59°			3.5260	73°			3.4494
46°			3.6068	60°	1.00000	0.71587	3.5200	74°			3.4442
47°			3.6001	61°			3.5145	75°			3.4400
48°			3.5934	62°			3.5090	76°			3.4398
49°			3.4867	63°			3.5035	77°			3.4296
50°	1.00067	0.72117	3.5800	64°			3.4980	78°			3.4294
51°			3.5740	65°			3.4925	79°			3.4188
52°			3.5680	66°			3.4870	80°	0.99767	0.70527	3.4130
53°			3.5620	67°			3.4815				

TABLE No. 17.—CALIBRATIONS OF SPERM OIL.

DATE.	TEMPERATURE, IN DEGREES, FAHRENHEIT.		$\frac{d}{D}$ INCHES SPERM FOR 1 IN., WATER.		
	Air.	Water.	Readings.	Average.	
1898.					
July 18..	81	.....	8.635	.....	Gauge had stood since morning 7 hours in draught in office.
" 18..	80½	.....	8.74	8.688	
" 19..	76	.....	8.908	.....	In cellar 7 hours.
" 19..	76½	.....	8.840	8.840	Same oil as above.
" 19..	77	.....	8.772	.....	
" 21..	71	.....	8.980	.....	Standing in basement since last calibration.
" 21..	71¼	.....	8.792	8.7915	Same oil specks.
" 21..	71½	.....	8.716	.....	
" 21..	72	.....	8.728	.....	
" 25..	74	.....	8.783	.....	Very careful.
" 25..	76½	.....	8.824	8.810	"
" 25..	77	.....	8.823	.....	"
Aug. 6..	72½	.....	8.95	.....	
" 6..	72½	.....	8.928	8.939	Gauge probably refilled during following experiment.
" 10..	68	.....	8.884	.....	Same as preceding observations on Brady tangent.
" 10..	70	.....	8.884	8.8893	
" 10..	70	.....	8.900	.....	
Sept. 23..	67	.....	8.993	.....	
" 23..	67	.....	8.90	8.9465	
Apr. 21..	35	.....	11.257?	.....	Calibrated out doors.
Sept. 17..	67⅛	.....	8.814	.....	
" 17..	67⅛	.....	8.821	.....	
1899.					
Mar. 9..	84	52–61	8.612	.....	Air had been at 70° for 3½ hours; water had been ½ hour from tap.
" 10..	80	79	8.57	.....	Air for 4 hours 80°; water not changed, and new water added, heated to 79°.
" 11..	60–60	61	9.05	.....	Room open all night; calibrations with window open.
" 11..	71–70	63–64	8.7626	.....	From 10 to 10.30 temperature of air was measured from 62° to 70°.

Sperm Oil.—Comparatively few field observations were made with sperm oil, and part of those were of such a nature that relative rather than actual results were sought. The object of its use was to obtain (by reason of its greater specific gravity, which approaches nearly to that of water itself, so that H_f in water is therefore greatly multiplied in the oil readings) more precise results than were possible with kerosene or gasoline. Although it may be used to advantage in the laboratory, it was discontinued in the field because it came to rest too slowly.

Twenty-five calibrations have been made, at various times and temperatures, which, as observed, are presented in Fig. 21, but no table has been prepared giving values for each degree of temperature.

The first twenty of these calibrations, and those of kerosene in 1898, were made by using two reservoirs made from 2-in. iron pipes

carefully turned off at their tops in a lathe, fastened in parallel positions on a board, and having a common inlet and separate outlets at the bottom, any or all of which could be shut off. The gauge to be calibrated was connected to the outlets of the two reservoirs. The zero was obtained by equalizing the columns when the two reservoirs were level and the water flowing freely over the tops of each, and these brought to rest by shutting off the inlet. The difference in head was then obtained by lowering the water in one reservoir and measuring its fall by a float consisting of a glass test tube loaded with shot and carrying a scale which was read across a straight-edge. The results would evidently be less accurate than those obtained by the method used with kerosene, already described.

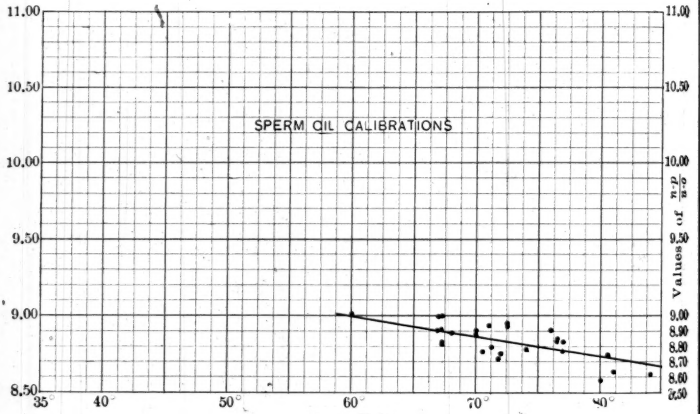


FIG. 21.

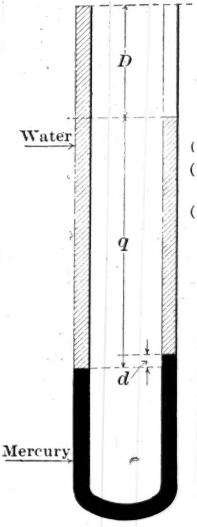
Mercury.—In the autumn of 1897 arrangements were made for calibrating the mercury that had been used previously in the various experiments. Gauge No. 1 was connected with a $\frac{3}{8}$-in. hose, which was led to the outside of the building and attached to a $\frac{1}{2}$-in. glass tube about 4 ft. long fastened vertically to the wall. Water was supplied through a hose from a second-story basin tap, and the corresponding heads of mercury in the gauge recorded for various heights of water in the glass tube.

Difficulty was encountered, however, as (1) no way was provided for balancing the mercury columns with two columns of water reading

zero, and thereby proving that the entire apparatus was in perfect working order, although when the equalizer on the gauge was applied, a reading of nearly zero was generally obtained; (2) the water as received from the tap contained large numbers of minute air bubbles, which were supposed to settle out in the glass tube before readings were taken.

In all, twenty-two series were taken, when it was found that the general average of the results obtained gave the mercury experimented upon a specific gravity 2% above that of pure mercury. It was then

CALIBRATION OF MERCURY



Let n = Sp. gr. of water.
" m = " " " mercury.
" p = " " " air
(1) $Dn + qn = dm + qn - dn + Dp$
(2) $Dn - Dp = dm - dn$.
If the weight of air Dp is neglected
(3) $Dn = dm - dn$
let $d = 1''$ as observed.
$Dn = m - n$ = Equivalent of 1" mercury in inches head of water at D

FIG. 22.

too late in the fall to continue the experiments, and, since then, it has not been thought necessary to construct new apparatus.

In January, 1901, the mercury which had been used was divided into two samples and tested in the experimental department of the laboratory of Parke, Davis & Co., at Detroit, and the specific gravity of each at 60° Fahr. found to be 13.571, which value has been used for reducing the mercury gauge observations.

Fig. 22 shows the principle by which mercury indicates the differ-

ence, Dn, in pressure between two columns of water. As the air, Dp, does not exist in actual field practice, Equation (3) shows the equivalent of 1 in. of mercury at d, in inches of water at D, and variations in the diameters of the tubes will not change the ratios here given. The cubical expansion of mercury is taken as 0.0001 per degree Fahrenheit, and in Table No. 18 are given the values of D for 1 in. of mercury at d for each degree of temperature.

TABLE No. 18.—MERCURY CALIBRATIONS.

$T.$, Fahr.	Specific Gravity.		1 in. Mercury $d = M - N.$	$T.$, Fahr.	Specific Gravity.		1 in. Mercury $d = M - N.$	$T.$, Fahr.	Specific Gravity.		1 in. Mercury $d = M - N.$
	Mercury $M.$	Water $N.$			Mercury $M.$	Water $N.$			Mercury $M.$	Water $N.$	
(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
39°	.....	.....	12.59851	50°	13.585	1.00067	12.58433	61°	.....	.....	12.56971
40°	13.598	1.00089	12.59711	51°	.....	.....	12.58297	62°	.....	.....	12.56842
41°	.....	.....	12.59583	52°	.....	.....	12.58164	63°	.....	.....	12.56713
42°	.....	.....	12.59455	53°	.....	.....	12.58041	64°	.....	.....	12.56584
43°	.....	.....	12.59327	54°	.....	.....	12.57898	65°	.....	.....	12.56455
44°	.....	.....	12.59199	55°	.....	.....	12.57765	66°	.....	.....	12.56326
45°	.....	.....	12.59071	56°	.....	.....	12.57632	67°	.....	.....	12.56197
46°	.....	.....	12.58943	57°	.....	.....	12.57499	68°	.....	.....	12.56068
47°	.....	.....	12.58815	58°	.....	.....	12.57366	69°	.....	.....	12.55939
48°	.....	.....	12.58687	59°	.....	.....	12.57233	70°	13.557	0.99893	12.55810
49°	.....	.....	12.58559	60°	13.571	1.00000	12.57100				

Reduction of Observations.

Correction for Diameter.—In order to reduce observed results in the same nominal size of pipe to the same basis of comparison, a correction should be made for variations in diameter, and is determined as follows:

Let d = diameter of standard pipe;
d_1 = diameter of any other pipe;
a = area of pipe of diameter d;
a_1 = area of pipe of diameter d_1;
v = velocity in pipe of diameter d;
v_1 = velocity in pipe of diameter d_1;
H_f = frictional loss in pipes of diameter d, of unit length:
h_f = frictional loss in pipes of diameter d_1, of unit length;
Q = constant discharge for all diameters;

$$\left.\begin{aligned} r &= \frac{d}{4} \\ r_1 &= \frac{d_1}{4} \end{aligned}\right\} = \text{hydraulic radius};$$

$$s = \frac{H_f}{\text{unit length} = l};$$

$$s_1 = \frac{h_f}{\text{unit length} = l}.$$

$$\left.\begin{aligned} Q &= a\,v = a\,c\sqrt{rs} \\ Q &= a_1\,v_1 = a_1\,c_1\sqrt{r_1\,s_1} \end{aligned}\right\} \text{(Chezy formula.)}$$

Assume, for small changes of diameter, $c = c_1$, then

$$a_1\sqrt{r_1\,s_1} = a\sqrt{rs};$$

but

$$a = \pi\left(\frac{d}{2}\right)^2$$

therefore,

$$\pi\left(\frac{d}{2}\right)^2\sqrt{rs} = \pi\left(\frac{d_1}{2}\right)^2\sqrt{r\,s.}$$

Substituting for r and s, and reducing,

$$d^2\sqrt{\frac{d}{4}}\sqrt{\frac{H_f}{l}} = d_1^2\sqrt{\frac{d_1}{4}}\sqrt{\frac{h_f}{l}}$$

$$\sqrt{d^5}\sqrt{H_f} = \sqrt{d_1^5}\sqrt{h_f}$$

$$d^5\,H_f = d_1^5\,h_f$$

or,

$$H_f = \left(\frac{d_1}{d}\right)^5 h_f.$$

From the last equation it is seen that the frictional loss in pipes is inversely proportional to the fifth power of their diameters, when c is constant, not otherwise; from which it follows that, for a constant loss of head, $Q \propto \sqrt{d^5}$. Owing to the method of derivation, this relation between Q and d is true only for conditions in which the value of c in the Chezy formula remains constant.

Thirty-Inch Cast-Iron Pipe.

Description of Line.

Conditions.—The line of 30-in. pipe experimented upon (Fig. 23) is situated about 4 miles from the pumping station, and is about 1 mile in length. It was laid in 1896–97. The route was selected to conform approximately to a contour line 50 ft. above the mean surface of the Detroit River, and, at the same time, avoid paved streets as much as possible.

The line thus located contained ten 90° turns, alternately to the right and left.

The maximum depth of cover was 7 ft , and the minimum depth 3 ft. 10 ins., the pipe being laid on a level grade.

Surveys.—An accurate transit line was run over the route selected, before construction was commenced, and all points of intersection were carefully set and preserved during the work. The pipe was laid as nearly on these lines as could be done with the eye, it being possible to obtain as accurate alignment for water pipe between certain points as for railway tracks, if the same care be taken. All horizontal measurements were made with a new 100-ft. steel tape of the best quality, which was not broken or repaired during the entire work; no other tape was used. All angles experimented upon were made exactly 90°, the very slight deviation therefrom being provided for in the joints at the junction of the 30-in. and 42-in. lines beyond the last section experimented upon.

It was decided to lay curves of different radii, and to make provision for observing the loss of head in each. The pipe line as completed is shown in Fig. 23, and the net lengths of tangents and curves are as shown in Table No. 19, and the characteristics of it are presented in Table No. 21.

TABLE No. 19.

Scott St. tangent*	12.42
90°—compound curve	62.92
Orleans St. tangent	193.59
90°—compound curve	62.83
Hale St. tangent	374.71
90°—60-ft. radius curve	94.24
Riopelle St. tangent	73.58
90°—40-ft. radius curve	62.84
Rowena St. tangent	441.17
90°—25-ft. radius curve	39.28
Russell St. tangent	136.79
90° small compound curve	22.25
Brady St. tangent	392.62
90°—15-ft. radius curve	23.56
Rivard St. tangent	1 032.33
90°—10-ft. radius curve	15.70
Alexandrine St. tangent	1 265.45
90°—6-ft. radius curve	9.42
Alley tangent	880.19
Total length	5 195.89

*The total length of Scott St. tangent, laid in 1880, is 1 991.2 ft., more of less, of which only 12.42 ft. are included in experimental work, this length being relaid.

Levels.—To establish bench-marks, a line of precise levels was run from U. S. B. M. "Detroit," at the foot of Mt. Elliott Avenue, to and along the proposed route, returning to the place of beginning, a total distance of 7 miles, with a closing error of 0.031 ft. After the pipe was laid and the joints caulked, but before covering, levels were taken on the top of each individual piece of pipe, curve, or special casting, near the spigot end. For this purpose a 15-ft. self-reading level rod was used.

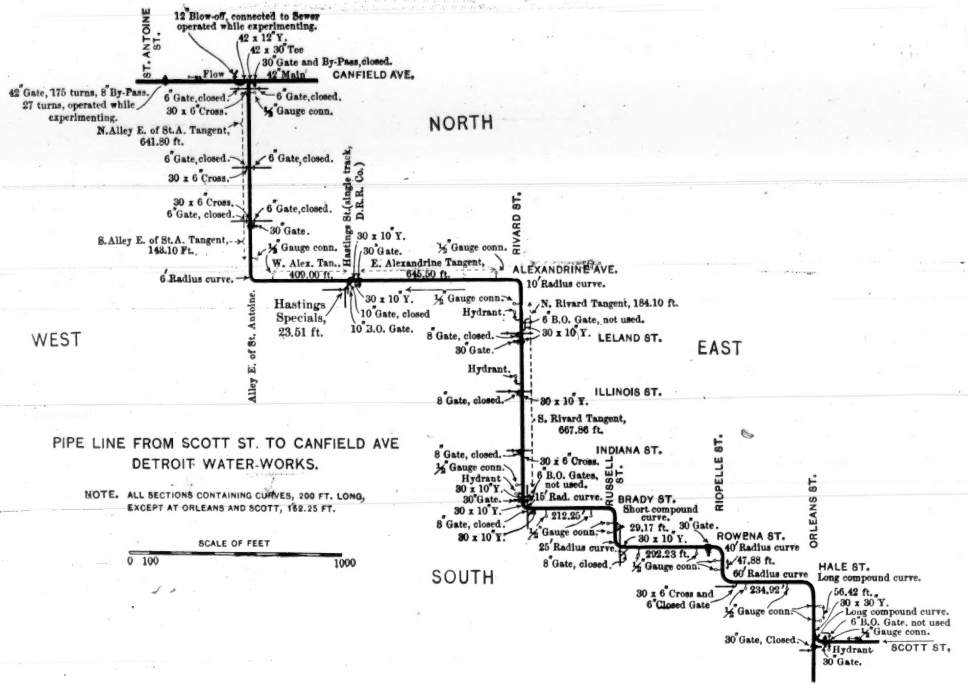

ST. ANTOINE ST.
12" Blow-off, connected to Sewer operated while experimenting.
42 x 12" Y.
42 x 30" Tee
30" Gate and By-Pass, closed.
Flow
42" Main
CANFIELD AVE.
42" Gate, 175 turns, 8" By-Pass, 27 turns, operated while experimenting.
6" Gate, closed.
30 x 6" Cross.
½" Gauge conn.
6" Gate, closed.
N. Alley E. of St. A. Tangent, 641.80 ft.
6" Gate, closed.
30 x 6" Cross.
30 x 6" Cross.
6" Gate, closed.
6" Gate, closed.
30" Gate.
S. Alley E. of St. A. Tangent, 148.10 Ft.
½" Gauge conn.
W. Alex. Tan., 409.00 ft.
6 Radius curve.
Alley E. of St. Antoine.
Hastings St. (single track, D. R. R. Co.)
30 x 10" Y.
30" Gate.
½" Gauge conn.
E. Alexandrine Tangent, 645.50 ft.
RIVARD ST.
ALEXANDRINE AVE.
10 Radius curve.
Hastings Specials, 23.51 ft.
30 x 10" Y.
10" Gate, closed
10" 2.0. Gate.
½" Gauge conn.
Hydrant.
N. Rivard Tangent, 184.10 ft.
6" B.O. Gate, not used.
8" Gate, closed.
30 x 10" Y.
LELAND ST.
30" Gate.
Hydrant.
ILLINOIS ST.
8" Gate, closed.
30 x 10" Y.
S. Rivard Tangent, 667.86 ft.
INDIANA ST.
8" Gate, closed.
30 x 6" Cross.
½" Gauge conn.
Hydrant.
6" B.O. Gates, not used.
30 x 10" Y.
15 Rad. curve.
30" Gate.
RUSSELL ST.
BRADY ST.
30 x 10" Y.
Short compound curve.
8" Gate, closed.
30 x 10" Y.
212.25'
29.17 ft.
30" Gate.
½" Gauge conn.
30 x 10" Y.
ROWENA ST.
25 Radius curve.
202.23 ft.
40 Radius curve
8" Gate, closed.
½" Gauge conn.
47.88 ft.
60 Radius curve
RIOPELLE ST.
ORLEANS ST.
HALE ST.
Long compound curve.
56.42 ft.
30 x 6" Cross and 6" Closed Gate
234.92'
30 x 30" Y.
½" Gauge conn.
Long compound curve.
6" B.O. Gate, not used
½" Gauge conn.
30" Gate, Closed.
Hydrant
30" Gate.
SCOTT ST.
NORTH
WEST
EAST
SOUTH
PIPE LINE FROM SCOTT ST. TO CANFIELD AVE
DETROIT WATER-WORKS.
NOTE. ALL SECTIONS CONTAINING CURVES, 200 FT. LONG, EXCEPT AT ORLEANS AND SCOTT, 182.25 FT.
SCALE OF FEET
0 100 1000

FIG. 23.

In all, 481 elevations were taken on the pipe, the maximum elevation being 621.30, and the minimum 620.78 ft., above mean tide in New York Harbor, making the difference in elevation between the highest and lowest points 0.52 ft. For the variation in elevation existing in any one experimental section, attention is called to Column 10, Table No. 21. As great care was taken in laying, there is only a very slight difference of elevation between any two consecutive lengths.

Diameter of Pipe.—The diameter of each 12-ft. length of pipe was measured in eight places, the measurements being taken in pairs at right angles to each other, at each end, and at a distance in from each end as far as could be conveniently reached (usually about 12 ins.) and read. For this purpose a 2-ft. two-fold, boxwood rule, having a 12-in. brass caliper sliding in one leg, and graduated to inches and sixteenths was used.

The open rule was firmly grasped near the center and one end placed against the pipe, when the slide could be extended by a movement of the thumb and slightly waved to find the greatest diameter, where it was read without removing. In this manner, diameters could be taken quite rapidly, one man calipering and another recording. Curves, short pieces of pipe, and special castings were also calipered. In all, more than 3 600 diameters were measured. Table No. 20 shows the method of keeping the notes.

TABLE No. 20.

No. of pipe.	Length.	Spigot.			Bell.			Weight.	Remarks.
		End.	Dist.	Diam.	End.	Dist.	Diam.		
181.....	12 ft. 2 ins.	30 30	12	30 30	$29\frac{7}{8}$ $29\frac{15}{16}$	12	$29\frac{13}{16}$ $29\frac{15}{16}$	3725	Somewhat uneven.
140.....	12 " 2 "	30 30	12	$30\frac{1}{16}$ $30\frac{1}{16}$	$29\frac{15}{16}$ $29\frac{13}{16}$	12	$30\frac{1}{16}$ $29\frac{13}{16}$	3505	
139.....	12 " 2 "	$29\frac{13}{16}$ 30	12	$30\frac{1}{16}$ $30\frac{1}{16}$	$29\frac{7}{8}$ $29\frac{13}{16}$	12	30 30	3520	

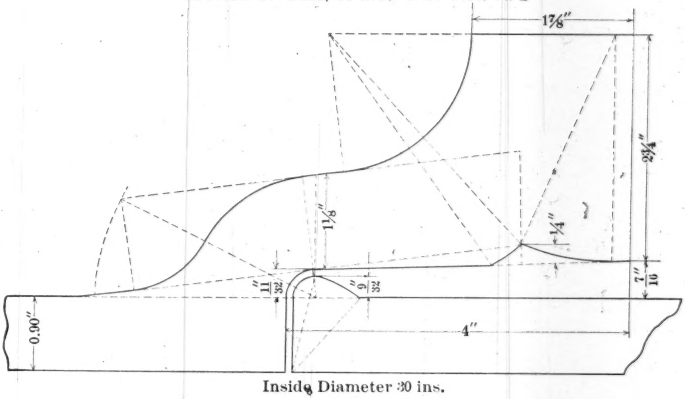
Columns 5, 6 and 7 of Table No. 21 give the mean, maximum and minimum diameters for each section experimented upon. The mean diameter is that used in all computations.

Description of Pipe.—The pipe used in the construction of the line was ordinary cast-iron water pipe. The straight pipe was cast to conform as nearly as possible with the dimensions shown in Fig. 24, the

weight averaging about 3 750 lbs. for each 12-ft. length. After being thoroughly cleaned, each pipe was heated over a fire in a closed apartment to about 350° Fahr., and was then dipped in coal-tar and allowed to remain about one minute, after which it was removed and allowed to cool. It was then subjected to a water pressure of 200 lbs. per square inch. The interiors of the finished pipe were quite smooth, the coating having a glassy or shiny appearance.

The design for the bells of curves was the same as for straight pipe, the body of the curve having a thickness of $\frac{7}{8}$ in. After being thoroughly cleaned, they were painted with two coats of graphite paint. The 6-and 15-ft. radius curves were not as smooth as ordinary

SECTION OF BELL, 30-INCH CAST-IRON PIPE



Inside Diameter 30 ins.

FIG. 24.

pipe, and, although dressed off fairly well, still contained many rough spots. One of the 6-ft. radius curves showed a crack during the test in the ditch, but was repaired with two wrought-iron bands. Two 15-ft. radius curves were condemned on the ground because of blow-holes and replaced with smoother ones. The 25 and 90-ft. radius curves were somewhat better, but were rougher than common pipe. The 10 and 60-ft. radius curves were good castings, and were fully as smooth as the ordinary pipe, while the 40-ft. curves were still better. One form of roughness, chiefly apparent in the 90-ft. radius curves, was caused by the contraction of the cores between the core ribs, thus forming depressions, in rings about $\frac{1}{16}$ in. deep, extending for about 120°

X.	Russell tangent		—29.17	No specials	29.953	30.163	29.750	32	3	0.08
XI.	Tangent	85.60								
	Curve, 15°–25 ft. rad.	6.54								
	" 30°–15 "	7.86		Testing box. No specials	29.822	30.125	29.656	160	21	0.10
	" 45°–10 "	7.85								
	Tangent	92.15								
		200.00								
XII.	Brady tangent		212.25	No specials	29.938	30.188	29.688	144	17	0.08
XIII.	Tangent	88.22		Sleeve open 5 ins. 3–30 × 10-in. Y's. Gate. 30 × 6-in. cross	29.962	30.281	29.688	162	24	0.16
	Curve, 90°–15 ft. rad.	23.56								
	Tangent	88.22								
		200.00								
XIV.	South Rivard tangent		667.86	30 × 8-in. cross. 2–30 × 6-in. Tees. 30 × 10-in. Y.	29.947	30.047	29.848	442	63	0.24
XV.	North Rivard tangent		184.10	Gate. 2–30 × 10-in. Y's. 30 × 6-in. cross	29.970	30.016	29.844	120	19	0.22
XVI.	Tangent	92.15		2 sleeves. 2 testing boxes.	29.946	30.219	29.750	164	24	0.25
	Curve, 90°–10 ft. rad.	15.70								
	Tangent	92.15								
		200.00								
XVII.	E. Alexandrine tangent		645.50	No specials	29.956	30.305	29.727	405	54	0.19
XVIII.	Specials		23.51	Gate. 2–30 × 10 in. Y's	30.068	30.086	29.844	28	6	0.02
XIX.	W. Alexandrine tangent		409.00	No specials	29.965	30.078	29.867	238	36	0.11
XX.	Tangent	95.29								
	Curve, 90°–6 ft. rad.	9.42								
	Tangent	95.29		No specials	29.951	30.125	29.813	140	18	0.10
		200.00								
XXI.	S. Alley tangent		143.10	30-in. sleeve. Gate	29.962	30.033	29.867	94	14	0.14
XXII.	N. Alley tangent		641.80	2–30 × 6-in. crosses	29.989	30.086	29.759	408	56	0.35
			5 195.89							

TABLE No. 21.

	Experimental section. (1)	Detailed length. (2)	Net length. (3)	Special obstructions. (4)	Average diameter. (5)	Mean diameter of max. cross-section. (6)	Mean diameter of min. cross-section. (7)	No. caliperings. (8)	No. joints. (9)	Max. variation in levels. (10)
I.	Tangent	12.42								
	Curve, 15°-90 ft. rad.	23.56								
	" 15°-60 "	15.71								
	" 15°-40 "	10.47		Gate. 30 × 30-in. Y.						
	" 15°-25 "	6.54		30 × 6-in. cross.	29.964	30.188	29.719	120	22	0.26
	" 15°-15 "	3.93		Sleeve open 4¼ ins.						
	" 15°-10 "	2.62								
	Length in excess of design	.09								
	Tangent	86.91								
			162.25							
II.	Orleans tangent		56.42	No specials	29.844	30.000	29.750	40	4	0.04
III.	Tangent	50.26		Test box, ring 29 ins. diameter,						
	Curve (shown in Sec. I)	62.83		and 3 ins. long, near middle	29.844	30.094	29.000	164	26	0.14
	Tangent	86.91		of one pipe						
			200.00							
IV.	Hale tangent		234.92	No specials	29.890	30.219	29.688	146	19	0.13
V.	Tangent	52.88								
	Curve, 90°-60 ft. rad.	94.24								
	Tangent	52.88		3-30-in. sleeves, 30 × 6-in. cross	29.910	30.188	29.625	160	25	0.12
			200.00							
VI.	Riopelle tangent		—47.88	No specials	29.839	30.000	29.750	40	4	0.05
VII.	Tangent	68.58								
	Curve, 90°-40 ft. rad.	62.84								
	Tangent	68.58		Gate. 30 × 10-in. Y	29.894	30.250	29.719	162	27	0.08
			200.00							
VIII.	Rowena tangent		292.23	No specials	29.969	30.156	29.719	178	24	0.12
IX.	Tangent	80.36		Testing box, 30 × 10-in. Y						
	Curve, 90°-25 ft. rad.	39.28		Sleeve open 4½ ins. bet. spig-	29.910	30.250	29.688	164	26	0.18
	Tangent	80.36		ots						
			200.00							

around the pipe. These depressions were from 6 to 8 ins. apart, but had no abrupt or well-defined edges.

After the line had been in use a few days, the sixth piece in the 60-ft. radius curve cracked near the bell, and was repaired by cutting out and sleeving in 18 ins. of straight pipe.

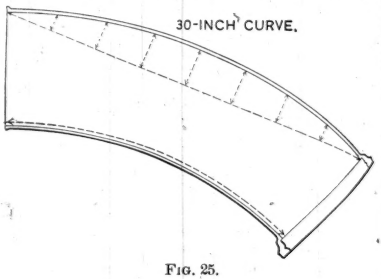


FIG. 25.

The radius of the curves was checked by a series of measurements from the chord, as shown in Fig. 25, and the interior side compared with a template. The variation from the designed radius was never found to be sufficient to affect one of these measurements $\frac{3}{8}$ in., therefore no corrections have been made for it.

Pipe Laying.—Pipe laying was begun in October, 1896, and completed in January, 1897. The soil is a yellow clay, well drained. The trench was usually 4 ft. wide on top (in a few cases 5 ft.), and was first excavated nearly to grade, stakes were set approximately 25 ft. apart near the top of the trench, levels were taken on top of them and the cutting calculated for each stake. As fast as necessary, stakes were set by the foreman in the bottom of the ditch at the proper elevation for the bottom of the pipe. By means of these, a 15-ft. straight-edge and a hand-level, the bottom was dressed up, bell-holes were dug, and the pipe lowered to its proper elevation. Either one or two derricks were used, in any case great care being taken to lower the pipe gently, but, if it became necessary, blocks of different thicknesses were placed under the pipes to bring them to their proper elevation. The joints, which were all made in the trench, were packed with hemp, leaving a space about $2\frac{1}{2}$ ins. in depth for lead, which was run in and caulked. Small sewers and water pipes were the only obstructions met, and these were either raised or lowered, as circumstances permitted. All pipes and curves were tested* in the open ditch, before covering, for 2 hours with a pressure of 80 lbs. per square inch, and, in order to allow back-filling to proceed as rapidly as possible,

* The testing of all pipe in the open ditch, to a pressure at least twice as great as it is expected to bear, is a practice introduced in these works in 1893 by one of the writers, and applied to all pipe laid since that time.

testing boxes were placed where needed. After being used, but before covering, a 2-ft. length of 30-in. pipe was adjusted in the testing box, (Fig. 26), in line with the pipe in either direction and concrete packed around it before bolting on the cover. The frictional loss in the testing boxes should not, therefore, exceed that of an equal amount of straight pipe.

All gates used on this line were made by the Michigan Brass and Iron Works, and are their standard, parallel face, 30-in. horizontal pattern. The disc is made in halves, which are forced apart by wedges when nearly in position, to seat. When open, the disc was generally

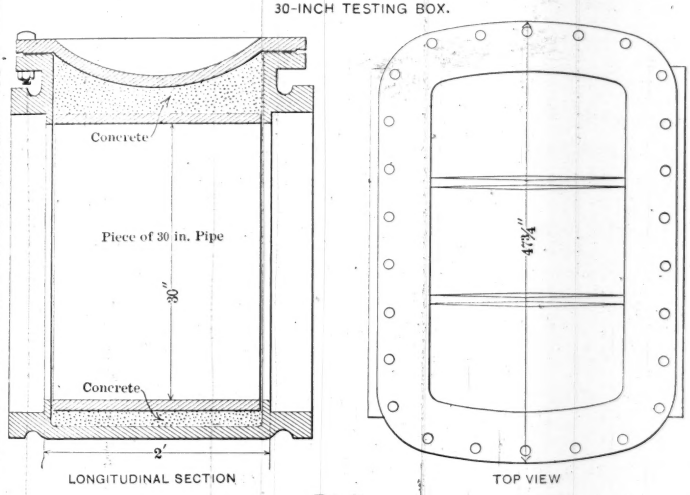


FIG. 23.

flush with the seats, and only in one or two cases did the end of the stems project into the clear opening when the gate was open, and then about 1 in. All gates were operated before laying. A 4-in. by-pass, set horizontally, and operated with a hand wheel, is placed over the gate, and during the experiments these were open except when closed to make a shut-off.

Blow-Offs.—In the design of the work blow-offs were provided for draining the sections of line between each pair of gates, each being connected to the sewer. Two of these blow-offs, being those draining the last two sections, were arranged with special nozzles, one of 9.216

ins. diameter and the other 7.215 ins. These nozzles were tapered and were turned out and carefully calipered. They were set to discharge into sewers in such a way that it was hoped they might be used to measure the flow, but a few trials immediately after construction showed the discharge to be somewhat obstructed, and as, in addition to the resulting uncertainty, it would be impossible to supply any territory through the line during experiment, these blow-offs were not used, as first intended, to measure the water, but simply to cause such velocities as were desired in the pipe during the experiments. The larger blow-off was at the extremity of the new line, and the smaller one in the middle of the tangent between the 10 and 6-ft. radius curves.

Piezometer Connections.—For experimental purposes, the line was divided into 22 lengths, as given in detail in Table No. 21. In order to compare results, enough tangent was included with each curve to make the total length of section 200 ft., measured on the axis of the pipe, the remainders of the tangents having odd lengths. The curves were divided into two parts of 45° each; the length of curved pipe was computed, and a sufficient length of tangent added to make 100 ft. on either side of the center of curvature.

After the tangents were laid, but before the curves were put in, the correct computed distance, depending upon the radius of curvature, was carefully measured from the point of intersection of the tangents, and marked on the inside of the pipe. At each point so marked a ½-in. hole was drilled in the top of the pipe by hand, from the inside, the drill being held in a vertical position, point up, by a special saddle, leveled with a spirit level. In this manner a sharp square edge on the inner surface was obtained.

Each hole was threaded from the outside and a ½-in. Payne tapping-machine cock inserted and screwed down. The driller remained on the inside of the pipe to be assured that the cock did not project, and also to carefully paint the exposed surface of the iron. A ½-in. lead connection was carried from the cock to a point within the curb and provided with a stop and waste cock, cast-iron box and wrought-iron stand pipe for hose connection; all as shown in Fig. 16.

In a few places the ½-in. lead pipe was carried into a gate-well, or the cock came within a well and required no lead pipe.

In addition to the vertical piezometers at the ends of each section,

taps were made from the inside on the horizontal diameter of the 6, 10, 15-ft. and small compound curves, at an angle of 55° from the beginning of the curve, measured in the direction of flow. These cocks were level with the center of the pipe and opposite each other.

Experiments of 1898.

Conditions.—The first experiments were made during the summer of 1898, the general plan of operation being to first close all side gates along the line, thus insuring an equal flow through its entire length. Differential Gauges Nos. 6 and 7 were then observed simultaneously on different sections to obtain the relative resistances in each, while the velocities were varied as desired, and measured by Pitot tubes either on the center or by traversing.

A majority of the experiments was conducted at night, in order to obtain, as nearly as possible, uniform temperature conditions as well as the greater range in velocities permissible at night, when domestic consumption was at a minimum.

Preliminary Series.—In the preliminary investigations, which are presented in Table No. 22, Experiments 1 to 43, observations were taken with practically constant flow at two velocities, the lower being that obtained when only the Canfield blow-off was open, and the higher that with both the Canfield and the Hastings blow-offs open. The latter condition produced nearly the same velocity in every case, but the former showed some variation between different dates, although fairly uniform for several hours during one set of experiments. The first experiments were made on May 30th, 1898, when Gauge No. 7 was connected to the two extremities of the second long compound curve section and Gauge No. 6 to those of the 60-ft. radius curve section, the Canfield blow-off being open. The losses in both curve sections, 200 ft. long, were observed for a considerable period, simultaneously, with water and air in the gauges, the loss of head being obtained directly in inches of water (Experiments 1, 2 and 3), after which the air in both gauges was replaced by kerosene and the observations continued. On moving Gauge No. 7 to the 40-ft. radius curve for comparison with the 60-ft. radius curve, it developed a leak and had to be discarded for the remainder of the day, and Gauge No. 6 was then moved from the 60-ft. radius curve, first to the 25-ft. radius and then to the small compound curve, the flow through the line remaining as nearly constant as the

24..	June 11.	Open.	Closed.	.....	.....	.....	P. M..	9.56 -10.15	10-ft. rad. curve.....	8.618	...	.....	.....	.....
25..	" 11.	"	"	P. M., 10.21 -10.34	Brady tangent.....	8.300	"	10.29 -10.47	" "	8.380	70°			
26..	" 11.	"	"	" 10.57 -11.09	15-ft. rad. curve....	9.696	"	11.01¼-11.04	" "	8.230	...			
27..	" 12.	"	"	" 11.42 -11.51	" "	9.850	"	11.44 -11.56	Small comp. curve.....	8.320	...			
28..	" 12.	"	"	A. M., 12.22½-12.26½	25-ft. rad. curve....	11.000	A. M.,	12.03 -12.21	" "	9.000	...	2.808	2.499	1.159
29..	" 12.	"	"	" 12.41 -12.50	" "	(11.748)	"	12.41 -12.50	40-ft. rad. curve.....	11.606	...			
30..	" 12.	"	"	.....	.....	.....	"	12.57 - 1.09	" "	11.670	...			
31..	" 12.	"	"	A. M., 1.19 - 1.34	60-ft. rad. curve....	12.455	"	1.19 - 1.24	" "	11.491	...			
32..	" 12.	"	"	" 1.48 - 1.57	" "	12.689	.....	.....	.....	.....	...	2.822	2.512	1.180
33..	" 12.	"	"	" 2.13 - 2.16	" "	(12.100)	A. M.,	2.11 - 2.16	Second long comp. curve.	12.200	...			
34..	" 12.	"	Open ..	" 2.25 - 2.36	" "	(22.200)	"	2.25 - 2.36	" " "	21.930	...			
35..	" 12.	"	" ..	" 3.04 - 3.06½	" "	22.200	"	2.58 - 3.12	40 ft. rad. curve.....	20.590	...			
36..	" 12.	"	" ..	" 3.38 - 3.41½	25-ft. rad. curve....	(20.600)	"	3.22 - 3.40	" "	20.010	...			
37..	" 12.	"	" ..	" 3.42 - 3.47½	" "	(20.800)	.....	.....	.....	.....	...			
38..	" 12.	"	" ..	" 3.55 - 4.01	" "	(20.600)	A. M.,	3.57 - 4.05	Small comp. curve.....	17.380	...	3.978	3.540	2.320
39..	" 12.	"	" ..	" 4.28½- 4.35½	15-ft. rad. curve....	20.770	"	4.27 - 4.30	" "	17.530	...			
40..	" 12.	"	" ..	" 4.44 - 4.50	" "	20.369	.....	.....	.....	.....	...			
41..	" 12.	"	" ..	" 5.08 - 5.14	Brady tangent.....	14.731	A. M.,	5.00 - 5.17	10-ft. rad. curve.....	16.410	...			
42..	" 12.	"	" ..	.....	.....	.....	"	5.32 - 5.49	" "	16.648	...			
43..	" 12.	"	Closed.	A. M., 5.59 - 6.14	6-ft. rad. curve.....	9.282	"	6.15 - 6.26	" "	9.300	.....	.....	.....	.....

NOTE.—Quantities in parentheses are considered questionable.

TABLE No. 22.—1898 Experiments. First Series.

No. of experiment.	Date.	Waste Gates. Canfield 12-in. B.-O.	Waste Gates. Hastings 10-in. B.-O.	Gauge No. 6. Time.	Gauge No. 6. Location.	Gauge No. 6. Av. read. H_f inches.	Gauge No. 7. Time.	Gauge No. 7. Location.	Gauge No. 7. Av. read. H_f inches.	Temp. Fahr.	Pitot Tube No. 3, at Hastings St. Obs. V_m Feet per second.	Pitot Tube No. 3, at Hastings St. True V_m Feet per second.	Pitot Tube No. 3, at Hastings St. True Hv_m Inches.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
					Water and Air.			Water and Air.					
1..	May 30..	Open.	Closed.	P. M., 12.00 -12.35	60-ft. rad. curve....	3.448	P. M.. 12.07 -12.35	Second long comp. curve..	3.211	.....	.....	.....	.....
2..	"	"	"	" 12.35½-12.56	" "	3.398	" 12.35½-12.56	" " ..	3.214	.....	.....	.....	.....
3..	"	"	"	" 12.56½- 1.09	" "	3.388	.....	.....	.....	.....	.....	.....	.....
					Water and Kerosene.			Water and Kerosene.					
4..	"	"	.....	.....	.....	.....	P. M., 1.13 - 1.24½	Second long comp. curve..	15.660	.....	.....	.....	.....
5..	"	"	"	P. M.. 1.25 - 1.50	60-ft. rad. curve....	16.873	" 1.25 - 1.50	" " ..	15.814	.....	.....	.....	.....
6..	"	"	"	" 2.08 - 2.45	" "	16.378	.....	.....	.....	78°	.....	.....	.....
7..	"	"	"	" 3.18 - 3.25½	25-ft. rad. curve....	13.506	.....	.....	.....	67°	.....	.....	.....
8..	"	"	"	" 3.26 - 3.36½	" "	13.441	.....	.....	.....	.....	.....	.....	.....
9..	"	"	"	" 3.37 - 3.46½	" "	13.450	.....	.....	.....	.....	.....	.....	.....
10..	"	"	"	" 3.47 - 3.57	" "	13.647	.....	.....	.....	.....	.....	.....	.....
11..	"	"	"	" 3.57½- 4.05½	" "	13.524	.....	.....	.....	.....	.....	.....	.....
12..	"	"	"	" 4.28 - 4.35½	Small comp. curve.	11.640	.....	.....	.....	.....	.....	.....	.....
13..	"	"	"	" 4.36 - 4.45½	" "	11.540	.....	.....	.....	.....	.....	.....	.....
14..	"	"	"	" 4.46 - 4.55½	" "	11.505	.....	.....	.....	.....	.....	.....	.....
15..	"	"	"	" 4.56 - 5.06½	" "	11.360	.....	.....	.....	.....	.....	.....	.....
16..	June 7.	"	"	" 9.48 -10.12½	15-ft rad. curve....	12.846	.....	.....	.....	.....	.....	.....	.....
17..	" 7.	"	"	" 10.12 -10.31	" "	13.050	P. M., 10.12 -10.31	Small comp. curve.....	(10.485)	72°	3.191	2.840	1.495
18..	" 8.	"	"	" 10.32 -11.03	" "	(13.323)	" 10.32 -11.03	" "	(10.030)	...			
19..	" 8.	"	Open ..	A. M., 12.23 -12.51	" "	20.046	A. M., 12.23 -12.51	" "	16.900	.....	3.972	3.535	2.318
20..	" 8.	"	" ..	" 1.20 - 1.51	" "	(19.429)	.....	.....	.....	.....	.....	.....	.....
21..	" 8.	"	" ..	.....	.....	.....	A. M., 1.51 - 2.21	40-ft. rad. curve.....	(18.706)	...	3.963	3.527	2.310
22..	" 8.	"	" ..	A. M., 2.23 - 2.48	Brady tangent.....	13.326	" 2.23 - 2.48	" "	(18.050)	...			
23..	.....	"	Closed.	" 2.54 - 3.07	"	8.375	" 2.54 - 3.07	" "	10.64	.....	.....	.....	.....

	July 16, 17, 1898.							Alexandrine Avenue, East of Hastings Street. Gauge No. 7, Kerosene.		Pitot Tube No. 3, on Center at Hastings and Alexandrine Streets (in Error).		Pitot Tube No. 4, Hastings St. and Alexandrine Avenue, oblique $12\frac{8}{16}$ N. of Center.			
								P. M.							
85		Closed.						11.10 -11.32	4.10	11.04 -11.24	0.343				
86		Open 3 *T.*						11.37½-11.45	4.60	11.34 -11.45	0.41				
87		" 7 *T.*						11.51 -11.58	7.40	11.50 -11.59	1.5536				
								A. M.							
88		" "						12.00½-12.12	7.80	12.00 -12.15.40	0.801				
89		" "						12.47½-12.56	7.83	12.44 -12.56	1.867				
90		" "						12.58 - 1.11	7.77	12.58 - 1.11	0.9421				
91		" "						1.11½- 1.39	7.51	1.12 - 1.39	1.2833				
92		" 15 *T.*						1.52 - 2.01	12.28	1.44 - 2.03	2.126				
93		" "								2.05 - 2.12	1.75				
94		Full open.						2.15 - 2.25	15.54	2.15 - 2.28.40	2.18				
95		" "						2.25½- 2.43	16.20						
													Column* 21×1.6	Gauge No. 6 on diam., 6-ft. radius curve, Alexandrine Ave. and Alley.	
96		" "						3.36 - 3.54	16.58	3.34 - 3.54	1.951				
97		Closed 20 *T.*						3.56 - 4.06	14.83	3.56 - 4.03	1.808	0.9896	1.593		
98		" 14 *T.*						4.10 - 4.14½	11.98	4.07 - 4.14	1.528	0.8333	1.333		
99		" 7 *T.*						4.16 - 4.24½	8.33	4.15 - 4.24	1.243	0.6483	1.037		
100		Closed.						4.30 - 4.37	3.37	4.27 - 4.39	1.139	0.25	0.400	4.30 -4.37	0.9
101	Open 3 *T.*	"						4.40 - 4.50	3.87	4.40 - 4.50	1.243	0.3194	0.511	4.40 -4.50	1.2
102	" 8 *T.*	"						4.59½- 5.05	8.40	4.51 - 5.05	1.614	0.5	0.800	4.59½-5.05	2.8
103	" 17 *T.*	"						5.16 - 5.20	16.27	5.06 - 5.20	2.377	1.019	1.630	5.16 -5.20	5.4
104	Full open.	"								5.21 - 5.28	3.280	1.5229	2.437		
105	" "	"						5.27¼- 5.35	23.52	5.29 - 5.35	3.142	1.56	2.496	5.27½-5.35	8.0
106	Closed 20 *T.*	"						5.42 - 5.45	19.74	5.37 - 5.45	2.617	1.178	1.885	5.42 -5.45	6.5
107	" 13 *T.*	"								5.45½- 5.50	2.373	1.0643	1.703		
108	" "	"						5.51½- 5.55	15.60	5.53 - 5.55½		1.017	1.627	5.51½-5.55	5.0
109	" 6 *T.*	"						6.02½- 6.06	9.12	5.56 - 6.03½	1.718	0.525	0.840	6.02½-6.06	2.8
110	" 4 *T.*	"						5.11½- 6.15	6.70	6.06 - 6.15	1.448	0.42	0.672	6.11½-6.15	2.2
111	Closed.	"						6.17½- 6.21	5.10	6.17 - 6.20	1.450	0.36	0.576	6.17½-6.21	1.5

*By previous traverses, $V_c = 1.6\ V$ at the $12\frac{8}{16}$-in. position.

TABLE No. 22—(*Continued*).—1898 EXPERIMENTS, SECOND SERIES.

Oil Gauge readings are in Inches of Oil and Water. Pitot Tube Gauge readings are in Inches of Water.

No. (15)	Condition of Waste Gates. (16)	(17)	T., Fahr. (18)	Time. (19)	Avg. Read. (20)	T., Fahr. (21)	Time. (22)	Avg. read. (23)	Time. (24)	Avg. read. (25)	Avg. read. (26)	(27)	(28)	(29)
June 24, 25, 1898.	Canfield 12-in. Blow-off	Hastings 10-in. Blow-off.		BRADY STREET TANGENT. 212.25 FT. Gauge No. 6, Kerosene.			NORTH RIVARD STREET TANGENT, 184.10 FT. Gauge No. 7, Kerosene.		PITOT TUBE NO. 3, ON CENTER AT HASTINGS.					
				P. M.										
65				9.35 –10.00				1.2960		0.2870				
66		Closed.		10.12 –10.19½	1.0531			1.1656		0.3178				
67		Open 3 *T.*		10.25 –10.40	1.4400	60°		1.5774		0.4790				
68		" "		10.40½–10.58	1.3500			1.4972		0.4274				
69		" 5 *T.*		11.01½–11.10	1.8000			1.9555		0.5570				
70		" 8 *T.*		11.11½–11.26	2.4680			2.5800		0.7650				
71		" "		11.27½–11.34	2.6250			2.8650		0.6971				
72		" 11 *T.*		11.38 –11.48	3.3000			3.4000		0.8490				
73		" "		11.49 –11.56	3.2600	59°		3.3100		0.8390				
				A. M.										
74		" 17 *T.*		12.06 –12.14	4.3500			4.4000		1.0860				
75		Full open.		12.17 –12.25	5.1257			5.1976		1.3460				
76		Closed 12 *T.*		12.38 –12.45½	4.6500			4.5894		1.2060				
77		" "	55°	12.54 –12.59	4.5090			4.5000		1.1740				
78		" "		12.59½– 1.11	4.4000			.....		1.1406				
79		" "		1.20 – 1.28	4.4800			4.5000		1.0940				
80		" 9 *T.*		1.30 – 1.39½	3.2325			3.1350		0.8000				
81		" 6 *T.*		1.43½– 1.49	1.7500			1.6000		0.4215				
82		" "		1.50½– 1.57½	1.5767			1.5267		0.3200				
83		Closed.		2.00 – 2.16	0.6803			0.7125		0.13145				
84		"		2.16 – 2.32										

June 18, 19, 1898.				Russell and Brady Streets, Compound Curve, 200 Ft. Gauge No. 6, Kerosene.			Rivard and Alexandrine 10-Ft. Radius Curve, 200 Ft. Gauge No. 7, Kerosene.	Pitot Tube No. 3. Traversing at Hastings and Alexandrine.		
								(38a) Mean Center Reading.	(39a) Obs. V_m Feet per Second.	(39b) True V_m Feet per Second.
				P. M.						
44	Closed.	Closed.	.....	10.13 -10.24	1.7118	64°	1.5000	0.162	(0.859)	0.765
45	Open 4½ *T.*	"	.....	10.50 -11.17	2.201	.....	1.9000	0.181	(0.567)	0.505
46	" 6½ *T.*	"	.....	11 50 -12.00	2.900	61°	2.7000	0.210	(0.609)	0.542
				A. M.						
47	Open 9½ *T.*	Closed.	61°	12.15 -12.40	3.855	.....	3.682	0.495	1.179	1.049
48	" 15½ *T.*	"	.....	12.47 -12.48½	5.600	.....	5.600	.....	.....	.....
49	" "	"	.....	1.20 - 1.44	5.807	60°	.....	1.077	2.014	1.792
50	Full open.	"	.....	2.02 - 2.10	9.500	.....	.....	1.992	2.767	2.463
51	"	"	.....	2.10 - 2.20	.....	.....	9.290	2.080	2.806	2.497
52	"	Open 7 *T.*	.....	2.34 - 2.55	12.239	.....	11.844	2.579	3.247	2.890
53	"	" 16 *T.*	.....	3.03 - 3.15	.....	.....	.....	.....	3.633	3.323
54	"	" "	.....	3.21 - 3.40	.....	.....	14.500	3.395	3.696	3.289
55	"	Closed.	.....	4.35 - 4.42	.....	.....	9.233	2.138	2.546	2.266
56	"	"	.....	4.44 - 5.03	.....	.....	9.050	2.000	.....	.....
57	"	"	.....	5.03½- 5.13	6-ft. rad. curve.	59°	.....	.....	.....	.....
58	Closed 12 *T.*	Closed	.....	5.17 - 5.27	8.1571	.....	8.120	1.775	.....	.....
59	" 6 *T.*	"	.....	5.33 - 5.40	6.801	.....	6.800	1.559	.....	.....
60	" 4 *T.*	"	.....	5.45 - 6.03	5.905	.....	6.022	1.319	.....	.....
61	" 3 *T.*	"	.....	6.06 - 6.13	5.000	.....	5.000	1.125	.....	.....
62	" 3 *T.*	"	.....	6.20 - 6.38	4.100	.....	4.1167	0.917	1.894	1.686
63	" 2 *T.*	"	.....	6.40 - 6.47	3.635	.....	3.5267	0.767	.....	.....
64	Closed.	"	.....	6.50 - 6.53	1.400	.....	1.200	0.240	.....	.....

TABLE No. 22—(*Continued*).

No. (30)	Condition of Waste Gates. (31)	(32)	T., Fahr. (33)	Time. (34)	Avg. read. (35)	T., Fahr. (36)	Avg. read. (37)	Avg. read. (38)	Avg. read. (39)
				Brady and Rivard Streets, 15-Ft. Radius Curve, 200 Ft.			Russell and Brady Streets, Compound Curve, 200 Ft.	Pitot Tube No. 4, on Center, at Leland and Rivard Streets.	Pitot Tube No. 5, on Center, at Hastings and Alexandrine Streets.
Aug. 20, 21, 1898.	Canfield 12-in. blow-off.	Hastings 10-in. blow-off.		Gauge No. 6, Kerosene.			Gauge No. 7, Kerosene.		
				P. M.					
209	Closed.	Closed.	.....	10.06½-10.25	1.419	.....	1.323	0.341	.....
210	Open 3 *T.*	"	.....	10.41 -11.00	.....	.....	1.478	(0.717)	0.3921
211	" 5½ *T.*	"	.....	11.16 -11.30	2.310	.....	2.072	(0.776)	0.6035
212	" 8 *T.*	"	.....	11.40 -11.54	3.267	.....	2.875	0.633	0.8321
213	" 12 *T.*	"	.....	11.58 -12.20	4.823	.....	4.255	0.9881	1.2580
				A. M.					
214	Open 17 *T.*	Closed.	.....	12.24 -12.45	6.402	.....	5.483	1.2357	1.7280
215	" 25 *T.*	"	.....	1.04 - 1.25	8.586	.....	7.400	1.5671	2.3405
216	Full open.	"	.....	1.30 - 1.52	10.636	.....	(9.727)	2.1246	(2.7644)
217	"	Open 5 *T.*	.....	1.58 - 2.08	12.562	.....	10.625	2.5000	3.3450
218	"	" "	.....	2.08 - 2.15	12.600	.....	10.793	(1.793)	3.3800
219	"	" 9 *T.*	.....	2.21 - 2.42	14.865	.....	12.622	2.936	3.9690
220	"	" 15 *T.*	.....	2 46 - 2.52	16.800	.....	14.100	3.2560	4.4416
221	"	" "	62°	2.55 - 3.11	17.539	.....	14.798	3.4734	4.6516
222	"	" 22 *T.*	.....	3.11½ - 3.19½	18.618	.....	15.621	3.697	.....
223	"	" "	.....	3.20 - 3.28	19.394	.....	16.388	3.803	5.2470
224	"	Full O. 31½ *T.*	.....	3.30 - 3.38	18.224	.....	15.503	3.650	4.8750
225	"	"	.....	3.40 - 3.55	.....	.....	17.168	4.113	.....
226	"	"	.....	3.59 - 4.06	20.200	.....	.....	4.063	.....
227	"	"	.....	4.07 - 4.15½	19.983	.....	16.578	4.04118	.....
228	"	Open 18 *T.*	.....	4.18 - 4.35	18.657	.....	15.756	3.738	.....
229	"	" 12 *T.*	.....	4.40 - 4.50	16.440	.....	14.062	3.2842	.....
230	"	" 6 *T.*	.....	4.55 - 5.02	9.940	.....	8.173	2.097	(3.220)
231	"	"	61°	5.03 - 5.08	(9.654)	.....	(11.895)	2.218	(2.855)
232	"	Closed.	.....	5.26 - 5.43	.....	.....	10.697	2.236	.....
233	"	"	.....	5.44 - 6.00	11.625	.....	9.462	2.0175	.....
234	Open 20 *T.*	"	.....	6.05 - 6.18	8.227	.....	7.334	1.5972	.....
235	" 13 *T.*	"	.....	6.28 - 6.48	6.840	.....	5.815	1.199	.....
236	" 8 *T.*	"	.....	6.52 - 6.58	5.1833	.....	4.235	0.946	.....
237	Closed.	"	.....	7.00	.....	.....	.....	.....	.....

Aug. 13, 14, 1898.				Orleans and Scott Streets, Compound Curve, 162.25 Ft. Gauge No. 6, Kerosene.			Hale and Riopelle Streets, 60-Ft Radius Curve, 200 Ft. Gauge No. 7, Kerosene.			Pitot Tube No. 4, traversing at Leland and Rivard Streets.			Pitot Tube No. 5, on Center at Hastings and Alexandrine Streets.
										(49a)	(49b) Obs. V_m Feet per Second.	(49c) True V_m Feet per Second.	(50a)
				P. M.									
163	Closed.	Closed.	.....	10.50 -11.08	1.60	.....	.....	1.496	.....	.....	.....	.....	0.348
164	Open 5 *T.*	"	62°	11.10 -11.30	2.4293	.....	.....	2.502	.....	.....	.....	.....	0.5352
165	" 9 *T.*	"	.....	11.39 -12.29	4.370	.....	.....	4.311	.....	.....	.....	.....	0.961
				A. M.									
166	" "	"	.....	12.21 -12.28½	(4.400)	.....	.....	(4.269)	0.81125	.....	.....	.....	0.964
167	" 16 *T.*	"	.....	12.34 -12.55	7.579	.....	.....	7.331	.....	2.240	1.926		1.643
168	" "	"	59°	1.00 - 1.25	.....	58°	.....	7.020	.....	.....	.....		1.723
169	" 24 *T.*	"	.....	1.34 - 2.00	.....	.....	.....	9.782	.....	2.551	2.194		2.231
170	Full " 38 *T.*	"	.....	2.03 - 2.30	.....	.....	.....	12.013	.....	2.847	2.448		2.719
171	" " "	Open 7 *T.*	.....	2.33 - 2.45	.....	.....	.....	15.304	.....	5.448	2.965		3.579
172	" " "	" "	.....	2.58 - 3.04	.....	.....	.....	15.600	2.9785	.....	.....		3.792
173	" " "	" "	.....	3.04 - 3.15	17.013	.....	.....	(15.746)	.....	.....	.....		3.7609
174	" " "	" 13 *T.*	.....	3.20 - 3.20½	20.07	.....	.....	(17.775)	.....	3.828	3.292		4.484
175	" " "	" 21 *T.*	.....	3.44½ - 3.50	19.20	.....	.....	(16.271)	3.925	.....	.....		4.283
176	" " "	" "	57°	3.53 - 4.03	21.671	.....	.....	.....	.....	3.933	3.382		4.729
177	" " "	" "	.....	4.14 - 4.21	20.987	.....	.....	19.77	.....	.....	.....		4.50
178	" " "	Full " 31½ *T.*	.....	4.21 - 4.26½	21.275	.....	.....	20.2625	3.908	.....	.....		4.591
179	" " "	" " "	.....	4.29 - 4.39	21.639	.....	.....	20.512	.....	3.988	3.430		4.667
180	" " "	" " "	.....	4.40 - 4.51	22.134	.....	.....	21.061	.....	.....	.....		4.844
181	" " "	Open 18 *T.*	57°	4.53 - 5.11	20.870	.....	.....	.....	.....	3.862	3.321		4.530
182	" " "	" "	.....	5.14 - 5.23	20.474	.....	.....	19.405	.....	.....	.....		4.439
183	" " "	" 8 *T.*	.....	5.31 - 5.50	16.700	.....	.....	15.901	.....	3.428	2.948		3.636
184	Closed.	Closed.	.....	5.55 - 6.14	12.92	.....	.....	12.259	.....	3.075	2.645		2.803
185	"	"	.....	6.14 - 6.20	13.100	.....	.....	12.454	2.204	.....	.....		2.846

TABLE No. 22—(*Continued*).

No. (40)	Condition of Waste Gates. (41)	(42)	T., Fahr. (43)	Time. (44)	Avg. Read. (45)	T., Fahr. (46)	Time (47)	Avg. Read. (48)	Avg. Read. (49)	Avg. Read. (50)
Aug. 19. 1898	Canfield 12-in. Blow-off.	Hastings 10-in. Blow-off.					ROWENA AND RUSSELL STREETS, 25-FT. RADIUS CURVE, 200 FT. Gauge No. 7, Kerosene.		PITOT TUBE No. 3, ON CENTER AT HASTINGS STREET.	PITOT TUBE No. 4, OBLIQUE REVERSED AT HASTINGS STREET.
							P. M.			So many variations that no observations were reduced.
186	Open 8 *T.*	Closed.	.....	.....	.....	.....	2.20 –2.25	6.732	1.414	.....
187	" 12 *T.*	"	.....	.....	.....	88°	2.30 –2.34	8.540	1.933	.....
188	" 17 *T.*	"	.....	.....	.....	.....	2.39 –2.49	10.741	2.088	.....
189	" "	"	.....	.....	.....	.....	2.55 –3.03	11.259	2.194	.....
190	" 24 *T.*	"	.....	.....	.....	.....	3.07 –3.13	13.296	2.550	.....
191	Full " 38 *T.*	"	.....	.....	.....	78⅛°	3.21 –3.34	14.765	3.024	.....
192	" 20 *T.*	"	.....	.....	.....	.....	3.37 –3.43½	13.079	2.546	.....
193	" "	"	.....	.....	.....	81°	3.52 –3.59	12.763	2.603	.....
194	" 14 *T.*	"	.....	.....	.....	.....	4.03 –4.09½	10.0786	1.896	.....
195	" 9 *T.*	"	.....	.....	.....	.....	4.11 –4.19	7.909	1.582	.....
196	" 6 *T.*	"	.....	.....	.....	.....	4.28 –4.33½	5.700	1.0375	.....
197	" 4 *T.*	"	.....	.....	.....	.....	4.35 –4.40	4.614	0.7046	PITOT TUBE No. 4, ON CENTER AT LELAND STREET.
198	Closed.	"	.....	.....	.....	.....	4.41½–4.50½	(3.126)	0.550	
199	"	"	.....	.....	.....	.....	5.05 –5.28	2.3714	0.483	
200	"	Open 5 *T.*	.....	.....	.....	.....	5.40 –5.50	.....	0.805	0.821
201	"	" 8 *T.*	.....	.....	.....	.....	5.55 –6.05	5.712	(1.082)	1.127
202	"	" 11 *T.*	.....	.....	.....	.....	6.07 –6.16	6.674	1.458	1.332
203	"	" 17 *T.*	.....	.....	.....	.....	6.18 –6.24	7.662	1.585	1.577
204	"	Full " 31½	.....	.....	.....	.....	6.30 –6.35	10.209	2.109	2.25
205	"	Open 18½	.....	.....	.....	.....	6.36 6.45	9.913	1.953	1.936
206	"	" 9½	.....	.....	.....	.....	6.48 –6.54	7.804	1.458	1.390
207	"	" 3½	.....	.....	.....	.....	6.58 –7.02	4.717	0.822	0.796
208	"	Closed.	.....	.....	.....	.....	7.04 –7.11	2.712	(0.540)	0.446

146.....	" "	" "	.....	3.35 – 3.36	17.800	.....	29.968	2.588	2.702
146.....	" "	" "	.....	3.36½ – 3.42	18.109	.....	30.592	2.669	2.777
147.....	" "	Closed.	.....	3.48 – 4.00	14.533	.....	24.782	2.204	2.203
148.....	" "	"	.....	4.02½ – 4.07	14.900	.....	25.114	2.147	2.248
149.....	Open 28 *T.*	"	.....	4 15 – 4.22	12.400	.....	20.800	1.752	1.852
150.....	" 20 *T.*	"	.....	4.23 – 4.26⅓	11.350	.....	19.200	1.582	1.695
151.....	" "	"	.....	4.26½ – 4.32	11.080	.....	19.060	1.620	1.688
152.....	" "	"	.....	4.32½ – 4.40	11.000	.....	18.480	1.525	1.648
153.....	" 15 *T.*	"	70°	4.42 – 4.47	9.500	.....	16.061	1.278	1.45
154.....	" "	"	.....	4.48 – 4.53	9.227	.....	15.682	1.296	1.391
155.....	" 11 *T.*	"	.....	4.58 – 5.03½	7.145	.....	12.000	0.982	1.056
156.....	" "	"	.....	5.04 – 5.07	7.400	.....	12.212	1.030	1.08
157.....	" 7 *T.*	"	.....	5.10 – 5.19	5.100	.....	8.490	0.654	0.705
158.....	" 4 *T.*	"	.....	5.22 – 5.26⅔	3.300	.....	5.338	0.346	0.42
159.....	" "	"	69°	5.27 – 5.29	3.400	.....	5.500	0.379	0.50
160.....	Closed.	"	.....	5.31 – 5.36	2.033	.....	(5.500)	0.325	0.372
161.....	"	"	.....	5.36½ – 5.42	1.900	.....	(3.100)	0.189	0.2806
162.....	"	"	.....	5.44	(1.850)	.....	.....	.....	0.308

TABLE No. 22—(*Concluded*).

No. (61)	Condition of Waste Gates. (62)	(63)	T., Fahr. (64)	Time. (65)	Avg. read. (66)	T., Fahr. (67)	Avg. read. (68)	Avg. read. (69)	Avg. read. (70)
				Rowena Street Tangent, 292.23 Ft. Gauge No. 6, Sperm Oil.			Hale Street Tangent, 234.92 Ft. Gauge No. 7, Sperm Oil.	Pitot Tube No. 3, on Center, at Superior Street.	Pitot Tube No. 5, on Center, at Hastings Street.
Sept. 21, 1898.	Canfield 12-in. Blow-off.	Hastings 10-in. Blow-off.		P. M.					
238.....	Closed.	Closed.	.....	1.33–1.44	.....	.....	5.1705	.....	0.870
239.....	"	"	.....	1.44–2.10	(5.786)	75½°	5.0441	.....	0.881
240.....	"	"	76°	2.11–2.53	(5.9012)	73½°	5.1267	0.752	0.840
241.....	"	"	.....	2.56–3.10	5.898	72½°	4.800	0.7207	0.869
242.....	"	"	77°	3.10–3.22	5.800	.....	4.769	0.69592	0.792
243.....	Open 5 *T.*	"	75°	3.30–3.42	8.124	.....	6.604	0.996	1.288
244.....	" 9 *T.*	"	75°	3.50–3.59	11.776	76°	9.730	1.457	2.1894
245.....	" 12 *T.*	"	75°	4.10–4.25	.....	.....	11.668	1.819	2.671
246.....	" "	"	72°	4.30–4.45	15.399	.....	.....	1.875	2.706
247.....	" "	"	.....	4.48–4.54	15.531	.....	12.419	1.861	2.731
248.....	" 16 *T.*	"	71°	4.56–5.10	18.156	.....	14.0143	2.162	3.241
249.....	" 21 *T.*	"	.....	5.11–5.19	21.400	.....	(16.0617)	2.630	3.381
250.....	" "	"	.....	5.20–5.25	21.264	.....	16.782	2.627	3.327
251.....	" 33 *T.*	"	.....	5.30–5.41	25.363	.....	19.2297	3.134	4.139
252.....	" 10 *T.*	"	.....	5.45–5.52	15.393	.....	12.500	1.693	2.420
253.....	Closed.	"	.....	5.56	.....	.....	.....	.....	.....

TABLE No. 22—(*Continued*).

No. (51)	Condition of Waste Gates. (52)	(53)	T., Fahr. (54)	Time. (55)	Avg. read. (56)	T., Fahr. (57)	Avg. Read. (58)	Avg. read. (59)	Avg. read. (60)
Aug. 6, 7, 1898.	Canfield 12-in. Blow-off.	Hastings 10-in. Blow-off.		Brady Street Tangent, 212.25 Ft. Gauge No. 6, Sperm Oil.			Alexandrine Avenue Tangent, 645.50 Ft. Gauge No. 7, Kerosene.	Pitot Tube No. 3, on Center, at Hastings Street.	Pitot Tube No. 4, on Center, at Leland Street.
				P. M.					
112.....	Closed.	Closed.	74°	9.11 - 9.31	.....	.....	.....	.....	0.46
113.....	Open 3 *T.*	"	.....	9.32½–10.19	.....	.....	.....	0.451	0.487
114.....	" "	"	.....	10.19½–10.30	.....	.....	6.003	0.392	0.5167
115.....	" "	"	.....	10.30½–10.42	.....	.....	5.369	0.378	0.4847
116.....	" "	"	.....	10.42½–10.47	2.750	.....	.....	0.407	0.467
117.....	" "	"	.....	10.47½–10.54½	2.800	.....	5.286	0.386	0.479
118.....	" 5 *T.*	"	.....	10.55 –10.58	3.700	.....	6.400	0.510	0.570
119.....	" "	"	.....	10.58½–11.04½	3.900	.....	6.695	0.531	0.5684
120.....	" "	"	.....	11.05 –11.09½	3.820	.....	6 627	0.543	0.5833
121.....	" 8 *T.*	"	.....	11.11 –11.17	5.415	.....	9.337	0.750	0.8167
122.....	" "	"	.....	11.17½–11.23½	5.315	.....	9.100	0.703	0.771
123.....	" 12 *T.*	"	.....	11.25 –11.36½	7.243	.....	11.985	1.029	1.093
124.....	" 17 *T.*	"	.....	11.38 –11.42½	9.067	.....	14.336	1.307	1.6909
125.....	" "	"	.....	11.45 –11.48½	10.000	.....	16.411	1.468	1.7636
126.....	" "	"	.....	11.49 –11.52	10.020	.....	16.843	1.545	1.840
127.....	" "	"	.....	11.57½–11.59½	10.700	.....	18.200	1.600	1.825
				A. M.					
128.....	" 24 *T.*	"	.....	12.01 –12.08	12.575	.....	20.900	1.902	2.1636
129.....	" "	"	.....	12.08½–12.13	12.327	.....	20.700	1.843	.....
130.....	" "	"	.....	12.13 –12.19	12.400	.....	20.900	1.844	.....
131.....	" "	"	.....	12.20 –12.30	12.476	.....	20.937	1.816	.....
132.....	" "	"	.....	12.31 –12.37	12.900	.....	21.706	1.910	.....
133.....	Full " 38 *T.*	"	.....	12.40 –12.50	14.900	.....	24.805	2.296	2.287
134.....	" "	"	72°	12.55 - 1.06	15.195	.....	25.219	2.301	2.306
135.....	" "	Open 3 *T.*	.....	1.07 - 1.15	15.6215	.....	25.682	2.380	2.432
136.....	" "	" "	71°	1.19 - 1.29½	15.869	72°	26.100	2.440	2.464
137.....	" "	" 6 *T.*	.....	1.31 - 1.34	16.300	.....	26.850	2.545	2.55
138.....	" "	" "	.....	1.35 - 1.42	18.4067	.....	29.458	2.880	2.8636
139.....	" "	" 9 *T.*	71°	1.50 - 1.59	.....	.....	.....	3.144	3.216
140.....	" "	Full " 31½ *T.*	.....	2.00½- 2.25	.....	.....	.....	3.814	(5.400)
141.....	" "	" "	.....	2.34 - 2.46	.....	.....	.....	4.401	4.4797
142.....	" "	Open 18 *T.*	.....	2.48 - 2.57	.....	.....	.....	3.961	4.0393
143.....	" "	" 7 *T.*	.....	2.59 - 3.12	.....	.....	.....	3.0295	3.155
144.....	" "	" 5 *T.*	.....	3.14 - 3.23	.....	.....	(31.400)	2.882	2.932

changes in the consumption of the city would permit. It was intended to observe these variations of velocity by the Pitot tube, but the point selected for observing with this instrument proved to be one where the velocities were highly distorted, and hence it was not considered safe to rely upon their indications. These experiments, Nos. 1 to 15, showed that the resistance in 200 ft. of pipe decreased as the radius of the curve at its center decreased, although the amount of decrease was somewhat indefinite, by reason of the small variation of flow from changes of consumption. Further observations were made on the night of June 7th and 8th, when the 40-ft., 15-ft. and 10-ft. radius curve sections and the Brady tangent were experimented upon at two velocities, and the velocities were measured by traverses with Tube No. 3 at Hastings Street and Alexandrine Avenue. On the night of June 11th and 12th the Brady tangent and all the curve sections, except the first long compound were observed, and all but the 6-ft. radius curve at two velocities. Pitot tube measurements of velocity were made as on the previous night. In the experiments of these two nights the gauges were set up on adjacent sections to be investigated, and were moved alternately past each other to the end of the line, the velocity remaining unchanged. On the last pair of curves, after the lower velocity had been observed, the Hastings blow-off was opened, and the sections similarly observed for the increased velocity. In this manner each curve section was compared by simultaneous observations with those next preceding and following it. While the reductions of these experiments give evidence of some erroneous observations, mainly due to the presence of air in the connecting hose, the series, as a whole, shows conclusively that for the two velocities in question, the least resistance is in the short-radius curves and the greatest in the longest radius curves.

The Second Series.—These results, however, were so contrary to accepted theory and practice that a more comprehensive and more accurate series of experiments was conducted covering several different velocities which were measured by means of the Pitot tubes, the data of which comprise the second portion of Table No. 22, numbered from 44 to 253, inclusive.

The readings observed were plotted with the times as abscissas and the gauge readings as ordinates; from this graphical log, periods of varying lengths were selected during which the velocities remained

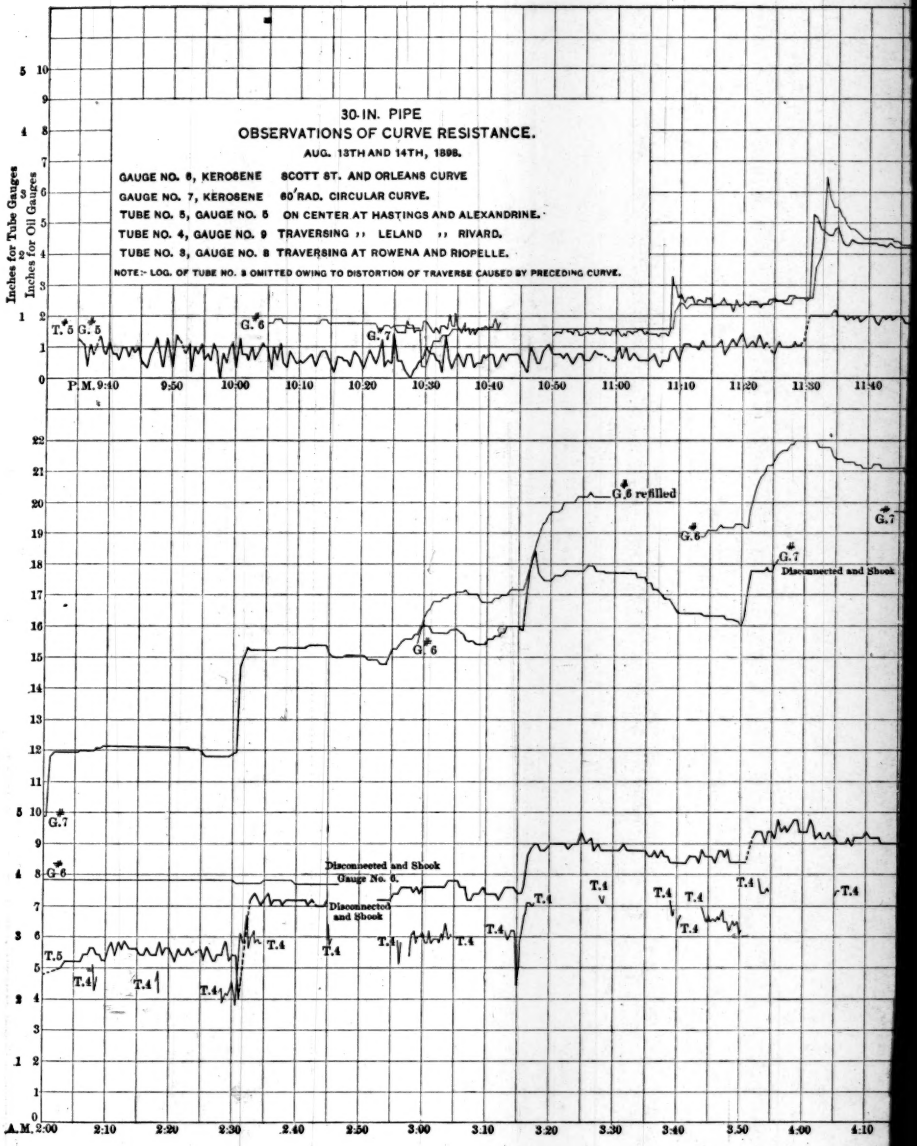
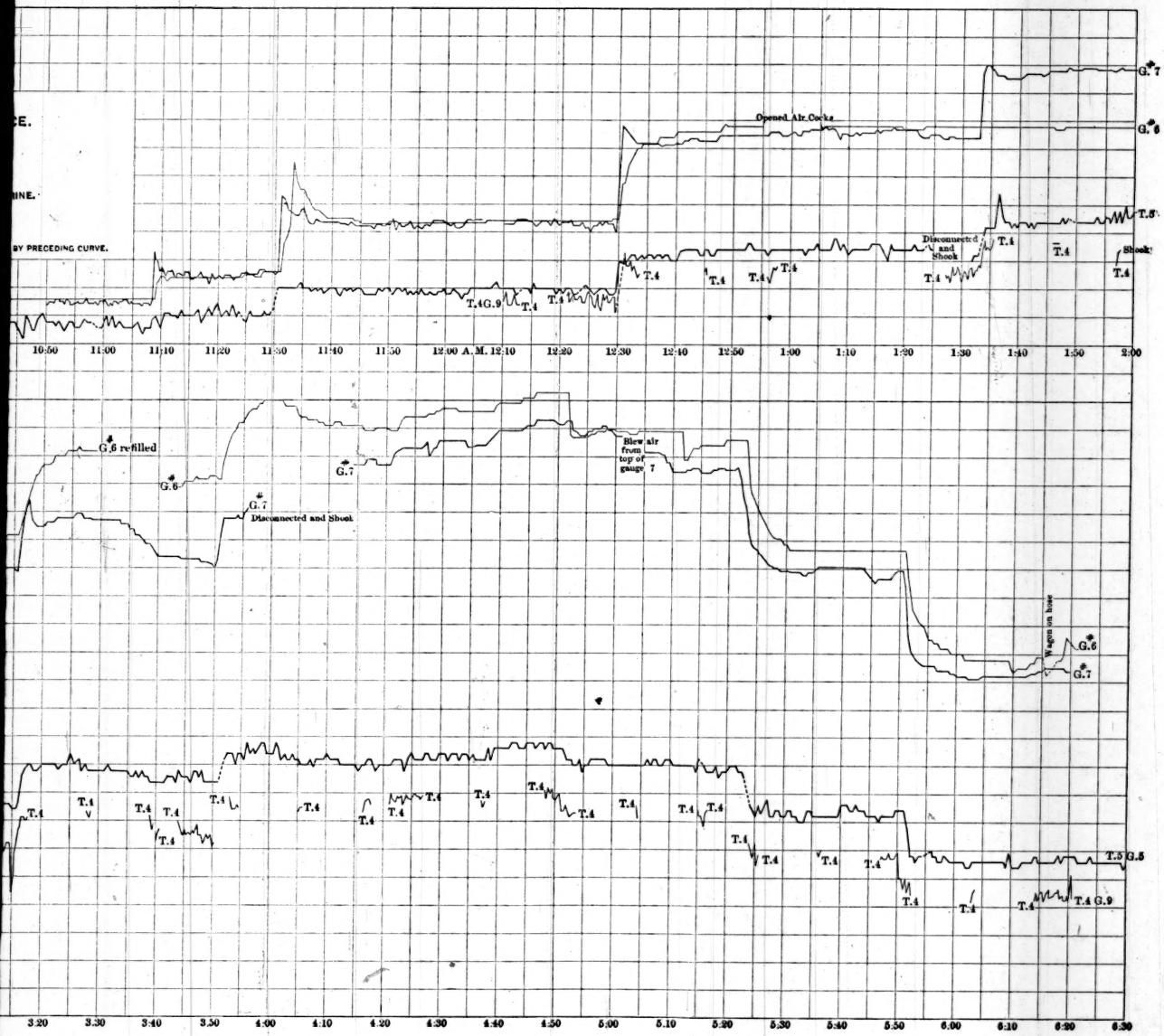

30-IN. PIPE
OBSERVATIONS OF CURVE RESISTANCE.
AUG. 13TH AND 14TH, 1898.
GAUGE NO. 8, KEROSENE SCOTT ST. AND ORLEANS CURVE
GAUGE NO. 7, KEROSENE 60' RAD. CIRCULAR CURVE.
TUBE NO. 5, GAUGE NO. 5 ON CENTER AT HASTINGS AND ALEXANDRINE.
TUBE NO. 4, GAUGE NO. 9 TRAVERSING ,, LELAND ,, RIVARD.
TUBE NO. 3, GAUGE NO. 8 TRAVERSING AT ROWENA AND RIOPELLE.
NOTE:- LOG. OF TUBE NO. 3 OMITTED OWING TO DISTORTION OF TRAVERSE CAUSED BY PRECEDING CURVE.
Inches for Tube Gauges
Inches for Oil Gauges
P.M. 9:40 9:50 10:00 10:10 10:20 10:30 10:40 10:50 11:00 11:10 11:20 11:30 11:40
A.M. 2:00 2:10 2:20 2:30 2:40 2:50 3:00 3:10 3:20 3:30 3:40 3:50 4:00 4:10
T.5
G.5
G.6
G.7
G.6 refilled
G.7 Disconnected and Shook
Disconnected and Shook Gauge No. 6.
Disconnected and Shook
T.4

PLATE VI.
TRANS. AM. SOC. CIV. ENGRS.
VOL. XLVII, No. 911.
WILLIAMS, HUBBELL AND FENKELL
ON FLOW OF WATER IN PIPES.



apparently constant, and then the readings were averaged for each period. Plate VI is a fair sample of one of the logs thus obtained, and Table No. 22 gives all averages obtained, with the date, temperature (where observed) and the condition of blow-off gate.

Reductions: Value of n.—The most simple form of expressing mathematically the relation between the loss of head, H_f, and the velocity, V, whether in a pipe or open channel, is $H_f = m\ V^n + b$, which is an equation applicable to any plane locus. If the locus passes through the origin of co-ordinates, *i. e*, if for $V = 0$, $H_f = 0$, the term represented by b disappears, and the equation becomes $H_f = m\ V^n$, in which, if m and n be determined, there is afforded all the information necessary to compute the discharge or loss of head in the pipes, or the coefficients in the Chezy, Kutter or any other flow formula. The writers have, therefore, adopted this simple expression as the fundamental form for use in their reductions. If all errors are eliminated from the work, it is assumed that $H_f = m\ V^n$, or, for logarithmic plotting by the ordinary method, log. $H_f = n$ log. $V +$ log. m.

To determine the value of n, the series taken on the night of August 20th and 21st, 1898, comparing H_f on Section XIII, 15-ft. radius curve, with Tube No. 5, on the center at Hastings and Alexandrine, has been selected as the one least likely to contain errors of any kind.

TABLE No. 23.—AUGUST 20th and 21st, 1898.

74° Fahr. Gauge No.6, 15-Ft. radius. Kerosene.	Log. Column 1.	Tube No. 5. On center Hastings and Alexandrine; blow-off operated at Canfield.	$\sqrt{\ }$ Column 3.	Mean Vel. Column 4. $\times$ 1.462*	Log. Column 5.
(1)	(2)	(3)	(4)	(5)	(6)
2.310	0.364	0.60850	0.777	1.135974	0.055
3.267	0.514	0.83214	0.912	1.333344	0.125
4.823	0.683	1.25800	1.121	1.638902	0.215
6.402	0.806	1.72800	1.311	1.916682	0.283
8.586	0.934	2.34050	1.529	2.235398	0.349
10.636	1.027	2.76440	1.662	2.429844	0.386
12.562	1.099	3.34500	1.829	2.673998	0.427
13.600	1.100	3.38000	1.839	2.688618	0.430
14.865	1.172	3.96900	1.990	2.909380	0.464
16.800	1.225	4.44160	2.107	3.080434	0.489
17.539	1.244	4.65160	2.156	3.152072	0.499
19.394	1.288	5.24700	2.290	3.347580	0.525
18.224	1.260	4.87500	2.208	3.228096	0.509

*From rating of Tube No. 5 (0.75) and the ratio of Vc to Vm (0.84), the multiplier to reduce the square root of reading on center observed with Tube No. 5, to true mean velocity is 1.462.

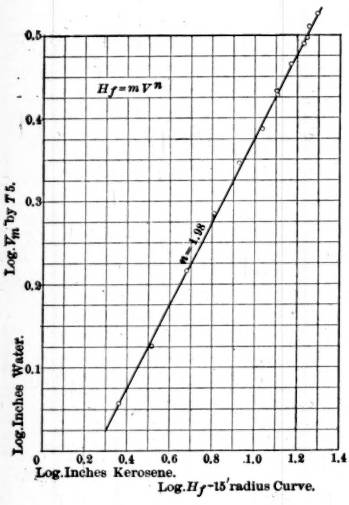


FIG. 27.

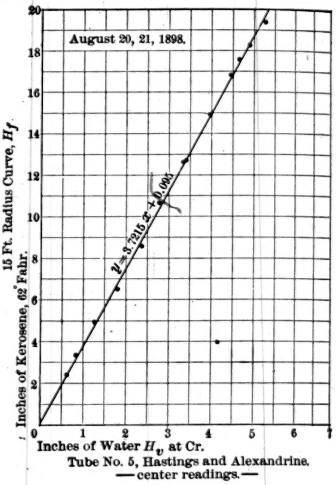


FIG. 28.

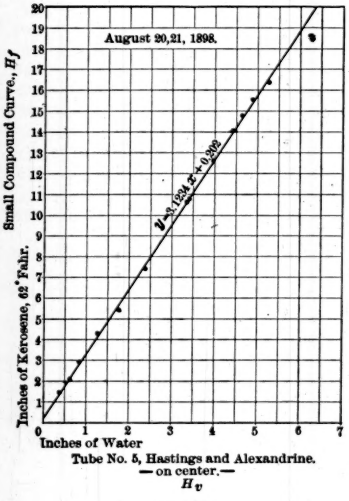


FIG. 29.

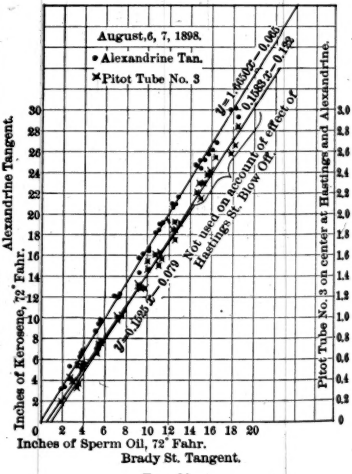


FIG. 30.

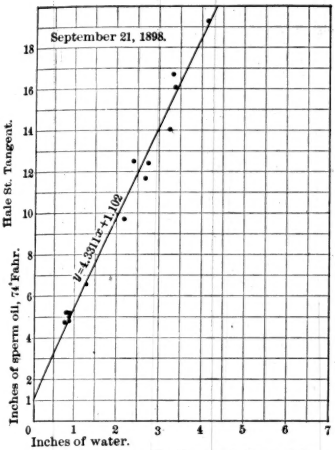


FIG. 31.

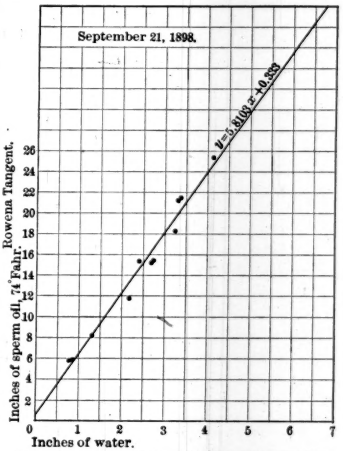


FIG. 32.

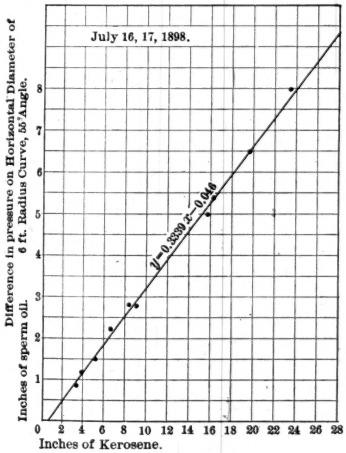


FIG. 33.

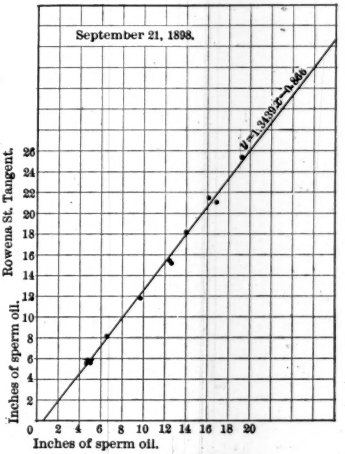


FIG. 34.

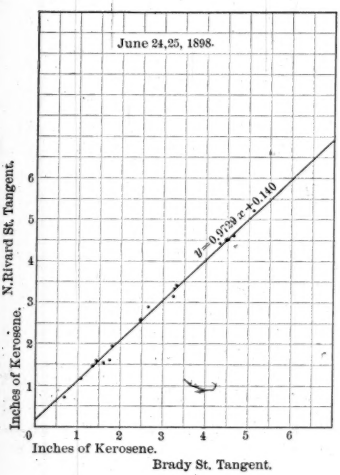


FIG. 35.

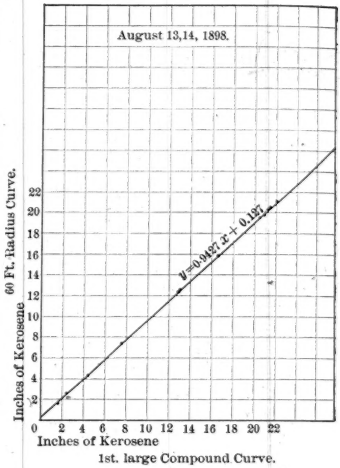


FIG. 36.

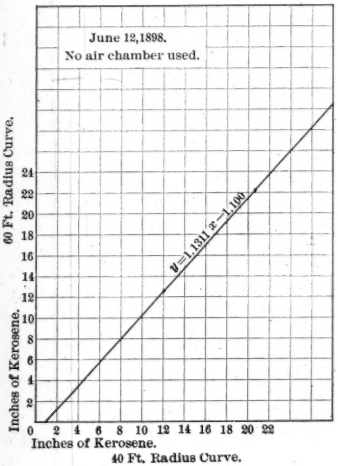


FIG. 37.

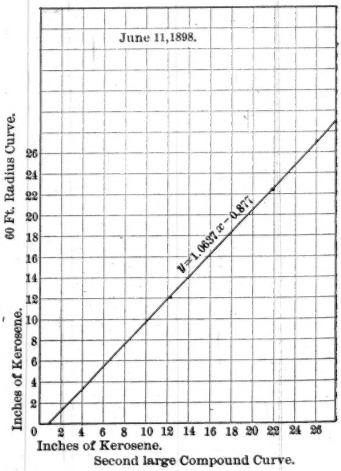


FIG. 38.

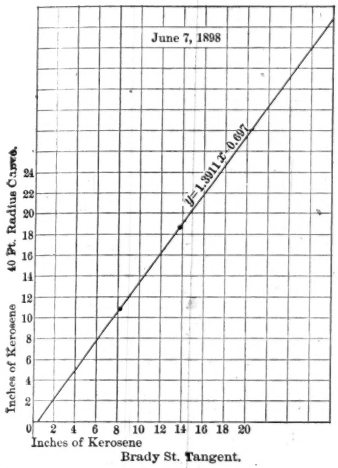
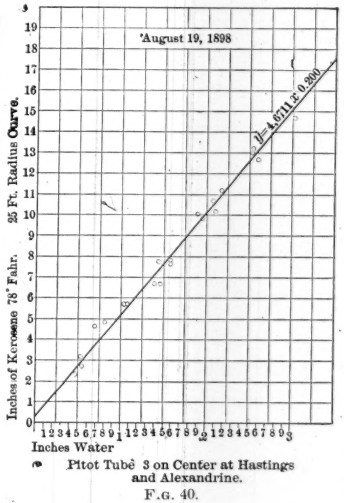


FIG. 39.



F.G. 40.

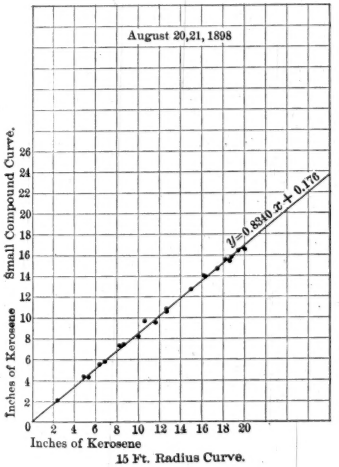


FIG. 41.

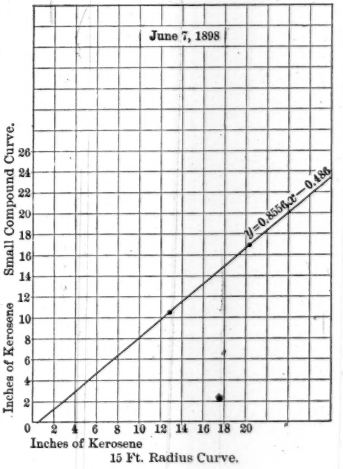


FIG. 42.

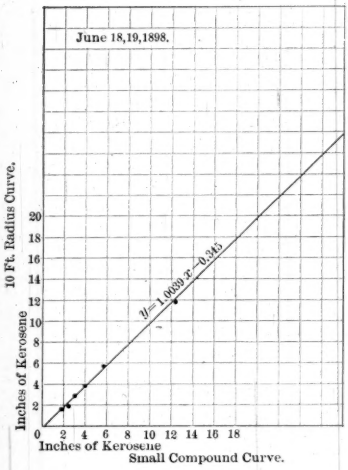


FIG. 43.

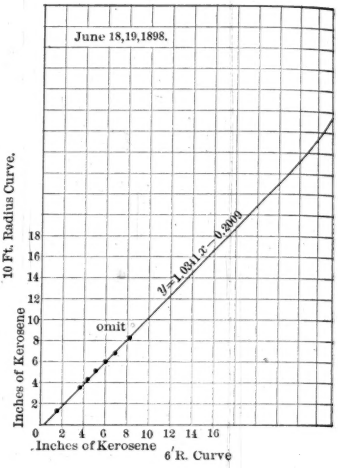


FIG. 44.

The observations and necessary reductions are given in Table No. 23, and the plotted results in Fig. 27, from which the value of n is found to be 1.98.

The value of n is so nearly 2 that it may be said, without material error, that $H_f \propto V^2$, but $V^2 = 2\,g\,h$; therefore, for purposes of comparison, it may be safely assumed that $H_v \propto H_f$. The values of H_v and H_f (Table No. 23) are plotted in Fig. 28 and, as near as the eye can judge, the locus is a straight line.

Reductions: The Straight-Line Method.—If a series of points plotted in this manner falls in a straight line which passes through the origin, n will equal 2. If the straight line of points does not pass through the origin, its equation becomes $y = a\,x \pm b$, and when plotted logarithmically a curve will result. For small values of $\pm\,b$ the curvature is so slight as to be not easily detected, and is usually overlooked, although it may be sufficient to cause a considerable variation in the second decimal place of the value of n.

In the 1898 experiments the flow of water through the pipe line was never completely stopped. Therefore, no observations were made to determine the initial error ($\pm\,b$) which is generally present in

differential gauge readings under a condition of no flow, as shown in later work. Such an error may have existed in the series selected, and, if such was the case, the average line should be moved parallel to itself until it passes through the origin, and the value of n would then be 2 instead of 1.98 as found. It is thus seen that the value 1.98 is a limiting rather than an actual value, and that the true value probably approaches 2.

The straight-line method of reduction has the great advantage of eliminating all initial errors which may exist, for, if $H_v \propto H_p$, the average line, representing observed points, should pass through the origin, and if it fails to do so its distance from the origin represents the initial error, which, however, does not affect the slope of the line.

It will be shown, later, that initial errors do exist and must be eliminated in order to reach correct conclusions. For these reasons, and because of the ease of its application to Pitot tube readings, which are observed in velocity head (H_f), the straight-line method has been adopted in reducing all 30-in. data. The average straight line is computed by first obtaining the average or center of gravity of all accepted points, thus dividing the series of points into two parts, an upper and a lower. The center of gravity of each portion is then found, and the three points thus computed will fall in a straight line if no errors have been made in computations. From the co-ordinates of these points the equation of the average line is computed. This method of averaging has been used by the writers in a large number of experiments, and is much more satisfactory than the method of averaging by eye with the aid of a fine thread. It involves much less labor than the method of least squares, and, with the exception that it does not give the probable error, is fully as satisfactory for straight-line loci.*

Reductions: Diagrams.—The observations given in Table No. 22 have not all been used in the reductions here given, but are published as taken in the course of the experiments. Figs. 27 to 44 inclusive, show the series which have been selected by the writers, from their familiarity with all the conditions, as least likely to contain error, and as adapted to show without exaggeration the phenomena investigated. The diagrams are generally self-explanatory, and illustrate at a glance the accuracy of the observations plotted.

*This method of reduction is further discussed in the paper "A Study in Hydraulics," by one of the writers, in *The Journal of the Association of Engineering Societies*, March, 1901.

Without entering into a detailed description of each diagram, the equations obtained from them are summarized in Table No. 24, Columns 1, 2 and 3.

Reductions: Relative Equations.—In order to reduce all results to the same basis for comparison, the equations obtained from the diagrams are combined to obtain the ratio of H_f in each experimental section, to H_f simultaneously observed in the Brady Street tangent. The results are shown in Columns 4 to 9 inclusive. Column 8 gives the multiplier to reduce H_f in the Brady Street tangent to H_f in the corresponding section of pipe as observed without correction for length. Column 9 shows the results corrected to a uniform length of 200 ft. There is no apparent inherent cause for the increase in frictional resistance in the Hale, Rowena, and Brady Street tangents, respectively, for these three tangents contain no special castings; the average diameters are so nearly alike that the variation may be neglected; and they are located relatively the same with respect to curves.

The presence of specials in the North Rivard Street tangent evidently causes a considerable increase in the frictional resistance, and the East Alexandrine Street tangent corresponds very closely to the Brady Street tangent. The results again persistently show that the curves of long radius offer more resistance to the flow of water than the curves of short radius. Some of the curve sections, however, contain gates and special castings which, as the results on North Rivard Street tangent prove, have an appreciable effect. This fact, together with the unaccountable increase in resistance between the first three tangents, made it seem to the writers desirable to extend the investigations further, and no general deductions will be made until the experiments which follow have been described.

Experiments of 1899.

Conditions.—All experiments made in June, 1899, were conducted in the day time. Instruments can be more easily connected and read by daylight than at night, and the presence of leaks or air in the connections is more likely to be detected, but day work has the disadvantage of greater variation in temperature, and in this case it was not possible to obtain as high velocities as in the previous experiments which were conducted at night. The line was thoroughly flushed to remove sediment before beginning experiments.

TABLE No. 24.

Fig. No.	Observed Equation.		Computed Ratio to Brady Tangent.					
			Experimental Section.		Equations combined.	Computation.	Ratio.	Column 8 corrected for length.
			No.	Location.				
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
28	(1) $\frac{15' \text{ R. K. } 62°}{\text{T. 5 Cr. Hastings}}$ =	3.7215	I	Orleans and Scott Comp. C.....	(10), (11), (13)	$\frac{1.1311 \times 1.3911}{0.9427}$ =	1.6691	.....
29	(2) $\frac{\text{S. Comp K. } 62°}{\text{T. 5 Cr. Hastings}}$ =	3.1234	III	Orleans and Hale Comp. C.....	(11), (12), (13)	$\frac{1.1311 \times 1.3911}{1.0637}$ =	1.4782	1.4782
30	(3) $\frac{\text{Alex. Tan. K. } 72°}{\text{Brady Tan. S } 72°}$ =	1.6650	IV	Hale Street Tangent.....	(5), (4)	$\frac{4.3311 \times 0.1525}{0.7109}$ =	0.9291	0.7997
30	(4) $\frac{\text{T. 3 Cr Hastings}}{\text{Brady Tan. S. } 72°}$ =	0.1525	V	Hale and Riopelle, 60' R. C.....	(13), (11)	1.3911×1.1311 =	1.5735	1.5735
31	(5) $\frac{\text{Hale St. Tan. S. } 74°}{\text{T. 5 Cr. Hastings}}$ =	4.3311	VII	Riopelle and Rowena, 40' R. C....	(13)	.....	1.3911	1.3911
32	(6) $\frac{\text{Rowena Tan. S. } 74°}{\text{T. 5 Cr. Hastings}}$ =	5.8103	VIII	Rowena Street Tangent.....	(6), (4)	$\frac{5.8103 \times 0.1525}{0.7109}$ =	1.2464	0.8530
33	(7) $\frac{\text{6' R. Hor. Diam.}}{\text{E. Alex. Tan.}}$ =	0.3339	IX	Rowena and Russell, 25' R. C.....	(14), (4)	$\frac{8.833 \times 0.1525 \times 4.671}{4.815}$ =	1.3068	1.3068
35	(8) $\frac{\text{Rowena Tan. (S.)}}{\text{Hale Tan. (S.)}}$ =	1.3439	XI	Russell and Brady Small C. C.....	(2), (4)	$\frac{3.1234 \times 0.1525 \times 8.833}{0.7109 \times 4.881}$ =	1.1409	1.1409
35	(9) $\frac{\text{N. Rivard Tan. (K.)}}{\text{Brady Tan. (K.)}}$ =	0.9729	XII	Brady Street Tangent.....	.....	.....	1.0000	0.9423
36	(10) $\frac{60' \text{ R. (K.)}}{\text{1st L. Comp. (K.)}}$ =	0.9427	XIII	Brady and Rivard, 15' R. C.....	(1), (4)	$\frac{3.7215 \times 0.1525 \times 8.833}{0.7109 \times 4.881}$ =	1.3626	1.3626
37	(11) $\frac{60' \text{ R. (K.)}}{40' \text{ R. (K.)}}$ =	1.1311	XV	North Rivard St. Tangent.....	(9)	.....	0.9729	1.0514
38	(12) $\frac{60' \text{ R (K.)}}{\text{2d L. Comp. (K.)}}$ =	1.0637	XVI	Rivard and Alexandrine, 10' R. C.	(17), (2), (4)	1.1409×1.0039 =	1.1453	1.1453
39	(13) $\frac{40' \text{ R. (K.)}}{\text{Brady Tan. (K.)}}$ =	1.3911	XVII	East Alexandrine Tangent.....	(3)	$1.665 \times \frac{8.833}{4.815}$ =	3.0543	0.9463
40	(14) $\frac{25' \text{ R. (K. } 78°)}{\text{T. 3 Cr. Hastings}}$ =	4.6710	XX	Alexandrine and Alley, 6' R. C.	(18), (17), (2) and (4)	$\frac{1.1453}{1.0341}$ =	1.1073	1.1073
41	(15) $\frac{\text{S. Comp. (K.)}}{15' \text{ R. (K.)}}$ =	0.8340	.....	.....	.....	.....	.....	.....
42	(16) $\frac{\text{S. Comp. (K.)}}{15' \text{ R. (K.)}}$ =	0.8556	.....	.....	.....	.....	.....	.....
43	(17) $\frac{10' \text{ R. C. (K.)}}{\text{S. Comp (K.)}}$ =	1.0039	.....	.....	.....	.....	.....	.....
44	(18) $\frac{10' \text{ R. (K.)}}{6' \text{ R. (K.)}}$ =	1.0341	.....	.....	.....	.....	.....	.....

Plan of Work.—Gauges Nos. 6 and 7 were filled with kerosene and water, the oil being of the same grade, taken from a gallon can which was filled before beginning the work, and was sufficient in quantity to allow the gauges to be refilled several times during the experiments, and afterward to be calibrated. Each day, Gauge No. 7 was placed on the Brady tangent and was read and recorded every 30 seconds, from the time of beginning in the morning until the closing down at night, except for a short time for lunch. Likewise, Pitot Tube No. 6, with Gauge No. 5, filled with air and water, was set at an angle of 45° with the vertical, with the point at the center of the 30-in. pipe, 2.83 ft. east of the 30-in. gate, and in the gate-well at Alexandrine and Hastings Streets, and was read and recorded every 30 seconds from about 9.00 A. M. until about 4.30 P. M. except for a short intermission for dinner.

Differential Gauge No. 6 was moved about from section to section, and, in the course of the work, covered all the curves and tangents from the beginning to the end of the line, except the Brady tangent, on which Oil Gauge No. 7 was located. Owing to the fact that the main was supplying the north and northwestern parts of the city, and that the work was being carried on in hot weather, when the demand for water was great, it was impossible to close down the line except for a few minutes; and, during that interval, the affected part of the city was supplied, through small pipes and from adjacent mains, with a limited quantity of water at a reduced head. The cooler of a brewery located at the corner of Alexandrine and Hastings Streets could be observed, and, as long as sufficient pressure existed in the side lines to supply that with plenty of water when the 30-in. pipe was shut off, it could remain closed.

The different velocities of flow through the 30-in. pipe were regulated and maintained by manipulating the 42-in. gate and its 8-in. by-pass on the east side of St. Antoine Street at Canfield Avenue, and by wasting into the sewer through the 12-in. blow-off at the alley east of St. Antoine Street and Canfield Avenue.

The work required the services of at least seven men. Each morning the watches were set alike, and comparisons were made before leaving at night. Whenever it was possible, Oil Gauges Nos. 6 and 7 were kept shaded either in the shadow of a tree or building, or under an umbrella, and a thermometer was attached to each and read at intervals

of from 5 to 10 minutes. Air chambers with double hose connections were used on the Brady tangent when Oil Gauge No. 6 was set on the curve at either end, and on June 21st, when using Oil Gauge No. 6, on the West Alexandrine tangent an extra air chamber was used at Hastings Street, in order not to interfere with the street car service. This chamber was set on the railway tracks, and was provided with an air-cock, and two hose connections with shut-off valves. When a car was seen approaching, both valves were closed, and the short piece of hose which crossed the track was disconnected and carried clear of it. After the car had passed, the hose was reconnected, its valve opened into the air chamber, and air allowed to escape through the air-cock. The air-cock was then closed and the valve leading to Oil Gauge No. 6 opened. During the observations on this tangent the connection was broken twenty times to allow cars to pass and only three readings at 30 seconds intervals were omitted in the entire series.

The hose used on each side of Oil Gauge No. 7, on the Brady tangent, consisted of 50 ft. of $\frac{1}{4}$-in. hose connected to the air chamber, then 50 ft. of $\frac{3}{8}$-in. hose, then 23 ft. of $\frac{1}{4}$-in. hose connected to the gauge. On each side of Oil Gauge No. 6, when used on the curves or short tangents, 25 ft. of $\frac{3}{8}$-in. hose was connected to the air chamber, then 50 ft. of $\frac{3}{8}$-in. hose, then 50 ft. of $\frac{1}{4}$-in. hose to the gauge. On the longer tangents additional 50-ft. lengths of $\frac{1}{4}$ and $\frac{3}{8}$-in. hose were added to give the required length, care being taken to keep the gauge as nearly as possible midway between the air chambers.

Table No. 25 gives an outline of the experiments conducted.

Date.	Gate.	Time. Began.	Opened.	Closed.	No. of turns operated.	Condition of gate.	Location of Gauge No. 6.	Remarks.
June 15	42-in.	9.19	.....	9.26	30	Closed.	Rowena tangent.	
	8-in. B. P.	9.26.30	.....	9.28	27	"	Began reading, 8.30.	
	"	9.39	9.40.30	.....	27	Open.		
	42-in.	10.00	10.02.30	.....	10	10 turns open.		
	"	10.17	10.21.30	.....	20	30 " "		
	12-in. B. O.	10.37	10.37.40	.....	5	5 " "	Stopped reading, 10.50.	
	"	11.16	.....	11.16.50	5	Closed.	Russell and Rowena curve, 25-ft. R.	
	42-in.	11.17.10	.....	11.25	30	"	Began reading, 11.10.	
	8-in. B. P.	11.25.30	.....	11.27.30	27	"	Dinner, 12.06 till 12.41.	
	"	11.37.30	11.39.30	.....	27	Open.		
	42-in.	11.55	11.59	.....	25	25 turns open.		
	12-in. B. O.	12.40	12.40.50	.....	6	6 " "	Quit reading, 1.01.	
	"	1.41	.....	1.41.30	6	Closed.	Russell tangent.	
	42-in.	1.43	.....	1.48	25	"	Began reading, 1.16.	
	8-in. B. P.	1.45.30	.....	1.50	27	"		
	"	2.00	2.02	.....	27	Open.		
	42-in.	2.17	2.21	.....	25	25 turns open.		
	12-in. B. O.	2.37	2.38	.....	6	6 " "	Quit reading, 2.53.	
	"	3.15	.....	3.15.30	6	Closed.	Russell and Brady curve, short compound.	Used air chamber on Brady curve, with double hose connection.
	42-in.	3.16	.....	3.22	25	"		
	8-in. B. P.	3.22.30	.....	3.24.30	27	"	Began reading, 3.04.	
	"	3.35	3.37	.....	27	Open.		
	42-in.	3.52	3.56	.....	30	30 turns open.	Quit reading, 4.06.	Quit with 8-in. B. P. open, 42-in. gate 30 turns open.
June 16	42-in.	9.30	.....	9.37	30	Closed.	Rivard and Brady curve, 15-ft. R.	Used air chamber on Brady curve, with double hose connection.
	8-in. B. P.	9.37.30	.....	9.39.30	27	"	Began reading, 9.00.	
	12-in. B. O.	9.49.30	9.50.10	.....	5	5 turns open.		
	"	10.04.30	10 05	.....	5	10 " "		
	"	10.20	10.20.30	.....	5	15 " "		Several complaints of short supply.
	"	10.30	.....	10.30.50	10	5 " "		
	"	10.45	.....	10.46.30	5	Closed.	Quit reading, 11.02.	
	8-in. B. P.	11.55	11.56.40	.....	27	Open.	Rivard and Alexandrine curve, 10-ft. R.	
	42-in.	12.55	12.58.38	.....	25	25 turns open.	Began reading, 11.50.	
	12-in. B. O.	1.13	1.15	.....	5½	3½ " "		
	"	1.30	.....	1.30.50	5½	Closed.	Quit reading, 1.42.	
	42-in.	2.52.30	.....	2.58	25	"	Alexandrine and Alley E of St. A curve, 6-ft. R.	
	8-in. B. P.	2.58.20	.....	3.02	27	"	Began reading, 2.38.	
	"	3.12	3.15	.....	27	Open.		
	42-in.	3.30	3.34	.....	20	20 turns open.		
	"	3.49	3.50	.....	5	25 " "		
	12-in. B. O.	3.51	3.52.15	.....	5½	5½ " "		
	"	4.07	.....	4.08	5½	Closed.	Quit reading, 4.18.	Quit with 8-in. B. P. open, 42-in. gate 25 turns open.

TABLE No. 25.

Date.	Gate.	Time. Began.	Opened.	Closed.	No. of turns operated.	Condition of gate.	Location of Gauge No. 6.	Remarks.
1899.								
June 13	42-in.	11.43	.....	11.51.30	50	Closed.	Orleans and Scott Compound Curve; began reading, 11.28.	Gate was only partly open.
	8 in. B. P.	11.52	.....	11.55	27	"		
	"	12.10	12.12	.....	27	Open.	Dinner, 12.15 till 12.46.	
	42-in.	12.45	12.49	.....	25	25 turns open.	Stopped reading, 1.06.30.	
	"	1.26	.....	1.32.30	25	Closed.	Orleans Tangent.	
	8-in. B. P.	1.33	.....	1.35	27	"	Began reading, 1.14.	
	"	1.50	1.51.50	.....	27	Open.		
	42-in.	1.52	1.53.30	.....	6	6 turns open.		
	"	2.06	2.11	.....	24	30 " "	Stopped reading, 2.31.30.	
	"	3.01	.....	3.10	30	Closed.	Orleans and Hale Compound Curve.	
	8-in. B. P.	3.11	.....	3.13	27	"	Began reading, 2.42.	
	"	3.28	3.30	.....	27	Open.		
	42-in.	3.43	3.45	.....	6	6 turns open.		
	"	3.59	4.04	.....	24	30 " "	Stopped reading, 4.08.	Quit with 8-in. B. O. open, 42-in. 30 turns open.
June 14	42-in.	10.04.30	.....	10.12	30	Closed.	Hale Tangent.	
	8-in. B. P.	10.13	.....	10.15	27	"	Began reading, 9.10.	
	"	10.30	10.32	.....	27	Open.		
	42-in.	10.33	10.36	.....	10	10 turns open.		
	"	10.55	11.00	.....	20	30 " "	Stopped reading, 11.14.	
	"	12.20	.....	12.27	30	Closed.	Riopelle and Hale Curve, 60-ft. R.	
	8-in. B. P.	12.27.30	.....	12.29	27	"	Began reading, 11.33.	
	"	12.40	12.42	.....	27	Open.	Dinner, 11.50 till 12.15.	
	42-in.	1.00	1.03	.....	10	10 turns open.		
	"	1.20	1.26	.....	20	30 " "	Stopped reading, 1.38.	
	"	2.12	.....	2.19.30	30	Closed.	Riopelle Tangent.	
	8-in. B. P.	2.20	.....	2.22	27	"	Began reading, 1.50.	
	"	2.35	2.37	.....	27	27 turns open.		
	42-in.	2.51	2.53.30	.....	10	10 " "		
	"	3.08	3.11.30	.....	20	30 " "	Stopped reading, 3.20.	
	"	3.42	.....	3.50	30	Closed.	Riopelle and Rowena Curve, 40-ft. R.	
	8-in. B. P.	3.50.30	.....	3.52	27	"	Began reading, 3.34.	
	"	4.00	4.02	.....	27	Open.		
	42-in.	4.15	4.17	.....	10	10 turns open.		
	"	4.30	4.35	.....	20	30 " "	Stopped reading, 4.40.	Quit with 8-in. B. O. open, 42-in. 30 turns open.

TABLE No. 25—(*Continued*).

Date.	Gate.	Time. Began.	Opened.	Closed.	No. of turns operated.	Condition of gate.	Location of Gauge No. 6.	Remarks.
June 21	42-in.	10.33	.....	10.42	25	Closed.	North tangent, Alley E. of St. A.	
	8-in. B. P.	10.42	.....	10.47	27	"	Began reading, 10.13.	
	"	11.01	11.04	.....	27	Open.		
	42-in.	11.20	11.26	.....	25	25 turns open.		
	12-in. B. O.	11.41	11.42	.....	5½	5½ " "	Stopped reading, 12.00.	
	"	11.57	.....	11.58	5½	Closed.		
	42-in.	2.59	.....	3.06	25	"	West Alexandrine tangent.	Single track street railway on Hastings Street, 4 minute service. Used extra air chamber and disconnected part of hose to allow cars to pass.
	8-in. B. P.	3.07	.....	3.10	27	"	Began reading, 2.41.	
	"	3.24.30	3.27	.....	27	Open.		
	42-in.	3.42	3.45	.....	25	25 turns open.	Broke connection 20 times.	
	12-in. B. O.	4.09	4.10	.....	5½	5½ " "		
	"	4.23	.....	4.24	5½	Closed.	Stopped reading, 4.19.	Quit with 8-in. B. P. open, 42-in. gate 25 turns open.
June 22	12-in. B. O.	10.23	10.24	.....	5¼	5¼ turns open	Did not use Gauge No. 6.	Took cross-section of pipe with Pitot Tubes Nos. 3 and 6 at Alexandrine and Hastings Streets. Gauge No. 7 on Brady, as usual.
	"	11.31	.....	11.32	5¼	Closed.		
	42-in.	11.33	.....	11.39	25	"		
	8-in. B. P.	11.40	.....	11.43	27	"		
	"	12 12	12.14	.....	27	Open.		
	42-in.	12 47½	12.55	.....	25	25 turns open.		
	12-in. B. O.	2.02	2.03	.....	5¼	5¼ " "		
	"	3.05	.....	3.06	5¼	Closed.		

TABLE No. 25—(*Continued*).

Date.	Gate.	Time. Began.	Opened.	Closed.	No. of turns operated.	Condition of gate.	Location of Gauge No. 6.	Remarks.
June 17	42-in.	10.05	.....	10.15	25	Closed.		
	8-in. B. P.	10.15	.....	10.18.15	27	"	North Rivard tangent.	
	"	10.37	10.40.30	....	21	21 turns open.	Began Reading, 9.55.	
	"	10.55	10.55.30	.....	6	Open.		
	42-in.	10.56	10.58.30	.....	10	10 turns open.		
	"	11.13.30	11.20	.....	20	30 " "	Stopped reading. 11.47.	
	"	1.44	.....	1.52.30	30	Closed.		
	8-in. B. P.	1.52.30	.....	1.58	27	"	South Rivard tangent.	
	"	2.19	2.21	.....	20	20 turns open.	Began reading, 1.30.	
	"	2.39	2.39.45	.....	7	Open.		
	42-in.	2.40	2.42.30	.....	11	11 turns open.		
	"	3.10	3.21	.....	64	75 " "	Stopped reading, 3.41.	Left 8-in. B. P. open, 42-in. gate 75 turns open.
June 19	42-in.	10.41	.....	10.57	75	Closed.		
	8-in. B. P.	10.58	.....	11.01	27	"	East Alexandrine tangent.	
	"	11.13	11.15	.....	27	Open.	Began reading, 10.01.	
	42-in.	11.30	11.35	.....	25	25 turns open.		
	12-in. B. O.	11.50.30	11.51.30	.....	5	5 " "	Stopped reading, 12.11.	
	"	12.06.30	.....	12.07.30	5	Closed.	Hastings Street specials.	4-in. B. P. to 30-in. gate partly open.
	42-in.	2.20	2.23	.....	10	35 turns open.	Began reading, 1.50.	
	"	2.46	.....	2.55	35	Closed.		
	8-in. B. P.	2.55	.....	3.00	27	"	Stopped because of fire on Rivard Street, 1.55 to 2.42.	
	"	3.10	3.12.30	.....	27	Open.		
	42-in.	3.27	3.37	.....	30	30 turns open.		
	12-in. B. O.	3.49	3.50	.....	5½	" "		
	"	4.05	.....	4.06	5½	Closed.	Stopped reading, 4.10.	Left 8-in. B. P. open, 42-in. gate 30 turns open.
June 20	42-in.	10.38	.....	10.44	30	"	South tangent, Alley E. of St. A.	At 11.04 found leak in connection at west end of Brady tangent.
	8-in. B. P.	10 45	.....	10.47	27	"		
	"	11.03	11.05	.....	27	Open.	Began reading, 10.27.	
	42-in.	11.15	11.18	.....	10	10 turns open.		
	"	12.58	.....	1.01	10	Closed.	Gauges Nos. 6 and 7 acted strangely all day.	Dug up, replaced wrought iron elbow, refilled trench in 45 minutes.
	8-in. B. P.	1.01	.....	1.04	27	"		
	"	1.44	1.46	.....	27	Open.	Tried different lengths of hose.	
	"	2.37	.....	2.40.30	27	Closed.	Spent nearly half the day, but could not find cause of trouble.	
	"	2.50	2.53	.....	27	Open.		
	42-in.	3.10	3.16	.....	25	25 turns open.		
	12-in. B. O.	3.31	3.32	.....	5½	5½ " "		
	"	3.46	.....	3.47	5½	Closed.	Stopped reading, 3.57.	Left 8-in. B. P. gate open, 42-in. gate 25 turns open.

TABLE No. 26.

JUNE 13TH, 1899.

Brady St. Tangent, 212.25 ft. Gauge No. 7, kerosene and water.						Orleans and Scott, Comp. curve, 162.25 ft. Gauge No. 6, kerosene and water.					Pitot Tube No. 6, Gauge No. 5. On center at Alexandrine and Hastings.			
No.	T., Fahr.	No. readings	Average inches, kerosene.	Equivalent reading at 60° Fahr.	Corrected for initial error ±.	T., Fahr.	No. readings.	Average inches, kerosene.	Equivalent reading at 60° Fahr.	Corrected for initial error ±.	No. readings.	Average reading, in inches, water.	Corrected for initial error ±.	Corrected equiva
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
	Deg.				+0.108	Deg.				−0.620			+0.369	
1	84	15	5.900	6.087	6.195	83	17	9.894	10.197	9.577	13	1.384	1.453	5.88
2	87	20	−0.105	−0.108	0.000	86	20	0.600	0.620	0.000	21	−0.069	0.000	0.00
3	87	12	0.892	0.923	1.031						13	0.123	0.192	0.77
4	89	19	4.495	4.663	4.771	93½	19	8.089	8.432	7.812	19	1.171	1.240	4.90
5	88	12	4.667	4.837	4.945	94½	12	8.445	8.812	8.192	12	0.808	Rej.	
JUNE 13TH, 1899.—Orleans Street Tangent, 56.42 ft.														
					+0.336					+0.304			+0.171	
6	89	23	4.813	4.993	5.329	87	23	1.039	1.076	1.380	23	1.208	1.379	5.53
7	89	25	−0.324	−0.336	0.000	88	13	−0.293	−0.304	0.000	17	−0.171	0.000	0.00
8	91	12	1.465	1.524	1.860	90	23	0.029	0.030		23	0.435	0.606	2.42
9	90	14	4.957	5.148	5.484	90	21	0.800	0.831	Rej.	15	1.278	1.449	5.80
10	86	15	4.893	5.060	5.396	90	12	0.800	0.831		15	1.260	1.431	5.76
11	92	19	4.963	5.165	5.501						18	1.318	1.489	6.00
JUNE 13TH, 1899.—Orleans and Hale, Compound Curve, 200 ft.														
					+0.483					−0.079			+0.158	
12	93	35	5.014	5.223	5.706	89	35	7.592	7.877	7.798	35	1.311	1.469	5.89
13	95	21	−0.463	−0.483	0.000	89	20	+0.076	+0.079	0.000	22	−0.158	0.000	0.00
14	94	24	0.600	0.626	1.109	90	23	1.596	1.658	1.579	24	+0.202	0.360	1.43
15	97	23	1.248	1.305	1.788	90	21	2.448	2.542	2.463	24	0.385	0.543	2.18
16	96	14	5.085	5.314	5.797	90	9	7.711	8.009	7.930	15	1.343	1.501	6.00
JUNE 14TH, 1899.—Hale Street Tangent, 234.92 ft.														
					+0.189					+0.017			+0.086	
17	82	87	5.421	5.580	5.769	81	87	5.561	5.718	5.735				
18	81½	13	5.483	5.641	5.830	80½	13	5.623	5.778	5.795	14	1.336	1.422	5.73
19	83½	18	−0.183	−0.189	0.000	81	18	−0.017	−0.017	0.000	18	−0.086	0.000	0.00
20	82½	27	2.259	2.327	2.516	82	27	2.400	2.470	2.487	27	0.520	0.606	2.44
21	84	25	5.276	5.544	5.633	83	25	5.624	5.796	5.813	25	1.408		
22	86	37	5.171	5.347	5.536						37	1.391	1.477	5.93
JUNE 14TH, 1899.—Hale and Riopelle, 60-ft. Radius Curve, 200 ft.														
					+0.223					−0.092			+0.052	
23	87	25	5.296	5.483	5.706	89½	25	8.936	9.285	9.193	25	1.426	1.478	5.95
24	89	13	−0.215	−0.223	0.000	88	15	+0.087	+0.092	0.000	15	−0.052	0.000	0.00
25	88½	29	0.959	0.994	1.217	88½	29	1.681	1.743	1.651	31	0.206	0.258	1.04
26	91	35	2.094	2.177	2.400	88	37	3.468	3.594	3.502	35	0.496	0.548	2.20
27	89	25	5.188	5.382	5.605	88½	25	8.556	8.872	8.780	25	1.350	1.402	5.65
28	90½	23	5.215	5.419	5.642						23	1.361	1.413	5.69
JUNE 14TH, 1899.—Riopelle Street Tangent, 47.88 ft.														
					+0.191					0.000			+0.016	
29	90	10	5.245	Rej.	Rej.	87½	21	1.181	Rej.		21	1.405		
30	88	18	5.269	5.461	5.652	87	24	1.300	1.346	1.346	24	1.415	1.431	5.76
31	88	25	−0.184	−0.191	0.000	88	21	0.000	0.000	0.000	29	−0.016	0.000	
32	90	23	0.987	1.025	1.216	89	25	0.252	0.261	0.261	22	0.200	0.216	0.87
33	92¾	25	2.284	2.379	2.570	88	21	0.509	0.528	0.528	21	0.590	0.606	2.44
34	97	19	5.453	5.704	5.895	88½	17	1.259	1.305	1.305	17	1.465	1.481	5.96
35	98	16	5.200	5.445	5.636						16	1.394	1.410	5.68
JUNE 14TH, 1899.—Riopelle and Rowena, 40-ft. Radius Curve, 200 ft.														
					+0.203					0.000			+0.013	
36	98	8	5.550	5.811	6.014						8	1.462	1.475	5.94
37	95	17	5.465	5.705	5.908	89	17	7.500	7.781	7.781	17	1.453	1.466	5.90
38	93	11	−0.195	−0.203	0.000	88	10	0.000	0.000	0.000	15	−0.013	0.000	0.00
39	93½	21	1.060	1.105	1.308	87½	21	1.524	1.579	1.579	25	0.282	0.295	1.19
40	95½	21	2.076	2.168	2.371	87	20	2.990	3.095	3.095	23	0.539	0.552	2.22
41	98	11	5.423	5.678	5.881	87	10	7.600	7.868	7.868	7	1.414	1.427	5.75

*H_{vc}, by Pitot Tube No. 6. at Alexandrine and Hastings = H_f in Brady Tangent ÷ 4.080.

TABLE No. 26—(*Continued*).

JUNE 15TH, 1899.

	Brady St. Tangent, 212.25 ft. Gauge No. 7, kerosene and water.					Rowena St. Tangent, 292.23 ft. Gauge No. 6, kerosene and water.					Pitot Tube No. 6, Gauge No. 5. On center at Alexandrine and Hastings.			
No.	T., Fahr.	No. readings.	Av'age inches, kerosene.	Equivalent reading at 60° Fahr.	Corrected for initial error ±	T., Fahr.	No. readings.	Av'age inches, kerosene.	Equivalent reading at 60° Fahr.	Corrected for initial error ±	No. readings.	Av'ge reading in inches, water.	Corrected for initial error ±	Corrected result × 4.060
(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)	(25)	(26)	(27)	(28)	(29)	(30)
	Deg.				+0.012	Deg.				+ 0.715			+0.051	
42	79	53	4.919	5.046	5.058	75½	53	5.717	5.839	6.654	.....	.....	.....	.....
43	82	17	—0.012	—0.012	0.000	76	12	— 0.700	— 0.715	0.000	17	— 0.051	0.000	0.000
44	82 7/10	29	1.138	1.172	1.184	76	28	1.011	1.033	1.748	29	0.211	0.262	1.056
45	83	10	2.190	2.257	2.269	76	10	2.510	2.565	3.280	10	0.425	0.476	1.918
46	85	15	2.027	2.094	2.106	76	15	2.100	2.146	2.861	15	0.412	0.464	1.870
47	84	24	4.550	4.695	4.717	76	25	5.484	5.604	6.319	31	1.076	1.129	4.542
48	85	21	5.971	6.168	6.180	76	21	7.167	7.324	8.039	21	1.495	1.546	6.230
49	82¾	18	6.061	6.245	6.257	.....	.....	.....	.....	.....	18	1.547	1.598	6.440
	JUNE 15TH, 1899.—Rowena and Russell Sts., 25-ft. Radius Curve, 200 ft.													
					+0.272					+ 0.259			+0.045	
50	80¾	21	6.247	6.421	6.693	.....	.....	.....	.....	.....	21	1.593	1.638	6.601
51	78½	13	6.292	6.451	6.723	77	10	8.430	8.616	8.875	12	1.617	1.662	6.698
52	78½	15	—0.267	—0.272	0.000	76½	15	— 0.253	— 0.259	0.000	15	— 0.045	0.000	0.000
53	78	22	0.745	0.763	1.035	77½	20	1.035	1.061	1.320	21	0.218	0.263	1.060
54	77	13	4.061	4.155	4.427	78	12	5.467	5.601	5.860	12	1.021	1.066	4.296
55	78½	18	3.756	3.850	4.122	80	18	5.061	5.197	5.456	17	0.973	1.018	4.103
56	78	15	6.067	6.216	6.488	80	15	8.127	8.346	8.605	15	1.563	1.608	6.480
57	79½	27	6.147	6.309	6.581	.....	.....	.....	.....	.....	28	1.579	1.624	6.465
	JUNE 15TH, 1899.—Russell St. Tangent, 29.17 ft.													
					+0.374					— 0.000			+0.134	
58	80½	14	6.129	6.298	6.672	78	15	0.747		.....	15	1.567	1.701	6.855
59	.....	.....	.....	.....	.....	79	23	0.800	Rejected.	.....	23	1.565	1.699	6.847
60	79½	11	6.254	6.419	6.793	79	10	0.800		.....	11	1.564	1.698	6.843
61	79	13	—0.365	—0.374	0.000	79	14	0.000	0.000	0.000	14	— 0.134	0.000	0.000
62	78½	25	0.818	0.839	1 213	78	25	0.200	0.205	0.205	25	0.204	0.338	1.362
63	78¾	29	4.369	4.477	4.851	78	31	0.681	0.698	0.698	29	1.014	1.148	4.626
64	83	25	6.308	6.501	6.875	78	21	1.000	1.025	1.025	22	1.573	1.707	6.879
	JUNE 15TH, 1899.—Russell and Brady Sts. Comp. Curve, 200 ft,													
					+0.341					+ 0.102			+0.113	
65	87	16	6.137	6.353	6 694	76½	16	7.169	7.331	7.433	17	1.573	1.686	6.795
66	81	14	—0.332	—0.341	0.000	76	13	— 0.100	— 0.102	0.000	15	— 0.113	0.000	0.000
67	81½	14	0.621	0.639	0.980	76	14	0.907	0.927	1.029	14	0.204	Rejected.	
68	81	14	4.457	4 583	4.924	76	14	5 300	5.416	5.518	14	1.114	1.227	4.945
	JUNE 16TH, 1899.—Brady and Rivard Sts. 15-ft. Radius Curve, 200 ft.													
					+0.096					+ 0 131			+0.035	
69	60½	55	4.484	4.487	4.583	58¾	55	5.753	5.723	5.854	.....	.....	.....	.....
70	63	6	4.300	4.318	4.414	59	7	5.529	5.521	5.652	7	0.905		
71	63	13	—0.096	—0.096	0.000	63	13	— 0.130	— 0.131	0.000	13	— 0.035		
72	66	13	0.662	0.668	(0.764)	.....	.....	.....	.....	.....	15	0.163	Rejected.	
										+ 0.201				
73	65¼	19	4.615	4.649	4.745	62	19	5.911	5.928	6.129	19	1.003		
													+0.079	
74	67	19	8.968	9.056	9.152	62	17	11.741	11.774	11.975	16	2.191	2.270	9.148
75	67	13	1.746	1.763	1.859	64	13	2.145	2.157	2.358	15	0.397	0.476	1.918
76	70	21	—0.080	—0.081	—0.015	64	21	— 0.200	— 0.201	0.000	17	— 0.079	0.000	0.000
77	69½	56	—0.086	—0.087	—0.009	.....	.....	.....	.....	.....	57	— 0.060	—0.019	0.077
	JUNE 17TH, 1899.—South Rivard St. Tangent, 667.86 ft.													
					+0.032					—0.215			+0.005	
78	81½	29	5.343	5.497	5.529	82½	29	17.110	17.649	17.434	29	1.378	1.383	5.573
79	82½	21	—0.031	—0.032	0.000	81¾	22	+ 0.209	+ 0.215	0.000	22	— 0.005	0.000	0.000
80	82	17	1.009	1.039	1.071	82	17	3.471	3.573	3.358	17	0.300	0.305	1.229
81	80¾	30	2.690	2.765	2.797	81	13	8.962	9.215	9.000	30	0.710	0.715	2.881
82	78½	20	2.673	2.740	2.772	81	19	9.200	9.459	9.244	21	0.714	0.719	2.898
83	77½	20	5.835	5.974	6.006	80½	20	17.850	Rejected.	.....	19	1.459	1.464	5.900
84	78	23	6.028	6.176	6.208	79	23	18.261		.....	21	1.505	1.510	6.085

TABLE No. 26—(*Continued*).

JUNE 17TH, 1899.

Brady St. Tangent, 212.25 ft. Gauge No. 7, kerosene and water.						North Rivard Street Tangent. 184.10 ft. Gauge No. 6, kerosene and water.					Pitot Tube No. 6, Gauge No. 5. On center at Alexandrine and Hastings.			
No.	T., Fahr.	No readings.	Average inches, kerosene.	Equivalent reading at 60° Fahr.	Corrected for initial error ±.	T., Fahr.	No. readings.	Average inches, kerosene.	Equivalent reading at 60° Fahr.	Corrected for initial error ±.	No. readings.	Average reading, in inches, water.	Corrected for initial error ±.	Corrected quantities.
(31)	(32)	(33)	(34)	(35)	(36)	(37)	(38)	(39)	(40)	(41)	(42)	(43)	(44)	(45)
	Deg.				−0.106					+0.008			+0.002	
85	85	20	5.793	5.984	5.878	76½	20	5.640	5.767	5.775	18	1.420	1.422	5.721
86	76	13	+0.104	+0.105	0.000	78	13	−0.008	−0.008	0.000	13	−0.002	0.000	0.000
87	74½	25	1.184	1.208	1.102	77	26	1.238	1.267	1.275	25	0.273	0.275	1.103
88	79	27	2.406	2.468	2.362	77	27	2.500	2.558	2.566	25	0.592	0.594	2.384
89	77¾	37	5.518	5.651	5.545	77	37	5.424	5.550	5.558	37	1.403	1.405	5.642
90	79	19	5.253	5.387	5.281	77	16	5.325	5.447	5.455	19	1.406	1.408	5.651
91	79¼	26	5.406	5.548	5.442	....	....	....	....	....	26	1.414	1.416	5.701
92	80	23	5.041	5.177	5.071	....	....	....	....	....	23	1.374	1.376	5.566
JUNE 16TH, 1899.—Rivard and Alexandrine Street, 10-ft. Radius Curve, 200 ft.														
					+0.155					−0.147			+0.091	
93	68¾	38	−0.130	−0.132	0.023	....	....	....	....	....	38	−0.072	0.019	0.357
94	67	11	−0.153	−0.155	0.000	70	11	+0.145	+0.147	0.000	11	−0.091	0.000	0.000
95	67	16	0.839	0.847	1.002	68½	19	1.337	1.353	1.206	19	0.141	0.232	0.701
96	69	31	0.886	0.897	1.052	71	31	1.261	1.280	1.133	28	0.117	0.208	0.588
97	69	28	3.801	3.841	3.996	71½	26	4.377	4.447	4.300	26	0.929	1.040	4.111
98	69½	21	5.333	5.404	5.559	72¼	21	6.348	6.447	6.300	21	1.307	1.398	5.494
99	70¾	37	3.892	3.950	4.105	....	....	....	....	....	33	0.985	1.076	4.328
JUNE 19TH, 1899.—East Alexandrine Street Tangent, 645.50 ft.														
					−0.221					−0.163			−0.022	
100	90	34	4.110	4.269	4.048	80	34	13.074	13.426	13.253	34	1.088	1.066	4.266
101	84	25	4.078	4.208	3.987	80	25	12.504	12.841	12.678	25	1.075	1.053	4.204
102	82	14	4.043	4.162	3.941	80½	13	12.262	12.600	12.427	14	1.074	1.052	4.200
103	82	13	+0.215	+0.221	0.000	82	7	+0.158	+0.163	0.000	13	+0.022	0.000	0.000
104	83	20	1.198	1.235	1.014	82½	20	2.990	3.080	2.917	20	0.300	0.278	1.110
105	85	22	4.016	4.148	3.927	83	22	11.386	11.734	11.571	22	0.991	0.979	3.905
106	86	23	5.369	5.552	5.331	83	23	15.009	15.468	15.305	18	1.299	1.277	5.100
JUNE 19TH, 1899.—Hastings Street Specials, 23.51 ft.														
					+0.163					+0.378			+0.017	
107	89	48	3.448	3.577	3.740	....	....	....	....	....	48	0.978	0.961	3.837
108	89	11	3.557	3.690	3.853	86	8	0.300	0.310	0.688	....	....	....	....
109	89	19	3.939	4.087	4.250	88½	16	0.375	0.389	0.767	21	1.059	1.042	4.159
110	89	17	−0.157	−0.163	0.000	88	17	−0.365	−0.378	0.000	17	+0.017	0.000	0.000
111	89	20	0.693	0.719	0.882	88	30	−0.187	−0.194	0.184	20	0.297	0.280	1.118
112	89½	25	3.884	4.032	4.195	88¼	23	0.343	0.316	0.734	25	1.078	1.061	4.235
113	90	29	5.457	5.668	5.831	89	27	0.600	0.622	1.000	30	1.487	1.470	5.864
114	90	8	3.788	Rej.	.....	89	9	0.400	Rej.	.....	7	1.100	Rej.	.....
JUNE 21ST, 1899.—West Alexandrine Street Tangent, 409 ft.														
					+0.306					+0.414				
115	73	27	3.857	3.916	4.122	75	23	7.230	7.379	7.793				
116	74	22	−0.300	−0.306	0.000	74	16	−0.406	−0.414	0.000				
117	73	26	0.755	0.769	1.075	73	24	1.783	1.815	2.229		Not read.		
118	72	35	3.929	3.954	4.260	72	31	7.638	7.765	8.179				
119	72	22	5.927	6.025	6.331	72	19	12.016	12.216	12.630				
JUNE 16TH, 1899.—Alexandrine and Alley, 6-ft. Radius Curve, 200 ft.														
					+0.148					0.000			+0.082	
120	72	55	3.994	4.060	4.208	....	....	....	....	....	55	1.002	1.084	4.167
121	72½	21	3.793	3.859	4.007	72	20	4.480	4.554	4.554	21	0.986	1.018	4.108
122	72	13	−0.146	−0.148	0.000	73	16	0.000	0.000	0.000	19	−0.032	0.000	0.000
123	72½	23	0.848	0.863	1.011	73	23	0.957	0.964	0.964	23	0.170	0.202	0.804
124	71½	28	3.458	3.513	3.661	72	30	3.700	3.762	3.762	29	0.777	0.809	3.260
125	73	11	6.263	6.376	6.524	72	10	6.820	6.933	6.933	11	1.536	1.568	6.319
126	73	11	5.595	5.796	5.944	72	9	6.178	6.280	6.280	11	1.386	1.418	5.715
127	73½	11	4.041	4.116	4.264	72	10	4.140	4.209	4.209	15	1.000	1.032	4.159

H_{vc}, by Pitot Tube No. 6, at Alexandrine and Hastings = H_f in Brady Tangent ÷ 4.080.

TABLE No. 26—(*Continued*).

JUNE 20TH, 1899.

Brady Street Tangent, 212.25 ft. Gauge No. 7, kerosene and water.						South Alley Tangent, 143.10 ft. Gauge No. 6, kerosene and water.					Pitot Tube No. 6, Gauge No. 5. On center at Alexandrine and Hastings.			
No.	T., Fahr.	No. readings.	Average inches, kerosene.	Equivalent reading at 60° Fahr.	Corrected for initial error.	T., Fahr.	No. readings.	Average inches, kerosene.	Equivalent reading at 60° Fahr.	Corrected for initial error ±.	No. readings.	Average reading, in inches, water.	Corrected for initial error ±.	Corrected result × 4.030.
(46)	(47)	(48)	(49)	(50)	(51)	(52)	(53)	(54)	(55)	(56)	(57)	(58)	(59)	(60)
	Deg.				+0.621	Deg.				+0.923			+0.012	
128	86	61	+0.271	0.280	0.901	....	....	....	....	....	61	+0.300	0.312	1.257
129	86½	25	+0.444	0.459	1.080	88	3	+0.300	0.311	1.234	26	+0.297	0.309	1.245
130	87	13	−0.600	−0.621	0.000	88	16	−0.900	−0.933	−0.010	21	−0.012	0.000	0.000
131	87½	25	+0.316	0.327	0.948	88	27	−0.167	−0.173	0.750	25	+0.284	0.296	1.193
132	88	29	3.457	3.583	4.204	87	25	1.700	1.760	2.683	28	1.001	1.013	4.082
133	88	16	5.306	5.499	6.120	87	12	2.900	3.002	3.925	16	1.499	1.511	6.089
134	88	10	5.540	5.741	6.362	87	13	3.100	3.209	4.132	9	1.556	1.568	6.319
135	88	31	3.723	3.858	4.479	87	20	2.000	2.070	2.993	26	1.088	1.100	4.433
JUNE 21ST, 1899.—North Alley Tangent, 641.80 ft.														
					+0.185					−0.524			+0.016	
136	....	....	....	....	....	70	14	12.986	13.167	12.643	17	0.914	Rejected.	
137	76	24	3.987	4.074	4.259	71	25	13.108	13.308	12.784	6	1.000	1.016	4.094
138	74½	21	−0.181	−0.185	0.000	72	20	+0.515	+0.524	0.000	17	−0.016	0.000	0.000
139	75	21	0.883	0.901	1.086	73½	21	3.962	4.036	3.512	21	0.301	0.317	1.278
140	76	21	3.971	4.058	4.243	74	21	13.329	13.586	13.062	21	1.017	1.033	4.162
141	76	20	5.783	5.910	6.095	74	15	18.933	19.299	18.775	19	1.442	1.458	5.876
142	75	10	4.000	4.083	4.268	74	8	13.000	13.251	12.727	7	1.025	1.041	4.195
					+0.288								−0.032	*
143	90	13	−0.277	−0.288	0.000	....	....	....	....	....	11	+0.032	0.000	0.000
144	91	8	0.744	0.774	1.062	....	....	....	....	....	7	0.236	0.204	0.264
145	....	....	....	....	....	....	....	....	....	....	52	1.169	1.137	....
146	93¾	26	4.027	4.193	4.481	....	....	....	....	....	....	....	....	1.112
147	93	26	4.031	4.200	4.488	....	....	....	....	....	....	....	....	1.114
148	93½	8	4.100	4.274	4.562	....	....	....	....	....	24	1.150	1.118	1.132
149	94	7	5.893	6.146	6.434	....	....	....	....	....	18	1.661	1.629	1.596
150	94	24	5.844	6.095	6.383	....	....	....	....	....	....	....	....	1.584
151	93½	27	5.922	6.173	6.461	....	....	....	....	....	....	....	....	1.603
152	93½	5	5.900	6.150	6.438	....	....	....	....	....	12	1.592	1.560	1.598
153	94	23	5.835	6.085	6.373	....	....	....	....	....	....	....	....	1.581
154	94½	26	6.031	6.293	6.581	....	....	....	....	....	12	1.639	1.607	1.633
155	95	10	6.115	6.384	6.672	....	....	....	....	....	....	....	....	1.656
											Pitot Tube No. 3, Gauge No. 5, Alexandrine and Hastings.			
					+0.218								−0.015	
156	83	16	4.325	4.457	4.675	....	....	....	....	....	....	....	....	1.160
157	84	7	4.257	4.392	4.610	....	....	....	....	....	18	1.028	1.013	1.144
158	82	25	4.256	4.381	4.599	....	....	....	....	....	....	....	....	1.141
159	85	24	4.169	4.306	4.524	....	....	....	....	....	....	....	....	1.123
160	90	12	4.129	4.288	4.506	....	....	....	....	....	42	1.002	0.987	1.118
161	92	8	6.115	6.364	6.582	....	....	....	....	....	24	1.516	1.501	1.632
162	92	28	5.882	6.122	6.340	....	....	....	....	....	....	....	....	1.573
163	92½	24	5.910	6.154	6.372	....	....	....	....	....	....	....	....	1.581
164	92	8	5.888	6.128	6.346	....	....	....	....	....	24	1.458	1.443	1.575
165	93	29	6.090	6.345	6.563	....	....	....	....	....	....	....	....	1.629
166	91¾	25	6.250	6.503	6.721	....	....	....	....	....	....	....	....	1.668
167	92	6	6.241	6.496	6.714	....	....	....	....	....	18	1.455	1.440	1.666
168	90	10	−0.210	−0.218	0.000	....	....	....	....	....	10	+0.015	0.000	0.000

* Estimated Reading, Tube No. 6, Col. 51 ÷ 4.03.

TABLE No. 26—(*Concluded*).

JANUARY 23D, 1901.

Pitot Tube No. 5, at Superior Street.			Pitot Tube C, at Hastings Street.		30-In. Gate at Hastings Street, Gauge No. 7, Kerosene and Water.	
Time.	No.	Average reading, in inches, water.	Average reading, in inches, water.	Corrected for initial error.	T., Fahr.	Average inches, kerosene.
(61)	(62)	(63)	(64)	(65)	(66)	(67)
P. M.				—0.500		
1.30 –1.45	169	Not observed.	1.623	1.123	32°	0.453
1.48 –2.00	170	"	1.599	1.099	.....	0.431
2.01 –2.11	171	1.3480	1.568	1.068	.....	0.456
2.11 –2.24	172	0.709 or 0.553 Reject.	0.850	0.350	.....	0.059
2.24 –2.27	173	0.000	0.416	—0.084	.....	0.000
2.43½–2.59½	174	0.000	Shaking.		.....	0.000
				—0.200		
2.58 –3.06	175	0.000	0.200	0.000	.....	0.062
3.08 –3.10½	176	0.510	0.633	Rejected.	.....	0.333 Rejected
3.11 –3.14	177	0.911	1.188	0.988	.....	0.406
3.17 –3.41	178	Traverse.	1.174	0.974	.....	0.428
3.41 –3.43	179	1.0110	1.083	Rejected.	.....	0.500
3.43 –3.54	180	Not read.	0.688	0.488	.....	0.200
3.55 –4.00	181	1.3040	1.440	1.240	.....	0.500
4.01 –4.26	182	Traverse.	1.328	1.128	.....	0.500
4.26 –4.30	183	1.275	1.300	1.100	.....	0.500

Daily Log.—All readings were plotted on cross-section paper in the usual manner, and the average of readings taken during periods of uniform velocity is given in Table No. 26. Plate VII is a fair sample of the daily plotted log of observations, as read in the field.

Initial Error.—At some period during each series all gates were closed down tight, and the gauges read with a condition of absolutely no flow, when there should be no frictional loss, provided sufficient time is allowed for the establishment of equilibrium. Seldom, however, did the gauges read exactly zero under such conditions.

The readings obtained are given in Table No. 27.

The reading of the differential gauge was zero in four, positive in seven, and negative in ten cases.

If readings were always of the same sign, or never came to zero, it might be argued that these readings represented actual conditions in the pipe; under the circumstances, however, the only rational explanation seems to be that an actual initial error does exist, and all observations in Table No. 26 have been corrected for the observed initial error.

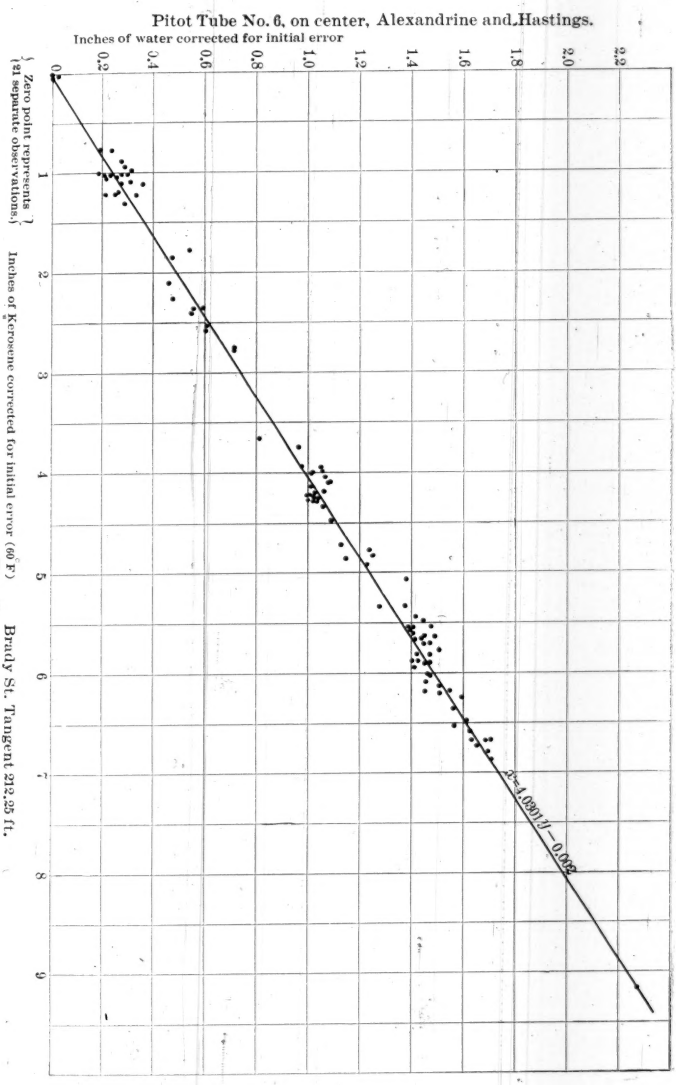

Pitot Tube No. 6, on center, Alexandrine and Hastings.
Inches of water corrected for initial error
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
Zero point represents (21 separate observations)
1
2
3
4
5
6
7
8
9
$y = 1.0017x - 0.002$
Inches of Kerosene corrected for initial error (60° F.)
Brady St. Tangent 212.25 ft.

FIG. 45.

TABLE No. 27.—INITIAL ERRORS.

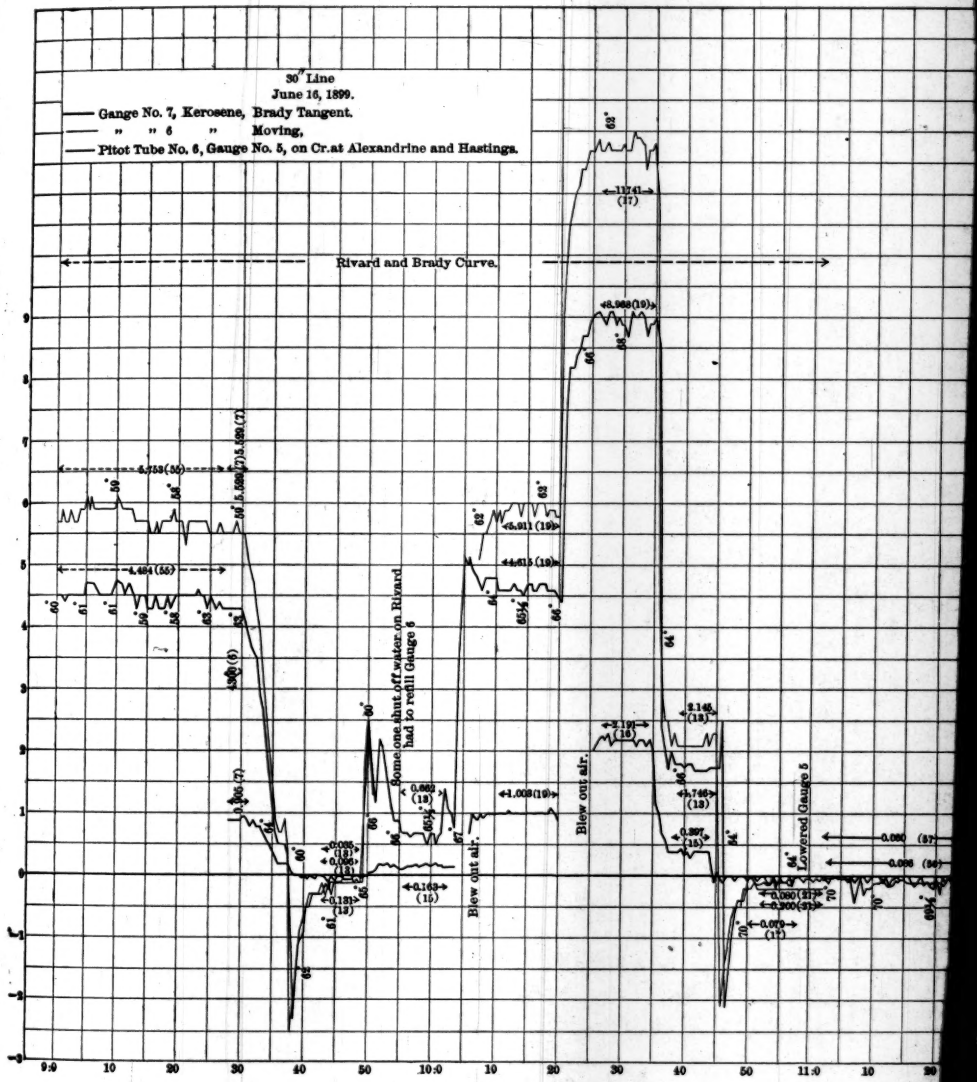
Sec.	Length, in feet.	Reading, kerosene.	Simultaneous reading on Brady Street. Kerosene.	Simultaneous reading of Pitot Tube No. 6 on Center.	Sec.	Length, in feet.	Reading, kerosene.	Simultaneous reading on Brady Street. Kerosene.	Simultaneous reading of Pitot Tube No. 6 on Center.
I....	162.25	+0.62	−0.108	−0.069	XII	212.25	.....	.....	.....
II....	56.42	−0.304	−0.336	−0.171	XIII...	200.00	−0.131	−0.96	−0.085
III...	200.00	+0.079	−0.483	−0.158	XIV....	667.86	+0.215	−0.032	−0.005
IV...	234.92	−0.017	−0.189	−0.086	XV	184.10	−0.008	+0.106	−0.002
V....	200.00	+0.092	−0.223	−0.052	XVI...	200.00	+0.147	−0.155	−0.091
VI...	47.88	0.000	−0.191	−0.016	XVII..	645.50	+0.163	+0.221	+0.222
VII..	200.00	0.000	−0.203	−0.013	XVIII.	23.51	−0.378	−0.163	+0.017
VIII.	222.23	−0.715	−0.012	−0.051	XIX ...	409.00	−0.414	−0.306	.. ....
IX...	200.00	−0.259	−0.272	−0.045	XX....	200.00	0.000	−0.148	−0.032
X....	29.17	0.000	−0.374	−0.134	XXI...	143.10	−0.933	−0.621	−0.012
XI...	200.00	−0.102	−0.341	−0.113	XXII..	641.80	+0.524	−0.185	−0.016

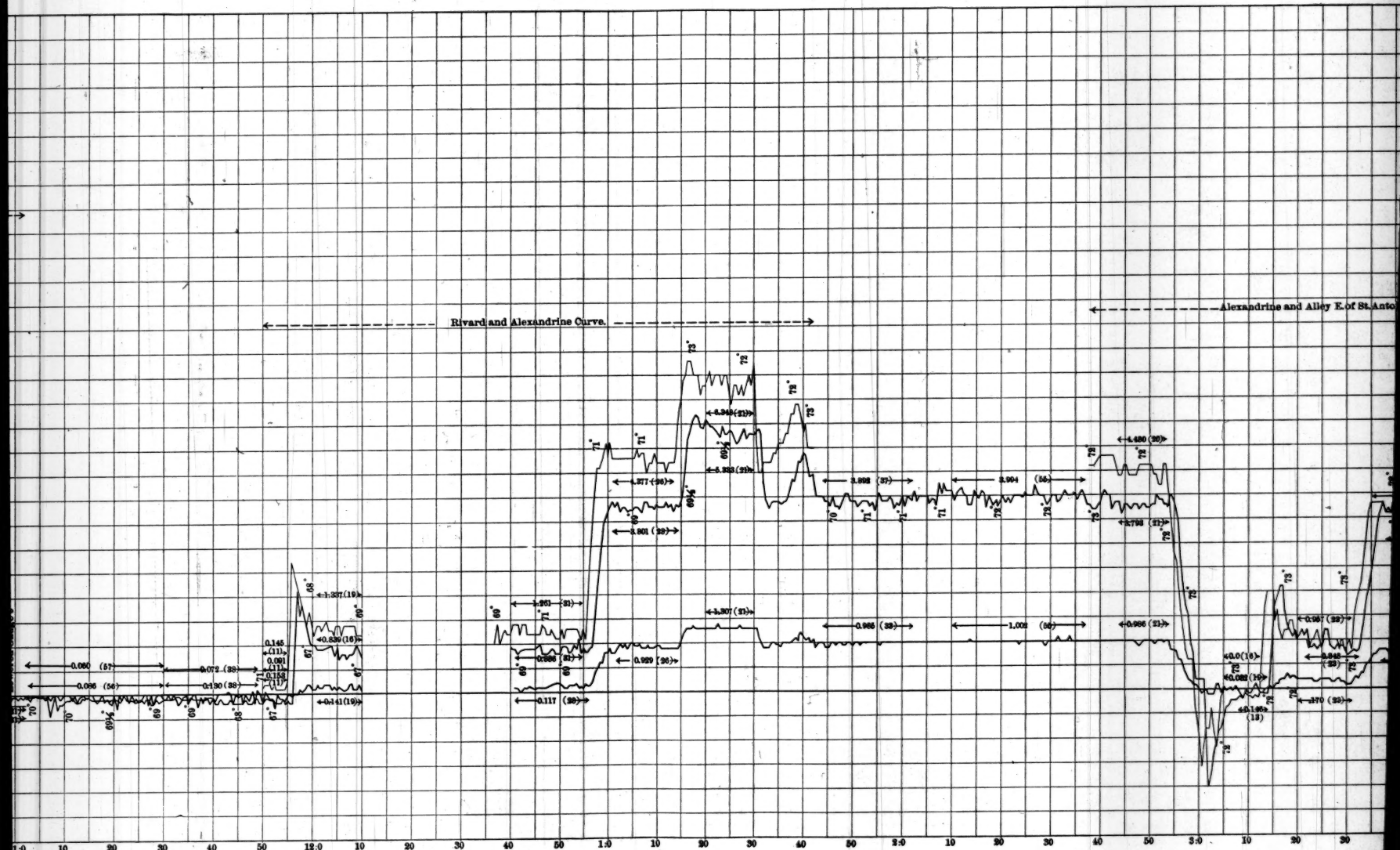
The Pitot tube readings, taken with no flow, are almost uniformly negative, which indicates that either the point read low or the side high, as all the readings were read by the same observer. This result may be due to the personal equation of error, or to the fact that the columns of water in the differential gauge did not always vibrate uniformly, one traveling through a greater distance than the other.

It is the opinion of the writers that this condition is due to the pulsations in the water itself, which, acting through the orifices in the side of the pressure chamber, tend by their impact to elevate the pressure column. As this vibration must be present at all times, its effect may be expected to be a constant, and the static readings, therefore, should be treated as initial errors.

Diagrams.—The ratio between Pitot Tube No. 6 on the center at Hastings and Alexandrine Streets and the frictional resistance in the Brady Street tangent, as computed and shown on Fig. 45, is 4.0301. This ratio is derived from experiments 1 to 142, inclusive, and holds good for all work done in 1899.

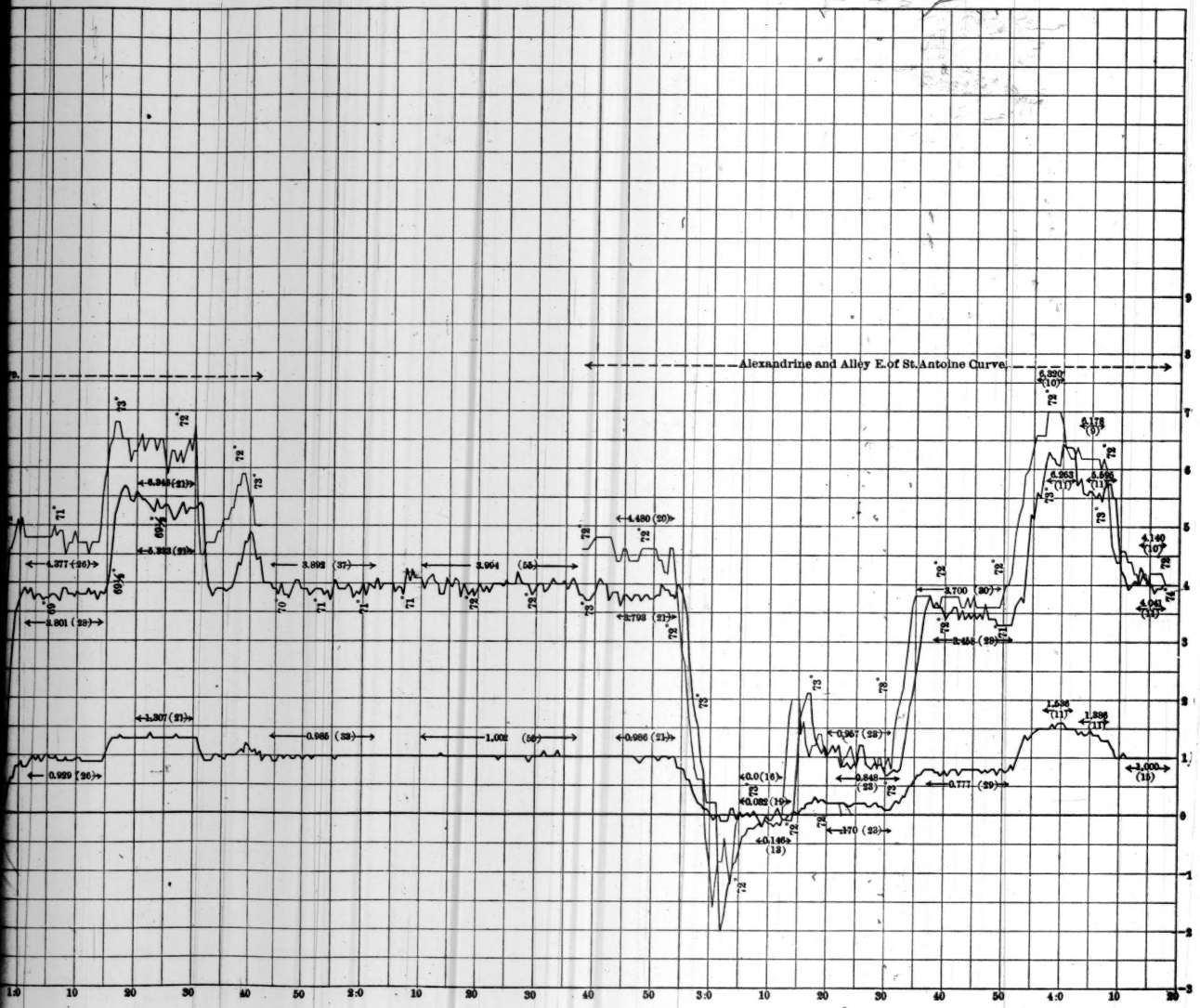
In Figs. 46 to 66 inclusive, the resistance, H_f, observed in each experimental section, is plotted with the resistance simultaneously observed in the Brady Street tangent, and also with the readings taken with Pitot Tube No. 6 on the center at Hastings and Alexandrine Streets, multiplied by 4.0301. The average line is computed from both series of points, thus making the results obtained practically an average of two series of observations.


30" Line
June 16, 1899.
Gange No. 7, Kerosene, Brady Tangent.
" " 6 " Moving,
Pitot Tube No. 6, Gauge No. 5, on Cr. at Alexandrine and Hastings.
Rivard and Brady Curve.
Some one shut off water on Rivard had to refill Gauge 6
Blew out air.
Blew out air.
Lowered Gauge 5
9:0
10
20
30
40
50
10:0
10
20
30
40
50
11:0
10
20



Rivard and Alexandrine Curve.
Alexandrine and Alley E. of St. Anto

PLATE VII.
TRANS. AM. SOC. CIV. ENGRS.
VOL. XLVII, No. 911.
WILLIAMS, HUBBELL AND FENKELL
ON FLOW OF WATER IN PIPES.



The following notes, referring to the diagrams, Figs. 46 to 66, inclusive, should be used also in connection with the values given in Table No. 26, inasmuch as the rejected points referred to are in most cases omitted from the diagrams.

Note (1) Fig. 46. Orleans and Scott Streets compound curve: Observation No. 5 on Pitot tube rejected.

Note (2) Fig. 47. Orleans Street tangent: Nos. 8, 9 and 10 on Orleans Street rejected because columns did not vibrate freely after shaking out hose; acted as if gauge was throttled.

Note (3) Fig. 48. Orleans and Hale Streets curve: It is apparent that the correction for initial error applied to Pitot tube readings is in excess of the true correction. This, however, does not affect the value of the coefficient in the equation.

Note (4) Fig. 50. 60-ft. radius curve: The observations on Brady Street tangent are somewhat irregular, but have been included in the average result, although a diagram of better appearance could have been obtained by using only Pitot tube results, in which case the ratio would have been approximately 1.52.

Note (5) Fig. 51. Riopelle Street tangent: No. 29 rejected; gauge not working as it should.

Note (6) Fig. 53. Rowena Street tangent: The zero observation on Rowena Street appears to be too low, but is included in the average. If omitted, the ratio would be decreased.

Note (7) Fig. 55. Russell Street tangent: Nos. 58, 59 and 60 are rejected as they were taken before the gauges were brought to zero, and, although there is nothing in the log to indicate that they are not reliable, a change in conditions evidently occurred in the Russell Street gauge when the gates were closed.

Note (8) Fig. 56. Small compound curve: No. 67, tube observation, rejected; not working well.

Note (9) Fig. 57. 15-ft. radius curve: Pitot tube observations Nos. 70 to 73, inclusive, rejected on account of air in connections.

Note (10) Fig. 58. S. Rivard Street tangent: Nos. 83 and 84 rejected, as log shows that gauge had not yet reached equilibrium.

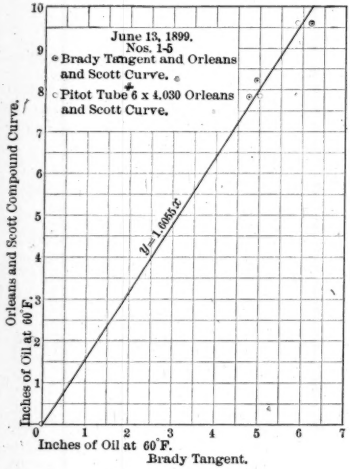


FIG. 46.

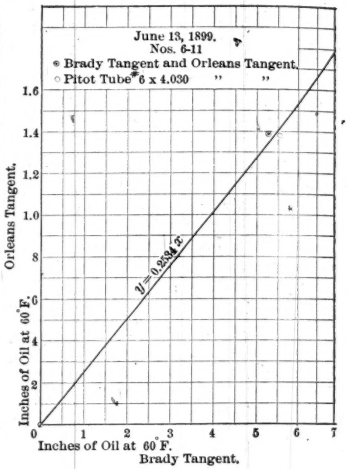


FIG. 47.

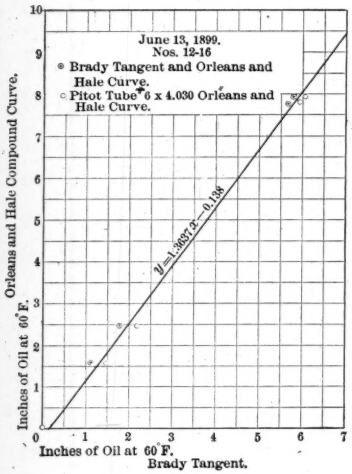


FIG. 48.

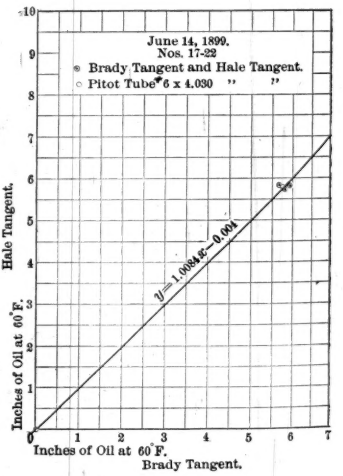


FIG. 49.

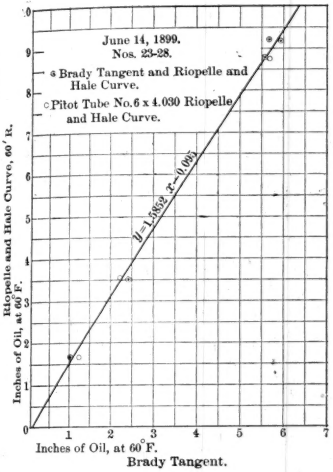


FIG. 50.

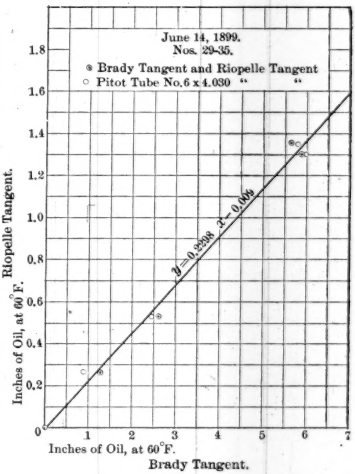


FIG. 51.

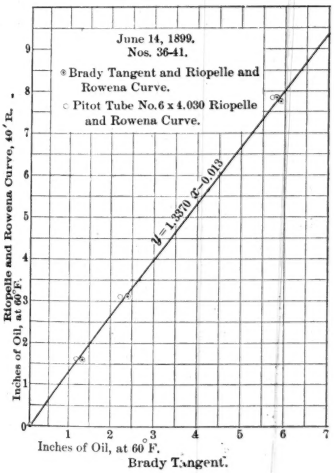


FIG. 52.

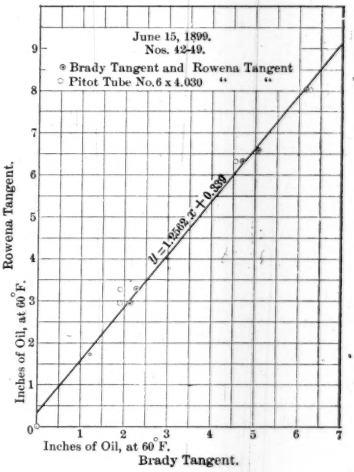


FIG. 53.

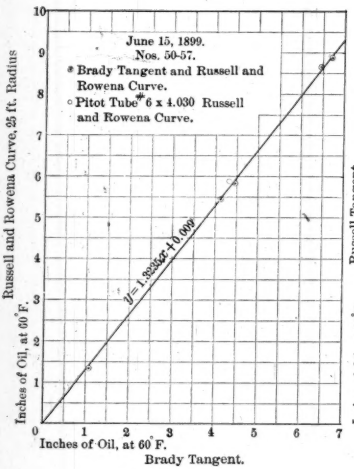


FIG. 54.

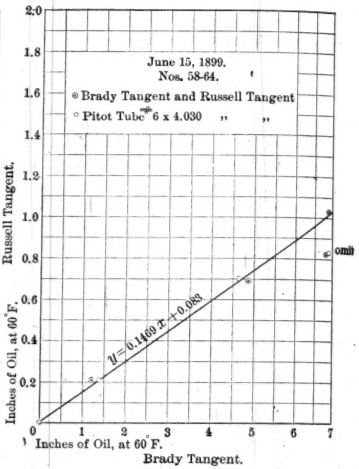


FIG. 55.

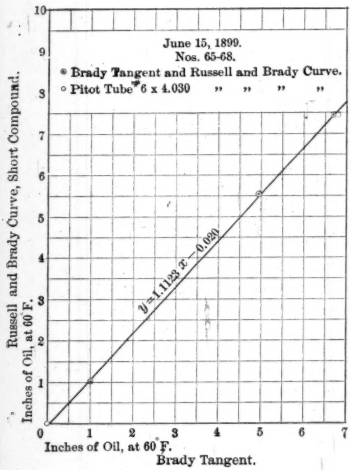


FIG. 56.

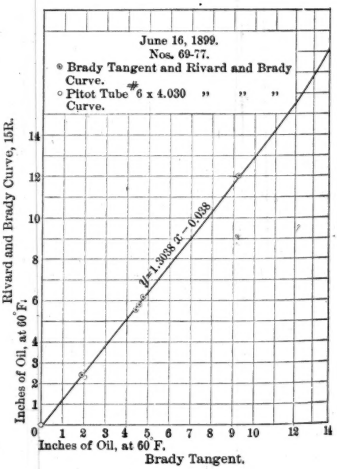


FIG. 57.

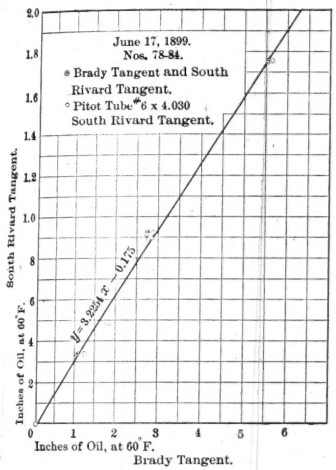


FIG. 58.

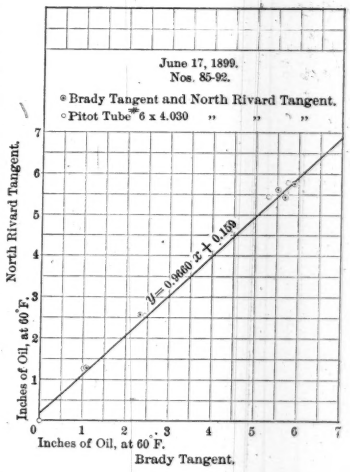


FIG. 59.

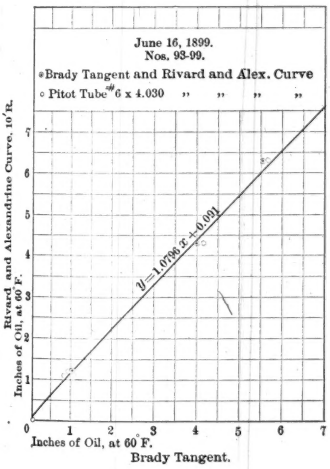


FIG. 60.

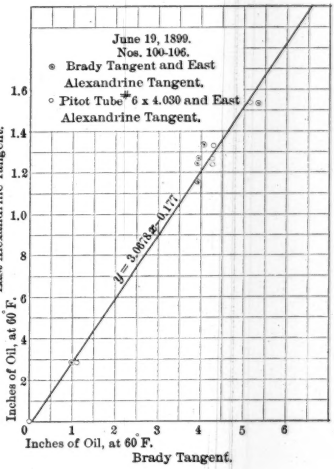


FIG. 61.

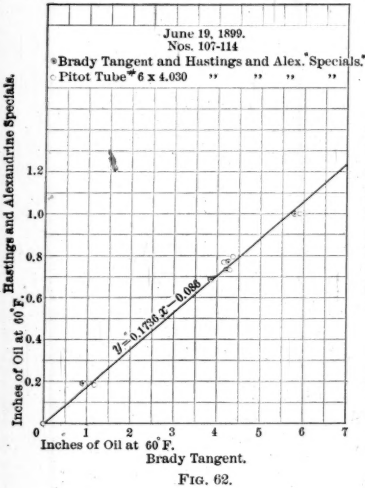


FIG. 62.

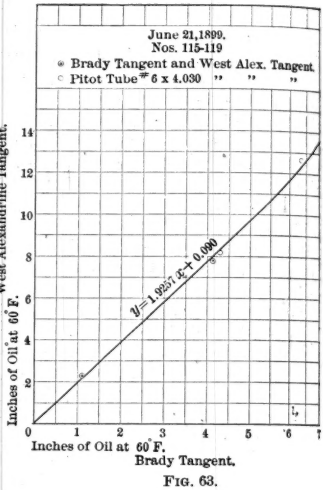


FIG. 63.

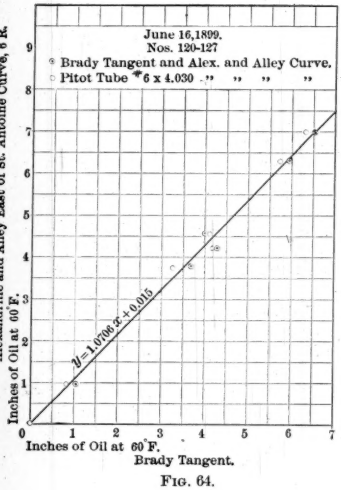


FIG. 64.

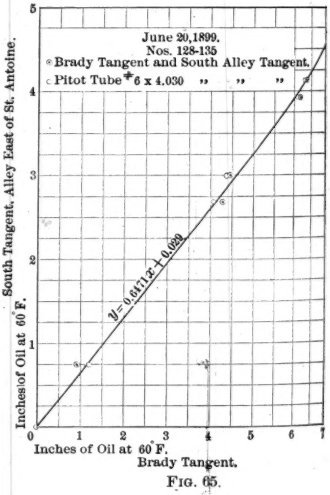


FIG. 65.

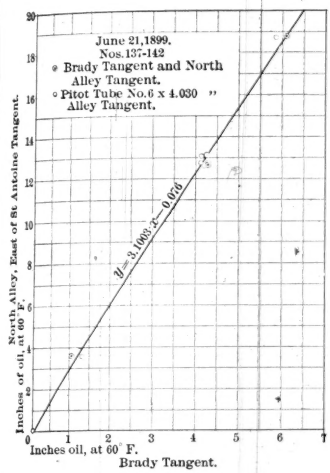


FIG. 66.

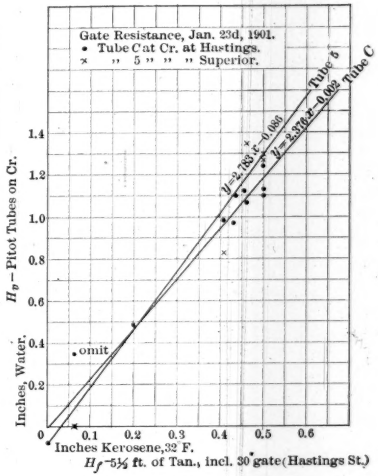


FIG. 67.

The ratios obtained from the diagrams are shown in Table No. 28, with corrections for length; and in Column 6 the results of work in 1898 are given for comparison.

Results.

Resistance of Specials.—The resistance in the 30-in. gate at Alexandrine and Hastings Streets was measured January 23d, 1901, with Gauge No. 7, using kerosene and water. The velocities at the same time were observed by Tube *C*, on center at Alexandrine and Hastings Streets, and by Tube No. 5, on center at Superior Street. The corrected results are shown in Fig. 67. As Tube No. 6 was used on all 1899 experiments, the ratio of gate resistance to Brady tangent is derived by means of the proper coefficients, as follows:

From the rating of Pitot Tubes *C* and No. 6, the true velocity head $(H_v) = 0.640 \times$ the velocity head observed with Pitot Tube *C*, and $0.7157 \times$ that observed with Pitot Tube No. 6; or, the equation expressing the relation between readings observed with these two tubes may be written in the following form:

Equation (1): H_v (Tube *C*) $= \frac{0.64}{0.7157} H_v$ (Tube No. 6), but from Fig. 45.

TABLE No. 28.

Fig. (1)	Experimental Section. No. (2)	(3)	Ratio to Brady Tangent. From diagrams. (4)	Column 4, corr. for length 200 ft. (5)	1898—Results. (6)	Mean. (7)	Value of m in $h_f = m\ V^2$ H_f in feet of water per 1000 ft. (8)	Value of c. $V = c\sqrt{rs}$. (9)	Length of tangent. Equivalent to excess curve resistance. (10)*
46.	I.	Orleans and Scott Comp. C....	1.6055	.....	.....	.....	.....	.....	239.0
47.	II.	Orleans tangent.....	0.2532	0.8264	.....	.....	.....	.....	.....
48.	III.	Orleans and Hale Comp. C....	1.3637	1.3617	1.4782	.....	.....	.....	143.6
49.	IV.	Hale St. tangent.....	1.0084	0.8611	0.7997	0.8304	0.1051	123.4	.....
50.	V.	Hale and Riopelle 60-ft. R. C..	1.5852	1.5852	1.5735	.....	.....	.....	180.4
51.	VI.	Riopelle tangant.....	0.2298	0.9081	.....	.....	.....	.....	.....
52.	VII.	Riopelle and Rowena 40-ft. R. C.....	1.3370	1.3425	1.3911	.....	.....	.....	123.0
53.	VIII.	Rowena tangent.....	1.2562	0.8598	0.8530	0.8564	1.084	121.5	.....
54.	IX.	Rowena and Russell 25-ft. R. C	1.3235	1.3284	1.3168	.....	.....	.....	103.4
55.	X.	Russell tangent.....	0.1469	1.0072	.....	.....	.....	.....	.....
56.	XI.	Russell and Brady small C. C.	1.1123	1.1123	1.1409	.....	.....	.....	49.6
...	XII.	Brady St. tangent.....	1.0000	0.9420	0.9423	0.9421	0.1192	115.9	.....
57.	XIII.	Brady and Rivard 15-ft. R. C..	1.3038	1.3038	1.3626	.....	.....	.....	88.0
58.	XIV.	South Rivard tangent.....	8.2254	0.9635	.....	0.9635	0.1220	114.6	.....
59.	XV.	North Rivard tangent.....	0.9660	1.0500	1.0514	1.0507	0.1330	109.6	.....
60.	XVI.	Rivard and Alexandrine 10-ft. R. C.....	1.0796	1.0796	1.1453	.....	.....	.....	34.6
61.	XVII.	E. Alexandrine tangent.....	3.0678	0.9504	0.9463	0.94835	0.1201	115.4	.....
62.	XVIII.	Hastings St. specials.....	0.1736	1.4768	.....	.....	.....	.....	.....
63.	XIX.	W. Alexandrine tangent.....	1 9257	0.9416	.....	0.9416	0.1192	115.9	.....
64.	XX.	Alexandrine and alley 6-ft. R. C.....	1.0706	1.0706	1.1073	.....	.....	.....	31.4
65.	XXI.	S. Alley tangent.....	0.6471	0.9044	.....	0.9044	0.1145	118.2	.....
66.	XXII.	N. Alley tangent.....	3.1003	0.9658	.....	0.9658	0.1223	114.4	.....

* Column 10 gives the length of tangent in which the value of H_f is equivalent to the excess resistance found in 200 ft. or 80 diameters of pipe, including curve; no correction being applied for the effect of specials or roughness.

Equation (2): H_c (Tube No. 6) $= \dfrac{H_f \text{ (in Brady Tan.)}}{4.0301}$.

And from Fig. 67:

Equation (3): H_f (for $5\frac{1}{2}$ ft. of pipe) + resistance in 30-in. gate $= \dfrac{H_c \text{ (Tube } C)}{2.376}$.

Substituting values from (1) and (2):

Equation (4): H_f ($5\frac{1}{2}$ ft. of pipe) + 30-in. gate resistance

$$= \frac{0.64 \times H_f \text{ (Brady Tan.)}}{0.7157 \times 4.0301 \times 2.376'}$$

$$= 0.0934\ H_f \text{ (Brady Tan.)}.$$

To eliminate the gate resistance from the $5\frac{1}{2}$ ft. of straight pipe,

assume that the resistance per foot of straight pipe is the same as in Brady Street tangent, or $\left(\frac{0.94}{200} = 0.0047\right) H_f$ (Brady Tan.); then, resistance of gate $= (0.0934 - 5\frac{1}{2} \times 0.0047)\ H_f$ (Brady Tan.)

$= 0.06843\ H_f$ (Brady Tan.).

But the total value of H_f for Sec. XVIII, Fig. 62, was found to be $0.1736\ H_f$ (Brady Tan.) in 1899. This section is 23.51 ft. long, and contains two 30 × 10-in. Y's, in addition to the same 30-in. gate for which the resistance is obtained above. Assuming the same value for H_f per foot of straight pipe, the resistance in one 30-in. gate and two 30 × 10-in. Y's is computed as follows: H_f for 23.51 ft. of straight pipe $= (23.51 \times 0.0047 = 0.1105)\ H_f$ in Brady Tan.; therefore, resistance in $\left\{\begin{matrix}\text{one 30-in. gate} \\ \text{two } 30 \times 10\text{-in. Y's}\end{matrix}\right\} = (0.1736 - 0.1105 = 0.0621)\ H_f$ in Brady Tan.

The two results do not agree, and appear to indicate, either that the gate resistance observed in 1901 is too great, or that the assumed value of H_f for 1 ft. of straight pipe is too low. To the writers, the latter case seems the more probable. However, the above assumed value of H_f for straight pipe is also the value actually found in Sec. XIX, which follows the section containing the gate and specials. To assume a lower value of H_f for straight pipe, therefore, leads to the logical conclusion that the effect of gates and specials extends into the following tangent for an undetermined distance, and that the resistance observed is less than the total resistance caused. Such an assumption also takes for granted that the observed value of H_f on tangents is a mean value only, and that the true value is not constant for the entire length of tangent observed. If it is farther considered that the maximum value of H_f in any tangent is at the end nearest the preceding curve, and equal to the value of H_f found in the curve section, then the minimum value of H_f in the tangent will be at the end farthest from the preceding curve section, and will be as much below as the maximum value is above the mean observed value of H_f, if the value diminishes at a uniform rate. On this basis, from Table No. 28, Column 5, the least possible value of H_f in the East Alexandrine tangent would be $0.8212 \times H_f$ in Brady tangent. It is believed, however, that H_f does not diminish uniformly, but rather in the form of a curve concave upward. The extreme limiting values of H_f immediately preceding the 30-in. gate are, in either case, 0.82, and $0.95 \times H_f$ in Brady tangent.

The value 0.86 has been selected as the most probable. Computed as before, the resistance in the 30-in. gate then becomes 0.0698 H_f in Brady tangent, and the resistance for the same gate and two 30 × 10-in. Y's is 0.07251 H_f in Brady tangent.

The last derived values do not include the resistance which takes place in the following West Alexandrine Street tangent, due to the gate and specials. Retaining the value, 0.86 H_f Brady Tan., as the proper coefficient to be applied to straight pipe, the additional resistance of the gate and two Y's may be computed from the observed value of H_f in the West Alexandrine Street tangent as follows:

Length of tangent = 409 ft.;

Observed coefficient per 200 ft. = 0.9416;

Assumed value per 200 ft. of straight pipe = 0.8600;

Special resistance in tangent

$$= \left(\frac{0.9416 - 0.8600 \times 409}{200}\right) H_f \text{ in Brady Tan.};$$

$$= 0.1669\ H_f \text{ in Brady Tan.}$$

These values are to be regarded as approximations, only.

Accuracy of Work.—The methods adopted in conducting the investigations afforded many opportunities to check the accuracy of the work, and the writers themselves have often been surprised at the exceedingly small limit of error found. It was feared, however, that the rating of Pitot Tube No. 6 might be somewhat in error, especially as the tank ratings gave results so entirely at variance with the rating finally adopted. In searching for an opportunity to check the accuracy of this rating it was recalled that on the night of August 6th and 7th, 1898, Pitot Tube No. 3 was read on center at Alexandrine and Hastings Streets, while the resistance in the Brady Street tangent was observed with Gauge No. 6 in sperm oil. The velocities in the lower portion of the series were obtained by operating the Canfield Avenue 12-in. blow-off. To produce the higher velocities, the Hastings Street 10-in. blow-off was also opened, but, in the comparison which follows, the higher readings are rejected, first, because it was feared that the proximity of the Hastings Street blow-off to the location of the Pitot tube (Fig. 68) might have produced a somewhat abnormal distribution of velocities in the section containing the tubes; and second, because the comparison was to be made with the observations made in 1899 between Tube No. 6 on center at Alexandrine and Hastings Streets, and the frictional loss in the Brady Street tangent, in

which the velocities were low; thus making it desirable to include in the comparison only the lower portion of the former series.

From Fig. 30, $\frac{H_v\,(T.3)}{H_f\,(B.S.\ 72^\circ)} = 0.1525$..... (1)

" Fig. 45, $\frac{H_v\,(T.6)}{H_f\,(B.K.\ 60^\circ)} = 4.0301$..... (2)

The coefficient of kerosene at 60° Fahr. = 4.895;

" " " sperm oil " 72° " = 8.833;

from which, for the temperatures here given, a reading taken in

kerosene $= \frac{4.895}{8.833} \times$ sperm oil,

$= 0.5542$ " "

ARRANGEMENT OF SPECIALS AT HASTINGS STREET.

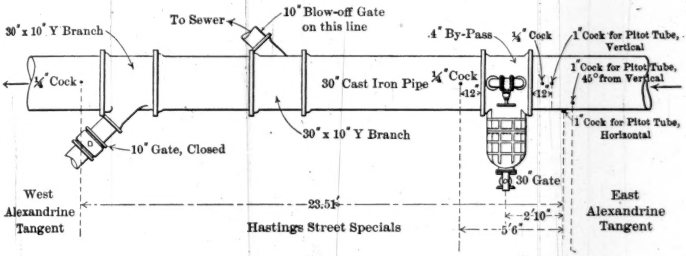


FIG. 68.

Substituting and transposing in Equations (1) and (2)

$H_v\,(T.3) = 0.1525\ H_f$..... (3)

$H_v\,(T.6) = \frac{0.5542}{4.0301}\ H_f$..... (4)

From Equations (3) and (4) the ratio of the value of H_v observed by both tubes becomes

$$\frac{H_v\,(T.3)}{H_v\,(T.6)} = \frac{0.1525 \times 4.0301}{0.5542} = 1.1092.$$

But, from the ratings, the coefficient to be applied to H_v observed by Tube No. 3 is 0.792, from which the coefficient of Tube No. 6 must $= 0.792 \div 1.1092 = 0.7142$, and the coefficient to be applied to velocities derived from Tube No. 6 will be $\sqrt{0.7142} = 0.845$, a difference of only 0.001 from the accepted rating. This check is considered to be remarkably close, being made between observations taken nearly a year apart, by different men and different instruments; and involves,

not only the rating of tubes, but the calibration of kerosene and sperm oil, together with a temperature correction. After obtaining this check the writers feel no hesitation in claiming for the experiments more than usual accuracy.

Sixteen-Inch Cast-Iron Pipe.

Description of Line.

Conditions.—In 1898 it became necessary to lay about 1.1 miles of 16-in. pipe to supply the rapidly increasing consumption in the western part of the city and the proposed water-works system of the adjoining village of Delray, which was to obtain its supply from Detroit. During the autumn of 1898 several series of experiments were conducted on a section of this line about 700 ft. long, containing two angles of 90° each, the first turning to the left and the second to the right. This part of the paper is devoted to a discussion of the methods used and results obtained from these experiments. All the 16-in. pipe mentioned by the writers, unless otherwise designated, will refer to this 700-ft. section.

Survey.—All tangents were carefully run with a transit on offset lines, care being taken that the stakes were not disturbed during the progress of the work, and that each angle was exactly 90 degrees. A bench-mark was established near by, grade stakes were set near the top of the trench at about 40-ft. intervals, and the foreman was furnished with a list of the cuttings at the various stakes necessary to obtain a level grade.

After the pipe was laid, but before being covered, an elevation was taken on each length. All horizontal measurements necessary to properly set the various piezometers and measure the lengths of tangents were made on top of the pipe, before being covered, with a new 100-ft. steel tape, and careful notes of the same preserved.

Diameter of Pipe.—Eight measurements were made on each length of pipe, two at each end 90° apart, and, likewise, two 12 ins. from each end. A 12-in. brass-trimmed boxwood rule having a brass slide dovetailed into one side was used for this purpose, and all readings were taken to $\frac{1}{10}$ in. The curves were measured in a like manner, and measurements were taken to determine the accuracy with which these castings were made. The dimensions used, as to radius of curvature,

TABLE No. 29.—Data of 16-Inch Line.

No.	Location.	Radius of curve.	Length in Feet = l.				Max. Variation of Elevation of Pipe from Level Grade, in Feet.		Experimental Section.							
			Curve on center line.	1st piez. to center of curve.	Center of curve to 2d piez.	1st to 2d piez.	Above.	Below.	Number of joints.	No. of caliperings.	Average diameter, inches.	Mean diameter of maximum cross-section.	Mean diameter of minimum cross-section.	$\frac{16}{D_1}$	$\left(\frac{16}{D_1}\right)^5$	Ratio: $\frac{220.29}{l}$
1	Porter Tan.	.....	....	.....	.....	500.11	0.116	0.103	45	334	16.0232	16.19	15.66†	0.9985	0.9925	0.4405
2	Cavalry and Porter, 5-ft. R.....	5.00	7.85	110.145	110.145	220.29	0.081	0.110	24 22	110	16.0415	16.34* 16.08*	15.75 15.78	0.9974	0.9870	1.0000
3	Cavalry and Army Comp.....	22½° 2-ft. *R.* " 3-ft. *R.* " 4-ft. *R.* " 5-ft. *R.*	5.49	110.145	110.145	220.29	0.000	0.176	22	158	16.0581			0.9963	0.9838	1.0000
4	Army Tan..	.....	....	.....	.....	219.34	0.124	0.146	18	152	16.0263	16.25	15.66	0.9984	0.9920	1.0043

*These sections occur in special curve castings.
†This section occurs in gate at Porter St. and Campbell Ave.

however, are theoretical ones, as the castings were found to conform very closely to the drawings from which they were made. In all, 810 caliperings were taken, and the mean of the calipered diameters was used for all computations involving diameter.

Pipe and Curves.—The pipe was of the regular hub and spigot pattern, quite smooth and free from defects, and no leaks were found when it was tested before being covered. It was coated and tested in a manner similar to the 30-in. pipe described previously. The pipes had a glossy appearance, and were quite smooth, although there was some unevenness due to the coating.

The curves were not as smooth as the straight pipe, and were painted with two coats of graphite paint. No ridges or depressions existed, but the surface of the iron felt rough to the hand. Some trouble was experienced in putting the pieces of the compound curves together properly, and it is probable that one or two of the joints were left somewhat "open." Foremen do not like to lay compound curves, although, if long enough to be properly marked out by the engineers, no difficulty will be encountered, but short ones usually require some solid joints.

TABLE No. 30.—PLAN OF 16-INCH EXPERIMENTS.

Date and series.	Pitot Tube No. 3 at Army Tan.	Pitot Tube No. 5 at Porter Tan.	Porter Tan.	Cavalry and Porter, 5-ft. Rad.	Cavalry and Army, Comp.	Army Tan.	Horizontal piezometers.
1st: Sept. 8, 1898.	*	.....	.....	6 *K.*	7 *K.*	.....	.....
2d " 13, " ..	*	*	.....	6 *K.*	7 *K.*	.....	.....
3d " 16, " ..	*	*	7 *K.*, Air.	.....	.....	.....	6 *S.*
4th " 17, " ..	.....	*	.....	6 *G.*	.....	7 *G.*	.....

* Pitot tube indicated used here on this series.
K. indicates that kerosene was used in gauge.
G. " " gasoline " " "
S. " " sperm oil " " "
Numbers 6 and 7 refer to Differential Gauges Nos. 6 and 7.

Pipe-Laying.—The pipe was laid by an experienced Detroit waterworks foreman and pipe gang. No bracing was necessary, and all work was very carefully executed. The ditch was about 6 ft. deep, in quite dry clay; and no water was encountered anywhere on the work.

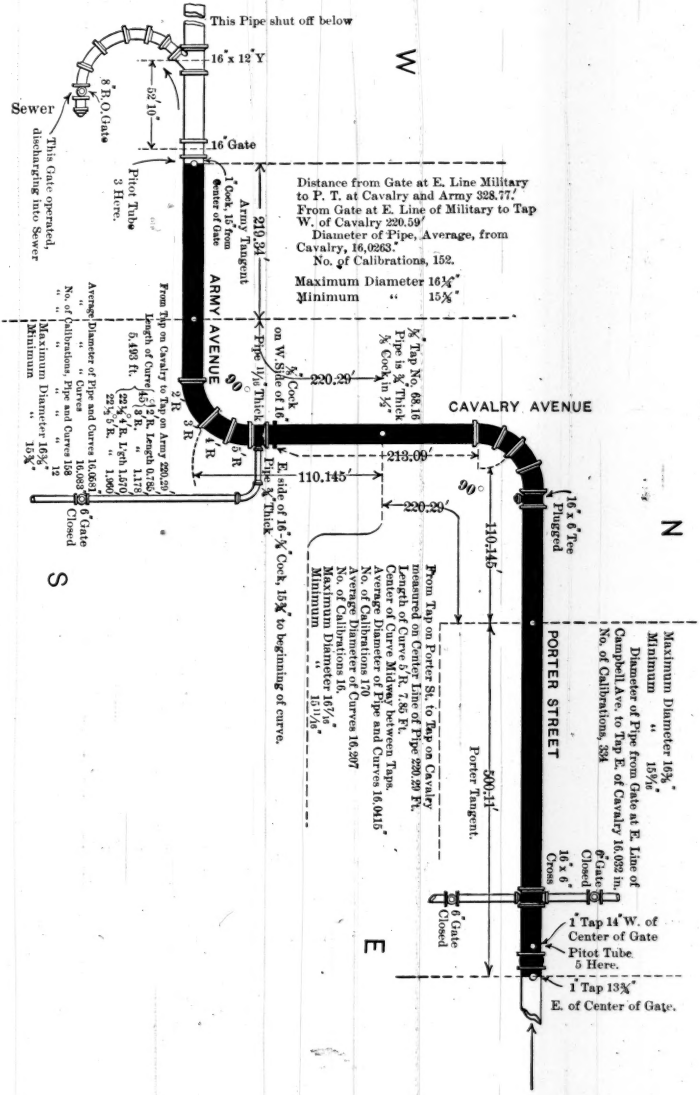

This Pipe shut off below
16" x 12" Y
W
Sewer
8" R.O. Gate
This Gate operated, discharging into Sewer
52' 10"
16" Gate
Pitot Tube 3 Here.
1" Cock, 15" from Center of Gate
Army Tangent
219.34'
Distance from Gate at E. Line Military to P. T. at Cavalry and Army 328.77.'
From Gate at E. Line of Military to Tap W. of Cavalry 220.59'
Diameter of Pipe, Average, from Cavalry, 16,0263."
No. of Calibrations, 152.
Maximum Diameter 16¼"
Minimum " 15⅝"
ARMY AVENUE
From Tap on Cavalry to Tap on Army 220.29'
Length of Curve 45° 2' R. Length 0.785
5.493 ft. 3' R. " 1.178
22½° 4' R. Lgth 1.570
22½° 5' R. " 1.960
Average Diameter of Pipe and Curves 16.0581
" " " Curves 16.083
No. of Calibrations, Pipe and Curves 158
" " " " " 12
Maximum Diameter 16⅜"
Minimum " 15¾"
2' R
3' R
4' R
5' R
90°
Pipe 11/16" Thick
⅝" Cock on W. Side of 16"
220.29'
⅝" Tap No. 68.16
Pipe is ¾" Thick
⅝" Cock in ½"
CAVALRY AVENUE
213.09'
110.145'
E. side of 16"-⅝" Cock, 15¾" to beginning of curve.
Pipe ¾" Thick
220.29'
90°
110.145'
16" x 6" Tee Plugged
N
S
From Tap on Porter St. to Tap on Cavalry measured on Center Line of Pipe 220.29 Ft.
Length of Curve 5' R. 7.85 Ft.
Center of Curve Midway between Taps.
Average Diameter of Pipe and Curves 16.0415"
No. of Calibrations 170
Average Diameter of Curves 16.207
No. of Calibrations 16.
Maximum Diameter 16 7/16"
Minimum " 15 11/16"
PORTER STREET
Maximum Diameter 16⅜"
Minimum " 15 9/16"
Diameter of Pipe from Gate at E. Line of Campbell Ave. to Tap E. of Cavalry 16.032 in.
No. of Calibrations, 334
500.11'
Porter Tangent.
6" Gate Closed
16" x 6" Cross
6" Gate Closed
1" Tap 14" W. of Center of Gate
Pitot Tube 5 Here.
1" Tap 13¾"
E. of Center of Gate.
E

FIG. 69.

Piezometers.—Eight corporation cocks (three 1-in. and five $\frac{5}{8}$-in.) were screwed into the pipe at various places, with a tapping machine; the pipe ends of the latter being plugged with brass, and having a $\frac{1}{8}$-in. hole tapped through the plug. All but two were set vertically on the top of the pipe, and none was allowed to project through the iron. Two $\frac{5}{8}$-in. cocks were set at the extremities of the horizontal diameter, $15\frac{3}{4}$ ins. up stream from the P. C. of the compound curve. The three 1-in. cocks were located in wells, and all others were carried to the side of the road with $\frac{1}{2}$-in. lead pipe. Each was provided with shut-off cock, drain, stop-box and stand-pipe.

Experiments.

Pipe Data.—In Fig. 69 are shown the relative positions of the tangents, curves and piezometers, together with calibrations and other general information; and in Table No. 29 a part of this is presented in tabular form, together with the coefficients for diameter corrections. As the experiments were made before the entire line and connections were completed, it was possible to waste large quantities of water without materially affecting pressures in the surrounding districts, and a mean velocity of 5.8 ft. per second was thus attained.

Series Nos. 1, 2, 3 and 4.—A plan of the entire experiments is shown in Table No. 30. Pitot Tube No. 3 was not observed in the fourth series, as it was necessary to use the 1-in. cock through which it was worked as the western piezometer for the Army tangent. As Gauge No. 6 was read on Cavalry and Porter Streets in the first, second and fourth series; Tube No. 3 on the Army tangent in the first, second and third; Tube No. 5 on the Porter tangent in the second, third and fourth, and Gauge No. 7 on Cavalry and Army Streets in the first and second series, it was believed that all the calibrations and ratings could be checked. This has been done in several ways, as will be shown in the reductions. The temperatures during the four series were unusually uniform, and in the various comparisons no corrections were made to reduce to a uniform temperature.

Table of Results.—All readings were first plotted on cross-section paper, with time as abscissas, and periods selected when it was apparent that a uniform condition of flow had become established. The readings in each of these periods were averaged into observations, and Table No. 31 contains the detailed results from such averages, together with the corrections, reductions and logarithms used in the diagrams.

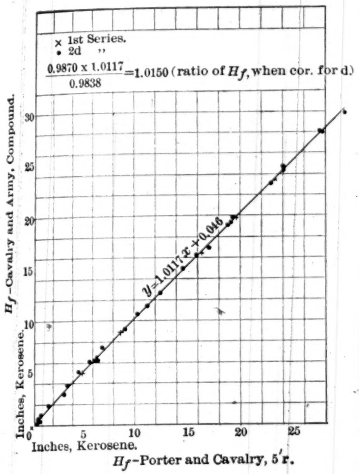


FIG. 70.

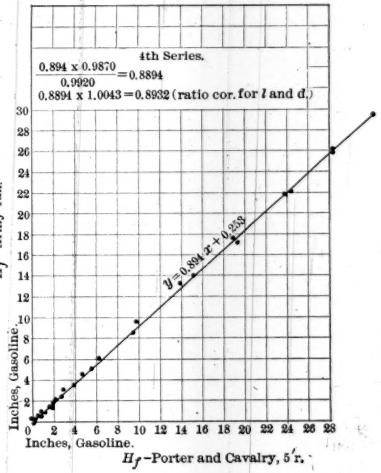


FIG. 71.

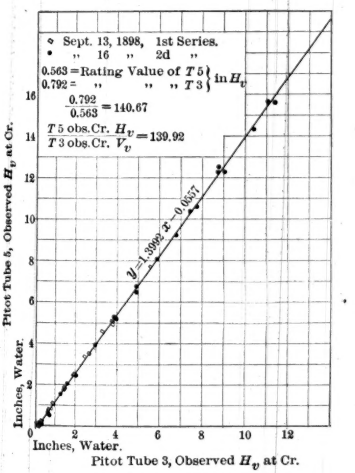


FIG. 72.

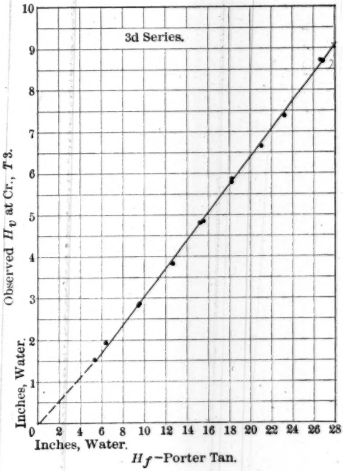


FIG. 73.

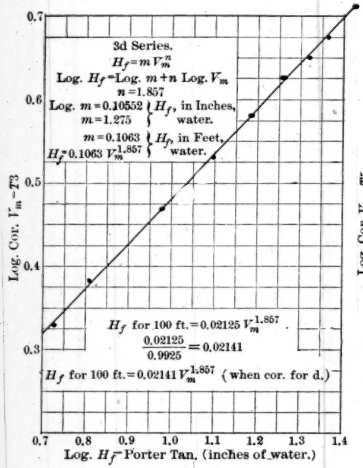


FIG. 74.

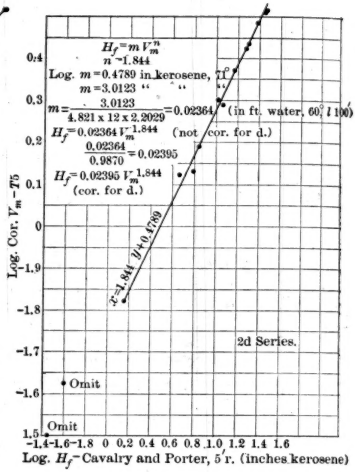


FIG. 75.

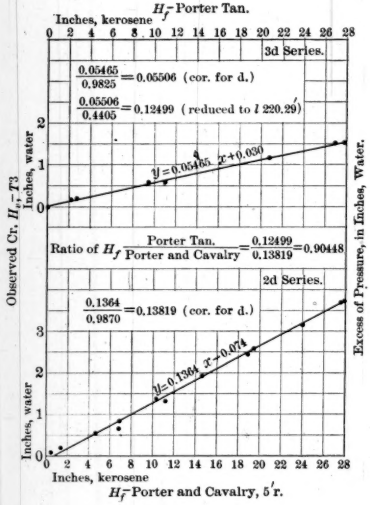


FIG. 76.

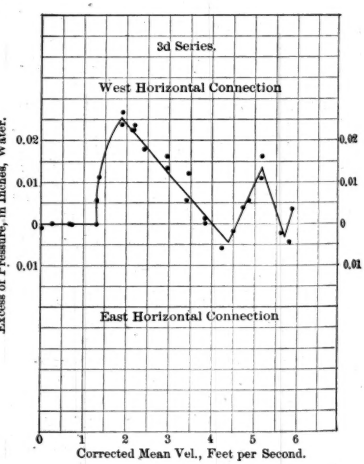


FIG. 77.

Reductions of Experiments.

Diagrams and Explanations.—Fig. 70 shows graphically the relative H_f for Cavalry and Army Streets, containing a 90° compound curve of 2, 3 and 4-ft. radius; and Porter and Cavalry Streets, containing a 90° 5-ft. radius curve, together with corrections for diameter, the length in each case being 220.29 ft.

Fig. 71 shows graphically the relative H_f for the Army tangent, and Porter and Cavalry Section, containing a 90° 5-ft. radius curve, together with the corrections for diameter and length.

Fig. 72 determines the relative value of readings taken simultaneously on center with Pitot Tubes Nos. 3 and 5, in Series 1 and 2, and has been used to determine the coefficient of Tube No. 5.

In Fig. 73, H_f, in inches of water on the Porter tangent, is plotted with the observed H_v at center by Pitot Tube No. 3. Before blowing the oil out of the differential gauge on Porter Street, a reading of practically zero in kerosene was obtained, thus proving that with a condition of no flow H_f would equal zero. As it is known that the line of points, as plotted on the diagram, if continued, would pass through the origin in a curved line, it is evident that H_f does not vary as H_v.

To determine, for the Porter tangent and Tube No. 3, the value of m and n, in the equation $H_f = m\ V_m^{\,n}$, when V_m = mean velocity, the log. of H_f is plotted with the log. of V_m in the usual manner, and corrections made for diameter, and, for purposes of comparison, m reduced for H_f in a length of 100 ft. of pipe (Fig. 74).

For the purpose of determining the value of n, in a section containing a curve, the log. of H_f for Cavalry and Porter Streets (5-ft. radius) is plotted with the log. of V_m from Pitot Tube No. 5, as given in the second series; and in a manner similar to that adopted in Fig. 74, m and n found for the equation, $H_f = m\ V_m^{\,n}$ (Fig. 75).

As the H_f in the Porter tangent is not directly comparable with the H_f in any other section, it becomes necessary to determine its relative frictional resistance by means of Pitot tube observations. Although it has been shown that H_f does not vary as $V_m^{\,2}$, but rather as $V_m^{\,1.85}$, for the purpose of this comparison, owing to the range being about the same, the V_m obtained by the same Pitot tube, the H_f in each case measured in kerosene, and the temperatures very nearly uniform, H_f is plotted with observed H_v at center by Tube No. 3, and

TABLE No. 31.

No.	Series and date.	Pitot Tube No. 3 at Army Tangent.					Pitot Tube No. 5 at Porter Tangent.					Differential Gauge No. 7. Porter Tangent.		
		No. Obs.	Observed H_v at center. Inches, water.	$\sqrt{H_v}$ as obs.	1.734 $\sqrt{H_v}$ = cor. Mean Vel. = V_m.		No. Obs.	Observed H_v at center. Inches, water.	$\sqrt{H_v}$ as obs.	1.462 $\sqrt{H_v}$ = cor. Mean Vel. = V_m.	Log. V_m.	T. Fahr.	No. readings.	Observed H_f. Inches, kerosene.
1														
2	1st Series...	2	3.75											
3	Sept. 8, 1898.	2	2.12											
4		12	2.66											
5		12	3.29											
6		7	0.614											
7		3	0.317											
8														
9	2d Series....		Gauge 5.											
10	Sept. 13, 1898	10	3.47											
11		12	2.22											
12		10	0.75											
13			Gauge 9.											
14		8	0.707											
15		9	0.533											
16		6	0.658											
17		9	0.722											
18		9	1.144											
19		13	1.520											
20		9	2.155											
21		7	2.557											
22		6	3.041											
23		5	4.080											
24		13	0.050											
25														
26		11	0.073				11	0.082	0.289	0.423	—1.6263			
27		9	0.189				9	0.211	0.459	0.671	—1.8267			
28		11	0.541				11	0.836	0.914	1.336	0.1258			
29		11	0.845				11	1.145	1.070	1.564	0.1942			
30		11	1.372				11	1.863	1.365	1.996	0.3002			
31		13	1.935				13	2.623	1.619	2.367	0.3742			
32		9	2.566				9	3.556	1.886	2.757	0.4404			
33		7	3.171				7	4.500	2.121	3.101	0.4915			
34		9	3.706				9	5.033	2.243	3.279	0.5157			
35		9	3.722				37	5.135	2.263	3.313	0.5202			
36		9	5.507				14	7.800	2.793	4.083	0.6110			
37		10	2.460				10	3.420	1.849	2.703	0.4318			
38		9	1.344				9	1.822	1.349	1.972	0.2949			
39		11	0.673				..	0.870	0.932	1.363	0.1345			
40								0.046	0.214	0.313	—1.4955			
41	3d Series....						13	0.054						
42	Sept. 16, 1898	6	0.017	0.134	0.232		8	0.125					7	—0.025
43		13	0.146	0.382	0.662		13	0.069					13	2.078
44		10	0.165	0.406	0.704		10	0.080					10	2.410
45		11	0.573	0.757	1.313		11	0.682						
46		10	0.570	0.755	1.309		10	0.710					5	9.400
47		13	0.604	0.777	1.347		11	0.582					68° 9	10.850
48		27	1.170	1.082	1.876		27	1.559						
49		9	1.161	1.078	1.869		9	1.578					10	20.678
50		12	1.551	1.245	2.159		53	2.081					24	27.896
51		8	1.506	1.227	2.128		8	2.063					8	26.950
52		19	1.505	1.226	2.126		19	2.047					Changed oil for air.	

TABLE No. 31—(*Continued*).

No.	Differential Gauge No. 6. Cavalry and Porter. 5-Ft. Radius.				Differential Gauge No. 7. Cavalry and Army Comp.			Differential Gauge No. 7. Army Tangent.			Differential Gauge No. 6. Horizontal Piezometers.				
	T. Fahr.	No. readings.	Observed H_f. Inches, kerosene.		T. Fahr.	No. readings.	Observed H_f. Inches kerosene.	T. Fahr.	No. readings.	Observed H_f.	T. Fahr.	No. readings.	Diff. in pressure.		
1	....	61	— 0.201	.....	....	65	0.032	....	....	.....	....	....	.....	.....	.....
2	....	.....	.....	.....	....	.....	.....	....	....	.....	....	....	.....	.....	.....
3	....	17	16.194	.....	....	17	16.288	....	....	.....	....	....	.....	.....	.....
4	....	11	19.418	.....	....	11	19.782	....	....	.....	....	....	.....	.....	.....
5	....	12	23.175	.....	....	12	23.421	....	....	.....	....	....	.....	.....	.....
6	....	19	8.647	.....	....	16	8.822	....	....	.....	....	....	.....	.....	.....
7	....	7	4.814	.....	....	8	4.869	....	....	.....	....	....	.....	.....	.....
8	....	27	— 0.256	.....	....	27	0.0685	....	....	.....	....	....	.....	.....	.....
9	79°	7	0.171	.....	75¾	7	0.164	....	....	.....	....	....	.....	.....	.....
10	....	5	23.920	.....	....	10	24.560	....	....	.....	....	....	.....	.....	.....
11	....	12	16.908	.....	....	12	16.870	....	....	.....	....	....	.....	.....	.....
12	....	.....	.....	.....	....	10	6.430	....	....	.....	....	....	.....	.....	.....
13	....	35	6.109	.....	....	35	6.154	....	....	.....	....	....	.....	.....	.....
14	....	7	5.986	.....	....	8	6.040	....	....	.....	....	....	.....	.....	.....
15	....	9	2.944	.....	....	9	2.900	....	....	.....	....	....	.....	.....	.....
16	....	8	3.437	.....	....	8	3.637	....	....	.....	....	....	.....	.....	.....
17	....	8	5.675	.....	....	9	6.083	....	....	.....	....	....	.....	.....	.....
18	....	6	8.867	.....	75¾	6	9.041	....	....	.....	....	....	.....	.....	.....
19	....	9	12.333	.....	....	13	12.438	....	....	.....	....	....	.....	.....	.....
20	....	7	15.943	.....	....	7	16.143	....	....	.....	....	....	.....	.....	.....
21	....	5	19.060	.....	....	6	19.483	....	....	.....	....	....	.....	.....	.....
22	....	6	22.900	.....	....	6	23.125	....	....	.....	....	....	.....	.....	.....
23	....	5	29.760	.....	....	5	29.920	....	....	.....	....	....	.....	.....	.....
24	....	9	0.322	.....	....	13	0.188	....	....	.....	....	....	.....	.....	.....
25	....	7	0.114	Log. Hf.	....	7	0.036	....	....	.....	....	....	.....	.....	.....
26	....	11	0.400	—1.6021	....	11	0.609	....	....	.....	....	....	.....	.....	.....
27	....	8	1.413	0.1501	....	9	1.867	....	....	.....	....	....	.....	.....	.....
28	....	7	4.529	0.6559	....	11	4.986	....	....	.....	....	....	.....	.....	.....
29	....	7	6.814	0.8334	....	11	7.227	....	....	.....	....	....	.....	.....	.....
30	....	7	10.229	1.0098	....	11	10.514	....	....	.....	....	....	.....	.....	.....
31	....	7	14.543	1.1627	....	13	14.927	....	....	.....	....	....	.....	.....	.....
32	....	5	19.340	1.2865	....	9	19.800	....	....	.....	....	....	.....	.....	.....
33	....	7	23.929	1.3789	....	7	24.321	....	....	.....	....	....	.....	.....	.....
34	....	6	27.583	1.4405	....	9	28.078	....	....	.....	....	....	.....	.....	.....
35	....	35	27.826	1.4444	....	37	28.038	....	....	.....	....	....	.....	.....	.....
36	....	.....	.....	.....	....	.....	.....	....	....	.....	....	....	.....	.....	.....
37	....	4	18.800	1.2742	....	11	19.064	....	....	.....	....	....	.....	.....	.....
38	....	7	11.157	1.0476	....	9	11.372	....	....	.....	....	....	.....	.....	.....
39	....	7	6.200	0.7924	....	11	6.05	....	....	.....	....	....	.....	.....	.....
40	....	11	0.263	—1.4200	....	11	0.218	....	....	.....	....	....	Inches, sperm.	Inches, sperm. corrected for initial error 0.05.	Difference in inches, water.
41	....	.....	.....	.....	....	.....	.....	....	....	.....	....	....	.....	.....	.....
42	....	.....	.....	.....	....	.....	.....	....	....	.....	66°	25	0.050	0.0	0.0
43	....	.....	.....	.....	....	.....	.....	....	....	.....	....	13	0.050	0.0	0.0
44	....	.....	.....	.....	....	.....	.....	....	....	.....	....	10	0.050	0.0	0.0
45	....	.....	.....	.....	....	.....	.....	....	....	.....	....	11	0.050	0.0	0.0
46	....	.....	.....	.....	....	.....	.....	....	....	.....	66°	6	0.100	0.050	0.0056
47	....	.....	.....	.....	....	.....	.....	....	....	.....	....	11	0.150	0.100	0.0112
48	....	.....	.....	.....	....	.....	.....	....	....	.....	....	28	0.286	0.236	0.0266
49	....	.....	.....	.....	....	.....	.....	....	....	.....	....	9	0.261	0.211	0.0238
50	....	.....	.....	.....	....	.....	.....	....	....	.....	....	57	0.258	0.208	0.0234
51	....	.....	.....	.....	....	.....	.....	....	....	.....	....	8	0.250	0.200	0.0225
52	....	.....	.....	.....	....	.....	.....	....	....	.....	....	19	0.250	0.200	0.0225

TABLE No. 31—(*Continued*).

No.	Series and date.	Pitot Tube No. 3 at Army Tangent.				Pitot Tube No. 5 at Porter Tangent.				Differential Gauge No. 7. Tangent.				Gauge Porter
		No. Obs.	Observed H_v at center. Inches, water.		Log. V_m.	No. Obs.	Observed H_v at center. Inches, water.			T. Fahr.	No. readings.	Observed H_f. Inches water.	Log. H_f.	
53		9	1.517	1.232	2.136 0.3296	9	2.089				9	5.311	0.725	
54		19	1.934	1.390	2.410 0.3820	19	2.579				19	6.397	0.806	
55		12	2.893	1.700	2.947 0.4694	49	3.996				49	9.494	0.977	
56		5	2.870	1.694	2.940 0.4683	5	3.960			69°	5	9.400	0.973	
57		15	3.843	1.960	3.398 0.5312	15	5.327				15	12.406	1.093	
58		10	4.835	2.199	3.813 6.5813	11	6.546				11	15.136	1.180	
59		8	4.869	2.205	3.823 0.5824	8	6.788				8	15.325	1.185	
60		10	5.895	2.428	4.210 0.6243	10	8.070				10	18.120	1.258	
61		12	5.847	2.418	4.232 0.6235	51	8.080			70°	51	18.177	1.259	
62		15	6.682	2.585	4.481 0.6514	14	9.278				15	20.867	1.319	
63		18	7.435	2.726	4.726 0.6745	18	10.411			72½°	19	23.074	1.363	
64		9	8.944			11	12.346							
65		10	8.770	2.960	5.132 0.7103	11	12.535				10	26.710	1.427	
66		12	8.727	2.954	5.123 0.7095	49	12.308				42	26.550	1.424	
67		26	10.356	3.218	5.580	24	14.383							
68		7	11.327	3.365	5.835	9	15.678							
69		12	11.030	3.321	5.759	45	15.691							
70		17	7.735	2.781	4.822	16	10.663							
71		3	3.933	1.987	3.449	14	5.256							
72		15	0.000	0.000	0.000	13	0.031							
73	4th Series.													
74	Sept. 17, 1898					9	0.111							
75						10	0.100							
76						7	0.129							
77						8	0.088							
78						9	0.311							
79						9	0.578							
80						9	0.789							
81						15	1.126							
82						13	1.946							
83						13	2.992							
84						13	4.146							
85						13	5.700	Tube turned 180° and back.						
86						13	6.308							
87						13	6.885							
88						5	8.380							
89						18	8.694							
90						19	6.947							
91						16	4.075							
92						12	2.350							
93						7	1.243							
94						11	0.764							
95						13	0.462							
96						9	0.378							
97						12	0.250							
98						13	0.269							
99						10	0.140							
100						8	0.138							
101						9	0.022							
102						8	0.062							

TABLE No. 31—(*Concluded*).

No.	Differential Gauge No. 6. Cavalry and Porter. 5-Ft. Radius.			Differential Gauge No. 7. Cavalry and Army Comp.			Differential Gauge No. 7. Army Tangent.			Differential Gauge No. 6. Horizontal Piezometers.				
	T. Fahr.	No. readings.	Observed H_f.	T. Fahr.	No. readings.	Observed H_f.	T. Fahr.	No. readings.	Observed H_f.	T. Fahr.	No. readings.	Diff. in pressure.		
			Inches, gasoline.						Inches, gasoline.					
53	....	....	....	....	....	....	....	....	....	....	9	0.250	0.200	0.0225
54	....	....	....	....	....	....	....	....	....	....	19	0.210	0.160	0.0181
55	....	....	....	....	....	....	....	....	....	67°	49	0.193	0.143	0.0161
56	....	....	....	....	....	....	....	....	....	....	5	0.170	0.120	0.0135
57	....	....	....	....	....	....	....	....	....	....	15	0.100	0.050	0.0056
58	....	....	....	....	....	....	....	....	....	....	11	0.063	0.013	0.0014
59	....	....	....	....	....	....	....	....	....	....	8	0.050	0.000	0.0000
60	....	....	....	....	....	....	....	....	....	....	19	0.000	−0.050	−0.0056
61	....	....	....	....	....	....	....	....	....	69°	51	0.000	−0.050	−0.0056
62	....	....	....	....	....	....	....	....	....	....	15	0.033	−0.017	−0.0019
63	....	....	....	....	....	....	....	....	....	....	19	0.087	0.037	0.0041
64	....	....	....	....	....	....	....	....	....	....	9	0.115	0.065	0.0073
65	....	....	....	....	....	....	....	....	....	73°	....	0.191	0.141	0.0161
66	....	....	....	....	....	....	....	....	....	....	49	0.147	0.097	0.0109
67	....	....	....	....	....	....	....	....	....	73	21	0.033	−0.017	−0.0019
68	....	....	....	....	....	....	....	....	....	....	9	0.083	0.033	0.0037
69	....	....	....	....	....	....	....	....	....	....	49	0.012	−0.038	−0.0043
70	....	....	....	....	....	....	....	....	....	....	17	0.097	0.047	0.0053
71	....	....	....	....	....	....	....	....	....	....	13	0.157	0.107	0.0121
72	....	....	....	....	....	....	....	....	....	70°	10	0.045	−0.005	−0.0006
73	81°	11	−0.086	....	....	....	....	2	0.200	....	....	....	....	....
74	....	9	−0.106	....	....	....	....	7	0.214	....	....	....	....	....
75	....	10	−0.140	....	....	....	....	10	0.275	....	....	....	....	....
76	....	7	0.193	....	....	....	....	7	0.514	....	....	....	....	....
77	....	8	0.625	....	....	....	....	8	0.919	....	....	....	....	....
78	....	9	1.672	....	....	....	....	9	1.933	....	....	....	....	....
79	....	9	2.772	....	....	....	....	9	3.000	....	....	....	....	....
80	....	7	4.507	....	....	....	....	9	4.528	....	....	....	....	....
81	....	13	6.169	....	....	....	....	15	6.023	....	....	....	....	....
82	....	10	9.800	....	....	....	79°	13	9.546	....	....	....	....	....
83	....	9	13.855	....	....	....	....	13	13.100	....	....	....	....	....
84	....	11	18.810	....	....	....	....	13	17.515	....	....	....	....	....
85	....	13	19.108	....	....	....	....	13	17.335	....	....	....	....	....
86	....	13	23.700	....	....	....	....	12	21.904	....	....	....	....	....
87	....	11	28.291	....	....	....	....	13	26.213	....	....	....	....	....
88	82°	21	28.212	....	....	....	79°	21	25.910	....	....	....	....	....
89	....	12	32.317	....	....	....	....	17	29.588	....	....	....	....	....
90	....	15	24.127	....	....	....	....	15	22.077	....	....	....	....	....
91	....	8	15.050	....	....	....	....	8	13.975	....	....	....	....	....
92	....	19	9.316	....	....	....	....	17	8.506	....	....	....	....	....
93	....	7	5.414	....	....	....	....	6	5.114	....	....	....	....	....
94	....	9	3.633	....	....	....	78°	11	3.427	....	....	....	....	....
95	....	13	2.454	....	....	....	....	13	2.315	....	....	....	....	....
96	....	9	1.778	....	....	....	....	9	1.639	....	....	....	....	....
97	....	12	1.200	....	....	....	....	12	1.250	....	....	....	....	....
98	....	13	1.300	....	....	....	....	13	1.250	....	....	....	....	....
99	....	10	0.900	....	....	....	....	10	0.885	....	....	....	....	....
100	....	8	0.563	....	....	....	....	8	0.600	....	....	....	....	....
101	74°	9	−0.050	....	....	....	....	10	0.020	....	....	....	....	....
102	....	8	−0.006	....	....	....	....	6	0.017	....	....	....	....	....

the relation between them determined in each case. When corrected for diameter and reduced to a uniform length of 220.29 ft., the ratio between the results obtained is the relative loss of head in equal lengths of Porter tangent and Porter and Cavalry section, containing a 5-ft. radius curve (Fig. 76).

Fig. 77 shows graphically the difference in pressure measured at two piezometers at the opposite extremities of the horizontal diameter of the 16-in pipe, $15\frac{3}{4}$ ins. up-stream from the P. C. of the compound curve, plotted with the mean velocities of flow as abscissas. No difference was observed until a velocity of about 1.30 ft. was reached, after which a very peculiar condition of affairs was found to exist. Whether these changes of pressure are due to the up-stream curve, 213 ft. away, to the down stream curve, $15\frac{3}{4}$ ins. away, or to the 6-in. branch, the center of which is 9 ins. down stream, it is not possible to say; nor is it clear that the changes are the results of the impacts of oblique currents at the high side, rather than of the diminution of pressure due to increased velocity in the layers of water next the opening on the low side; but the experiment establishes the fact that the pressures recorded by orifices in the pipe wall are not necessarily the same at all points around the circumference of the same cross-section.

Conclusions.—From Figs. 74 and 75, by means of Tube No. 3 in the first, and Tube No. 5 in the second (H_f in inches of water and H_f in inches of kerosene), the following values are derived, after corrections are made for diameter:

H_f for 100-ft. Porter Tangent $= 0.02141\ V_m^{1.857}$.

H_f " " " Porter and Cavalry 5-ft. Radius $= 0.02395\ V_m^{1.844}$.

By means of Fig. 76, however, it was found that for equal lengths,

$$\frac{\text{Porter Tangent}}{\text{Porter and Cavalry}} = 0.9045,$$

$$\frac{0.02395 \times 0.9045}{0.02141} = 101.2$$ or there is a difference of 1.2 %, due to discrepancies in calibrations, ratings, measurements, diameters, etc.

H_f is found to be slightly greater in the Porter tangent than in the Army tangent, which is accounted for by the fact that the former contained a 16-in. gate, 16 × 6-in. cross, and Pitot Tube No. 5 extending to the center of the pipe.

In Table No. 32 are given the relative losses of head for the various experimental sections (reduced to a uniform length of 220.29 ft.), the values of m in the equation, $H_f = m\ V_m^{1.85}$, for the two tangents, and the values of c, in the Chezy formula, at velocities of 1, 2, 3, 4 and 5 ft. per second.

From the relative values of H_f, assuming the Army Street tangent as a standard, the resistance introduced by a 90° 5-ft. radius curve, 16 ins. in diameter, appears to be equivalent to the resistance in 26.35 ft. of 16-in. straight pipe, and the resistance in the compound radius curve to that in 30.03 ft. of 16-in. straight pipe; and, on the same basis, the total resistance due to one gate, one 16 × 6-in. cross, and Pitot Tube No. 5 extending to the center of the pipe, would be equivalent to 6.35 ft. of straight pipe. These values, if correct, apply to all velocities, and are independent of the values of c in the Chezy formula.

TABLE No. 32.—Results of 16-Inch Pipe Experiments.

Comparative H_f and H_f per 1 000 ft., with corresponding values of c (Chezy Formula).

Location.	Relative values of H_f for $l = 220.29$ ft.		Value of m in $H_f = m\ V^n$ in feet of water per 1 000 ft. $n = 1.857$	Values of c, in $V = c\sqrt{rs}$, for Given Values of V.				
				1	2	3	4	5
Army Tan	0.8932	1.0000	0.2114	119.3	125.8	129.4	132.2	134.5
Porter Tan	0.9045	1.0127	0.2141	118.3	124.7	128.5	131.3	133.6
Cavalry and Porter 5-ft. Rad.	1.0000	1.1196	.....	.....	.....	.....	.....	.....
Cavalry and Army, comp.	1.0150	1.1363	.....	.....	.....	.....	.....	.....

Twelve-Inch Cast-Iron Pipe.

Description of Line.

Conditions.—The village of Delray is adjacent to and southwest of Detroit, on the Detroit River. Its population in 1898 was approximately 2 000, and was rapidly increasing. Several large manufacturing establishments were located within its limits, with prospects of more in the near future, and, in order to maintain proper fire-protection and provide a liberal water supply for domestic use, a system of works was

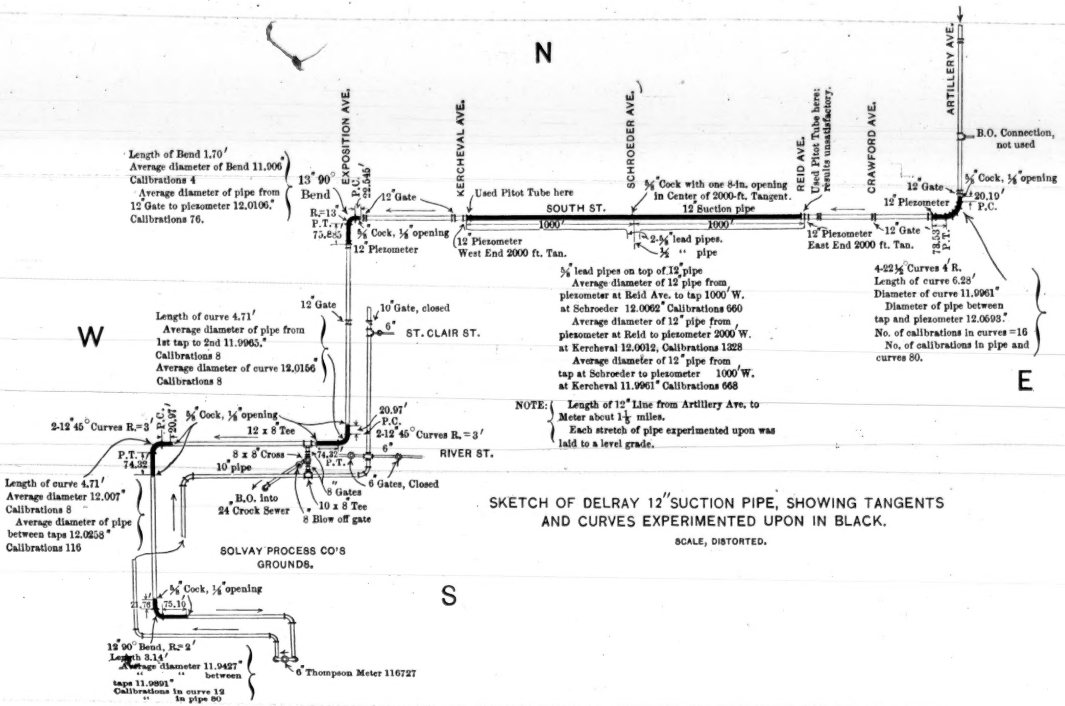
installed, construction beginning in July, 1898, and final connections at the pumps being completed in the spring of 1899. The supply of water was obtained from Detroit, at a pressure of about 28 lbs., by laying a 12-in. cast-iron main from the city limits through a portion of Delray to the boiler house of the Solvay Process Company, a distance of about $1\frac{1}{5}$ miles, where two 6-in. meters were installed, and the water, after passing through these meters, entered the Delray system.

This 12-in. supply line was laid without any side connections whatever, and no taps were allowed to be made to it, its object being to supply water to the fire-pumps at the Solvay plant, where steam is obtainable at all times, day and night. All experiments on 12-in. pipe discussed in this paper, unless otherwise noted, were made on this main.

Survey.—The route of the 12-in. pipe line was carefully staked by transit on offset lines sufficiently to the side to insure the line stakes remaining until after the pipe was laid. In all, five angles were introduced, two of which were exactly 90°, and in the remaining three the tangents were swung slightly to also make right angles, in one case just before, and in two cases immediately after, leaving the experimental station.

All linear measurements were carefully made with a new 100-ft. steel tape, and it is believed that no appreciable errors were made in measuring the length of either tangents or curves. The levels were carefully run and checked, and bench-marks were established about every 1 000 ft. During pipe laying, grade stakes were set at about 40-ft. intervals, and each pipe was lined and leveled as carefully as could be done by experienced pipemen under fairly good conditions. After being laid and caulked, an elevation was taken on each pipe near the spigot end, or, an elevation for every joint. The highest length of pipe in any of the experimental sections was found to be 0.11 ft. above grade, and the lowest 0.10 ft. below grade, which may be considered very good work, inasmuch as a part of the trench was in quicksand. Although the entire line is not level, each section experimented upon was laid at a level grade.

Pipe laying was begun (see Fig. 78) at Artillery Avenue, and continued through on South Street, and thence on Exposition Avenue and River Road, and a short distance in the Solvay Process Com-



SKETCH OF DELRAY 12″ SUCTION PIPE, SHOWING TANGENTS AND CURVES EXPERIMENTED UPON IN BLACK.

SCALE, DISTORTED.

FIG. 78.

pany's grounds, in the summer and fall of 1898, by one gang of men under the same experienced foreman; the work was therefore of uniform character.

Diameters of Pipe.—Before being placed in the ditch, each length of pipe was calipered in eight places, at each end on two diameters at right angles with each other, and similarly 12 ins. from each end. In short pieces, only the end caliperings were taken. A 10-in. brass-bound boxwood rule having a thin brass slide dove-tailed in one side, and reading to $\frac{1}{16}$ in., was used. The entire 12-in. line was calipered in this way, and, although some of the stretches of tangents have never been used for experimental purposes, it was thought best at that time to leave no gaps, as any part might sometime be needed. Altogether, 4 349 caliperings were taken. Each pipe was numbered with white lead before being calipered, and in all subsequent levels and locations was referred to by this number. The caliperings were made on the curves in the same manner as on the straight pipe, and a template was cut to fit some of the curves to test their radius.

Pipes and Curves.—The pipes were of the regular hub and spigot pattern, cast bell up, and $\frac{6}{10}$ in. in thickness. They were coated and tested in a manner similar to the 30-in. pipe. The coating gave the pipe quite a shiny appearance, and most of the roughness of surface seemed to be due to slight inequalities in the tar coating.

The curves were good smooth castings, about the same as the straight pipe, and were painted with two coats of graphite paint, which gave a good finish, although not as glossy as found on the pipe.

Pipe Laying.—The soil encountered was mostly gravel, with some sand and clay near Artillery Avenue. At Reid Street the bottom of the trench was damp; at Schroeder Avenue water was first noticed, and from there to the River Road from 2 to 10 ins. of water would rise over night in the trench. Bell holes were dug for caulking, and hand pumps were provided to keep the trench as dry as possible, all joints being made in the trench. The water caused considerable caving, and on South Street, near Exposition Avenue, the trench was fully 12 ft. wide on top and 7 ft. deep. After being laid and caulked, but before being covered, a test of 80 lbs. per square inch was applied, and all sand holes or joints found sweating were properly attended to.

Piezometers.—Two kinds of piezometers were used on this line, the locations of which are shown on Fig. 78. The $\frac{5}{8}$-in. taps were

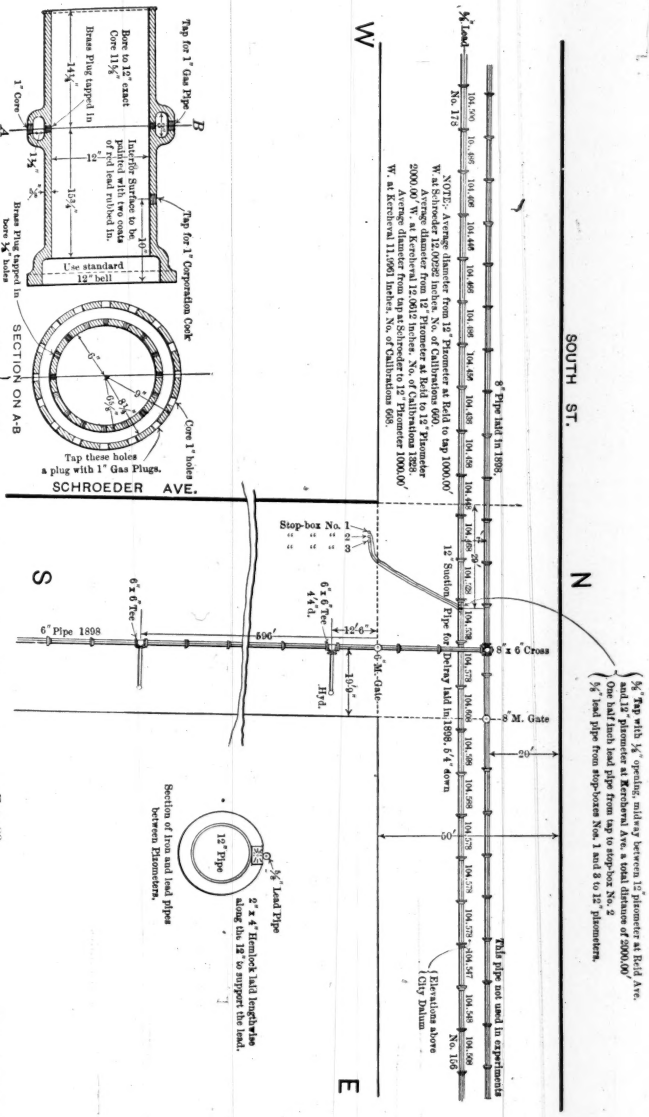

SOUTH ST.
N
W
E
S
SCHROEDER AVE.
8" Pipe laid in 1898.
8" x 6" Cross
8" M. Gate
12" Suction Pipe for Delray laid in 1898, 5'4" down
Stop-box No. 1
" " " 2
" " " 3
6" Pipe 1898
6" x 6" Tee
6" x 6" Tee 4'4" d.
6" M.-Gate
Hyd.
596'
12'-6"
19'-9"
20'
50'
No. 178
No. 156
Elevations above City Datum
This pipe not used in experiments
NOTE:- Average diameter from 12" Pizometer at Reid to tap 1000.00' W. at Schroeder 12.00282 inches. No. of Calibrations 660.
Average diameter from 12" Pizometer at Reid to 12" Pizometer 2000.00' W. at Kercheval 12.0612 inches. No. of Calibrations 1328.
Average diameter from tap at Schroeder to 12" Pizometer 1000.00' W. at Kercheval 11.9961 inches. No. of Calibrations 668.
Tap for 1" Gas Pipe
Tap for 1" Corporation Cock
Bore to 12" exact Core 11⅝"
Brass Plug tapped in
1" Core
Interior Surface to be painted with two coats of red lead rubbed in.
Use standard 12" bell
Brass Plug tapped in bore ¼" holes
Core 1" holes
Tap these holes a plug with 1" Gas Plugs.
SECTION ON A-B
Section of Iron and lead pipes between Pizometers.
12" Pipe
⅝" Lead Pipe
2" x 4" Hemlock laid lengthwise along the 12" to support the lead.

FIG. 79.

FIG. 80.

ordinary ⅝-in. brass corporation cocks, plugged with brass, flush at the end, and a ⅛-in. hole drilled through the plug in the center. These were tapped into the pipe with an ordinary Mueller tapping machine, care being taken that the inner end of the cock should not project into the pipe beyond the inside surface of the iron, and that the taps be located exactly on top of the pipe. In four cases, however (see 12-in. "piezometer" on Fig. 78 for location, and Figs. 79 and 80 for details), a special piezometer casting was set. Pressures were obtained through sixteen holes opening from the circumference of the pipe into a chamber, which in turn was connected with a ½-in. lead pipe carried to the side of the road, in the same manner as described and shown in Fig. 16. The ⅝-in. taps likewise were carried clear of the roadway. Two of the large piezometers were set on the South Street tangent exactly 2 000 ft. apart, and a ⅝-in. lead pipe was carried on top of 2 x 4-in. hemlock strips, laid on the pipe, from each of these piezometers to a point midway between them, and thence to the side of the road. At a point equally distant from these piezometers, a ⅝-in. cock with a ⅛-in. opening was inserted, and it, too, was carried to the side of the road near the ⅝-in. lead connections.

Three connections were thus brought side by side, one being connected with a ⅝-in. lead pipe leading 1 000 ft. east, one in a like manner leading 1 000 ft. west, and a third, a ½-in. lead pipe, to a tap exactly midway between. Fig. 80 shows the arrangement of pipes at Schroeder and South Streets, which is the center of the 2 000-ft. tangent. No trouble from freezing any of the connections along the 12-in. line has occurred to date, March, 1901.

Experiments.

Pipe Data.—Tables Nos. 33 and 33a give the location, sizes and lengths of the various 12-in. curves and tangents experimented upon, together with the proper correction for slight differences in calibrations, in order to bring all to a uniform diameter of 12 ins. It will be observed that the sizes are very uniform.

Table No. 35 gives the general synopsis of the experimental work carried on, together with locations of the gauges and Pitot tubes for each series.

First and Second Series.—Early in October, 1898, preparations were made to begin experimental work, and most of the instruments were

TABLE No. 33.

No.	Location.	Length, in Feet.				Maximum Variation of Elevation of Pipe from Grade, in Feet.		Curves.		First to Second Piezometer.					
		Radius of curve, in feet.	Curves, on center line.	First piezometer to center of curve.	Center of curve to second piezometer.	First to second piezometer.	Above.	Below.	No. of pipe joints, first to second piezometers.	No. of caliperings.	Average diameters, in inches.	No. of caliperings.	Average diameter, in inches.	Mean diameter of max. cross-section.	Mean diameter of min. cross-section.	Ave. dia. / 12 = $\frac{D_1}{D}$.	$\left(\frac{D_1}{D}\right)^5$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)
1...	Artillery and South curve.	4	6.28	23.33	76.67	100.0	0.02	0.05	12	16	11.9961	690	12.0338	12.22	11.91	1.0049	1.0247
2...	East 1 000-ft. tangent.	....	.....	.....	.....	1000.0	0.10	0.07	83	.....	.....	660	12.0028	12.22	11.75	1.0010	1.0030
3...	West "	....	.....	.....	.....	1000.0	0.10	0.10	83	.....	.....	660	11.9961	12.19	11.81	0.9997	0.9885
4...	Total tangent 2 000 ft.	....	.....	.....	.....	2000.0	.....	.....	166	.....	.....	1 308	12.0012	12.22	11.75	1.0001	1.0005
5...	Exposition and South curve	1.08	1.70	23.33	76.67	100.0	0.00	0.04	10	4	11.906	76	12.0106	12.19*	11.81*	1.0009	1.0045
6...	" " River - "	3	4.71	23.33	76.67	100.0	0.00	0.11	10	8	12.0156	80	11.9985	12.16	11.87	0.9999	0.9965
7...	Solvay " "	3	4.71	23.33	76.67	100.0	0.07	0.11	10	8	12.007	116	12.0258	12.37	11.87*	1.0021	1.0106
8...	" grounds "	2	3.14	23.33	76.67	100.0	0.06	0.00	10	12	11.9427	80	11.9891	12.16	11.87	0.9991	0.9955

*These sections occur in the special curve castings.

TABLE No. 33a.

12-INCH LINE. (See Figs. 78, 79 and 80.)

	FEET.
From P. I. of preceding curve to B. O. on Artillery Ave.	3 110.6
" B. O. on Artillery Ave. to center of gate at Artillery Ave. and South St.	282.2
" center of gate at Artillery Ave and South St. to first piezometer of 4-ft. radius curve section	1.467
" first to second piezometer of 4-ft. radius curve section	100.00
" second piezometer of 4-ft. radius curve section to center of gate at Crawford St.	441.7
" gate at Crawford St. to gate at Reid St.	373.8
" gate at Reid St. to first piezometer of 2 000-ft. tangent	25.75
" first to middle piezometer of 2 000-ft. tangent	1 000.0
" middle to last piezometer of 2 000-ft. tangent	1 000.0
" last piezometer of 2 000-ft. tangent to center of gate at Kercheval St.	68.0
" gate at Kercheval St. to gate at Exposition Ave.	591.2
" center of gate at Exposition Ave. to first piezometer of 1.08-ft. radius curve section	1.457
" first to second piezometer of 1.08-ft. radius curve section, South St. and Exposition Ave.	100.0
" second piezometer of 1.08-ft. radius curve section to center of gate at St. Clair St.	557.7
" center of gate at St. Clair St. to first piezometer of first 3-ft. radius curve section	635.4
" first to second piezometer of first 3-ft. radius curve section, Exposition Ave. and River St.	100.0
" second piezometer of first 3-ft. radius curve section to 12 x 3-in. Tee (near B. O.)	33.0
" 12 x 8-in. Tee (near B. O.) to center of 12-in. gate	2.61
" center of 12-in. gate (near B. O) to first piezometer of second 3-ft. radius curve section	284.4
" first to second piezometer of second 3-ft. radius curve section, River St. and Solvay Grounds	100.0
" second piezometer of second 3-ft. radius curve section to first piezometer of 2-ft. radius curve section	810.4
" first to second piezometer of 2-ft. radius curve section, Solvay Grounds.	100.0
" second piezometer of 2-ft. radius curve section to P. I. of 90° bend, near meters	78.3

stored in a building near where the work was to be conducted. It was not, however, until November 4th that the first series was obtained, and it was then found that Mercury Gauge No. 2 had burst, from freezing, and could not be used on this work.

At this time the meters had not yet been set, and the velocities in the first series were read with Pitot Tube No. 3 vertical and Gauge No. 8, at Kercheval and South, in the piezometer special at the lower end of the 2 000-ft. tangent; and in the second series, in piezometer special at the upper end of the 2 000-ft. tangent, near Reid and South Streets. The Pitot tube connection in the upper piezometer (Reid and South Streets) is 25 ft. 9 ins. below a 12-in. gate, and, although all gates were taken apart shortly after the line was laid, and examined to see

TABLE No. 34.

First Series, November 4th, 1898.

Note.—All temperatures in parentheses are interpolated.

Pitot Tube No. 3.—Gauge No. 8, at Kercheval and South.								Gauge No. 1.—On 2 000-Ft. Tangent.				Gauge No. 7.—Artillery and South, 4-Ft. Radius.				Gauge No. 6.—Exposition and South 1.08-Ft. Radius.					
(1) No.	(2) No. readings on center.	(3) Observed inches, water.	(4) + 0.073.	(5) Cal. Cr. Vel.	(6) Cal. Cr. Vel. × 0.89 = true Cr. Vel.	(7) True Cr. Vel. × 0.84 = mean Vel.	(8) Log. of mean Vel.	(9) No. readings.	(10) Inches, mercury.	(11) Temp., degs. Fahr.	(12) Inches, water, 60° Fahr.	(13) No. readings.	(14) Inches, gasoline.	(15) Temp., degs. Fahr.	(16) Inches, water, 60° Fahr.	(17) No. readings.	(18) Inches, gasoline.	(19) Temp., degs. Fahr.	(20) Inches, water, 60° Fahr.	(21) Log. inches, water, 60° Fahr.	(22) †Average pressure, pounds per square inch, Crosby test gauge.
1..	.....	.....	.....	.....	.....	.....	.....	.....	.....	55	.....	63	14.042	(58)	3.9757	9	14.750	60°	4.1903	.....	16.353
2..	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	87	13.973	(59)	3.9628	41	13.396	(61°)	4.0957	.....	16.025
3..	.....	.....	.....	.....	.....	.....	.....	25	5.650	61	71.019	18	13.983	(60)	3.9724	25	14.608	(62°)	4.1630	.....	.....
4..	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	28	14.082	(61)	4.0063	22	14.632	(63°)	4.1747	.....	16.607
5..	.....	.....	.....	.....	.....	.....	.....	49	5.686	62	71.464	53	14.006	(62)	3.9920	53	14.609	64°	4.1764	.....	16.491
6..	.....	.....	.....	.....	.....	.....	.....	71	5.755	(63)	72.324	85	13.668	(63)	3.9007	85	14.155	63°	4.0397	.....	16.651
7..	10	3.300	3.373	4.262	3.793	3.186	0.5032	6	5.740	(65)	72.120	7	13.685	(64)	3.9122	7	14.443	(63°)	4.1207	0.6150	16.375
8..	9 (*T*)*	3.350	3.423	4.292	3.819	3.208	0.5062	45	5.622	66	70.630	44	13.713	(65)	3.9258	47	14.263	(64°)	4.0775	0.6104	16.427
9..	44	3.442	3.515	4.350	3.872	3.252	0.5122	43	5.521	66	69.369	46	13.719	(65)	3.9275	45	14.244	64°	4.0720	0.6098	16.840
10..	33	4.917	4.990	5.283	4.702	3.950	0.5966	33	7.497	(63)	94.216	33	18.618	(63)	5.3134	33	19.460	(63°)	5.5537	0.7446	13.500
11..	16 (*T*)	4.828	4.901	5.136	4.571	3.940	0.5955	49	7.563	61	95.065	49	18.932	(61)	5.3861	49	19.711	62°	5.6173	0.7495	13.610
12..	19	5.242	5.315	5.348	4.760	3.998	0.6018	28	7.640	59	96.063	28	19.350	(59)	5.4878	28	20.212	58°	5.7225	0.7576	14.181
13..	27	7.167	7.240	6.243	5.556	4.667	0.6690	23	10.647	58	133.857	27	26.396	(57)	7.4607	27	28.111	(57°)	7.9454	0.9001	8.535
14..	24	—0.073	00	00	00	00	.....	.....	.....	55	.....	81	0.030	(55)	0.0084	16	0.043	56°	0.0121	.....	.....

*(*T*) indicates that traverses were made during this interval of observation.
†Pressures taken near blow-off, with Crosby test gauge, No. 357 646, carefully tested.

TABLE No. 34—(*Continued*).

THIRD SERIES, JUNE 6TH, 1899.

PITOT TUBE No. 3, GAUGE No. 9, AT KERCHEVAL AND SOUTH.						6-IN. METER, No. 116 727.						GAUGE No. 7, WEST 1 000 FT. OF TANGENT.						GAUGE No. 6, EAST 1 000 FT. OF TANGENT, 1 FT. RADIUS.					
(40)	(41)	(42)	(43)	(44)	(45)	(46)	(47)	(48)	(49)	(50)	(51)	(52)	(53)	(54)	(55)	(56)	(57)	(58)	(59)	(60)	(61)	(62)	(63)
No.	No. readings.	Observed inches, water.	—0.148.	Cal. Cr. Vel.	Cal. Cr. Vel., $\times 0.89$ = True Cr. Vel.	Cubic feet per minute.	Cubic feet per second.	Mean Vel. = Cubic feet per second. $\div 0.7854$.	(Mean V.)^2	Velocity head = H_v, in inches.	Log. of Vel.	Temp., deg. Fahr.	No. readings.	Inches, kerosene.	Kerosene, 60° Fahr.	+ 0.159.	Log.	Temp., deg. Fahr.	No. readings.	Inches, kerosene.	Kerosene, 60° Fahr.	—0.117.	Log.
25...	.....	.....	.....	.....	.....	0.495	0.00825	0.0105	0.000110	0.00002	.....	(90)	33	—0.153	—0.159	0.000	.....	90	35	0.170	0.177	0.060	.....
26...	.....	.....	.....	.....	.....	0.844	0.01407	0.0179	0.000320	0.00006	.....	(90)	18	0.028	0.029	0.188	.....	90	17	0.217	0.225	0.108	.....
27...	.....	.....	.....	.....	.....	7.133	0.11888	0.1514	0.02292	0.0043	−1.1801	(91)	28	0.545	0.567	0.726	−1.8609	91	28	0.836	0.869	0.752	−1.8762
28...	11	0.155	0.007	0.196	0.174	6.231	0.10365	0.13196	0.017398	0.0032	−1.1201	(91)	29	0.455	0.473	0.632	−1.8007	91	29	0.786	0.817	0.700	−1.8451
29...	27	0.195	0.047	0.503	0.448	15.357	0.25595	0.32586	0.106145	0.0198	−1.5130	(93½)	29	2.345	2.444	2.603	0.4155	93½	26	2.477	2.582	2.465	0.3918
30...	.....	.....	.....	.....	.....	29.765	0.49608	0.63158	0.398919	0.0743	−1.8004	(92¼)	31	7.871	8.194	8.353	0.9218	92¼	28	7.996	8.324	8.207	0.9142
31...	7	0.471	0.323	1.318	1.173	40.450	0.67417	0.85831	0.736679	0.1372	−1.9333	(91½)	41	13.590	14.187	14.296	1.1552	91½	40	13.950	14.511	14.394	1.1582
32...	.....	.....	.....	.....	.....	50.278	0.83797	1.06685	1.1385	0.2121	0.0281	(93⅜)	36	20.375	21.247	21.406	1.3305	93⅜	36	20.514	21.392	21.275	1.3279
33...	.....	.....	.....	.....	.....	51.167	0.85278	1.08571	1.1793	0.2197	0.0357	(89⅛)	39	21.172	21.968	22.127	1.3449	89⅛	36	21.150	21.946	21.829	1.3390
34...	.....	.....	.....	.....	.....	50.222	0.83603	1.06498	1.1321	0.2109	0.0271	(85½)	21	20.619	21.310	21.469	1.3318	.....	.....	.....	.....	.....	.....
35...	25	0.919	0.771	2.087	1.813	70.857	1.18095	1.50351	2.2620	0.4214	0.1771	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
36...	14	1.418	1.270	2.615	2.327	91.000	1.51667	1.9307	3.7276	0.6945	0.2857	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....

TABLE No. 34—(*Continued*).

SECOND SERIES, NOVEMBER 16TH, 1898.

		Pitot Tube No. 8, Gauge No. 8, at Reid and South.					Gauge No. 1, on 2 000-Ft. Tangent.			Gauge No. 2, Exposition and South, 1.06-Ft. Radius.			
	(23)	(24)	(25)	(26)	(27)	(28)	(29)	(30)	(31)	(32)	(33)	(34)	(35)	(36)	(37)	(38)	(39)
	No.	No. readings on center.	Observed inches, water.	+ 0.250	Cal. Cr. Vel.	Cal. Cr. Vel. × 0.89 = True Cr. Vel.	True Cr. Vel. × 0.84 = mean Vel.	Log. of mean Vel.	No. readings.	Inches, mercury.	Temp. deg. Fahr.	Water, 60° Fahr.	No. readings.	Inches, gasoline.	Temp. deg. Fahr.	Inches, water, 60° Fahr.	Average pressure, pounds per square inch.
15.		.....	.....	.....	.....	.....	.....	.....	71	0.1554	48°	1.956	117	00	45°	00	31.81
16.		.....	.....	.....	.....	.....	.....	.....	49	0.2704	49°	3.903	97	00	44°	00	31.45
17.		.....	−0.332	−0.082	1.545	1.375	1.155	0.0636	9	0.2597	45°	3.270	....	.....	.....	.....	29.04
18.		39	−0.332	−0.082	1.545	1.375	1.155	0.0636	23	0.5362	(44°)	6.777	41	1.900	42°	0.5228	29.15
19.		17 (*T*)	0.194	0.444	2.838	2.580	2.177	0.3379	24	2.678	(44°)	33.721	43	6.723	41°	1.8470	25.53
20.		10 (*T*)	1.315	1.560	2.838	2.580	2.177	0.4688	25	4.525	43°	56.985	49	12.443	41°	3.4184	21.89
21.		9 (*T*)	2.690	2.880	3.987	3.504	2.943	0.4688	22	6.797	(42°)	85.605	43	18.197	(40°)	4.9731	18.05
22.		9 (*T*)	3.967	4.117	4.707	4.189	3.519	0.5464	21	9.718	(41°)	122.406	39	26.387	40°	7.2853	11.80
23.		9 (*T*)	5.540	5.700	5.584	4.970	4.155	0.6186	23	12.417	(40°)	156.418	1	33.000	(39°)	9.0312	7.07
24.		3	−0.167	+0.083	6.412	5.707	4.794	0.6907	9	0.2198	(39°)	2.763	19	0.005	37°	0.0014	31.35

TABLE No. 34—(*Continued*).

FIFTH SERIES, JUNE 7TH, 1899.

PITOT TUBE No. 3, GAUGE No. 9, AT KERCHEVAL AND SOUTH.						6-IN. METER No. 116 727.							GAUGE No. 6, RIVER AND SOLVAY, 3-FT. RADIUS.				GAUGE No. 7, SOUTH AND EXPOSITION 1.08-FT. RADIUS.						
(88)	(89)	(90)	(91)	(92)	(93)	(94)	(95)	(96)	(97)	(98)	(99)	(100)	(101)	(102)	(103)	(104)	(105)	(106)	(107)	(108)	(109)	(110)	(111)
No.	No. readings.	Observed inches. water.	Cal. Cr. Vel.	Cal. Cr. Vel. × 0.89 = true Cr. Vel.	True Cr. Vel. × 0.842 = Mean Vel.	No. readings.	Cubic feet per minute.	Cubic feet per second.	Mean Vel. in 12 ins.	Vel. ^{2}.	Vel. head = H_v, inches.	Log. of Vel.	Temp., degs. Fahr.	No. readings.	Inches, kerosene.	Inches, kerosene, 60° Fahr.	Temp., degs. Fahr.	No. readings.	Inches, kerosene.	Inches, kerosene, 60° Fahr.	— 0.235.	Inches, water, 60° Fahr.	Log. inches, water.
46...	.....	.....	.....	.....	.....	9	0.441	0.00740	0.00941	0.0001	0.0	.....	86	25	—0.090	—0.093	94⅜	12	0.225	0.235	.....	.....	.....
47...	29	0.319	1.311	1.167	0.983	14	41.357	0.68928	0.87755	0.7701	0.1435	—1.9432	(87)	26	1.392	1.441	94	25	1.872	1.952	1.717	0.3507	—1.5449
48...	15	1.350	2.636	2.400	2.020	9	90.000	1.50000	1.90971	3.6470	0.6794	0.2810	(87)	22	6.409	6.635	94	20	7.115	7.420	7.185	1.4678	0.1667
49...	.....	.....	.....	.....	.....	14	90.143	1.50238	1.91274	3.6586	0.6816	0.2817	(88)	26	6.511	6.748	95	27	7.237	7.555	7.555*	1.5434	0.1884
50...	.....	.....	.....	.....	.....	15	108.20	1.80333	2.29589	5.2712	0.9820	0.3609	88½	31	9.400	9.744	(95½)	28	10.139	10.587	10.587*	2.1628	0.3351
51...	.....	.....	.....	.....	.....	10	85.00	1.41667	1.80362	3.2530	0.6060	0.2561	88½	21	6.009	6.230	(95½)	21	6.619	6.912	6.912*	1.4120	0.1498
52...	.....	.....	.....	.....	.....	17	0.200	0.00333	0.00424	0 0	0.0	.....	88	23	—0.111	—0.115	(95)	33	0.151	0.158	.....	.....	.....

* No correction made.

TABLE No. 34—(*Continued*).

Fourth Series, June 7th, 1899.

Pitot Tube No. 3, Gauge No. 9, at Kercheval and South.						6-In. Meter No. 116 727.							Gauge No. 6, River and Exposition, 3-Ft. Radius.				Gauge No. 7, South and Exposition, 1.08-Ft. Radius.						
(64)	(65)	(66)	(67)	(68)	(69)	(70)	(71)	(72)	(73)	(74)	(75)	(76)	(77)	(78)	(79)	(80)	(81)	(82)	(83)	(84)	(85)	(86)	(87)
No.	No. readings.	Observed inches, water.	Cal. Cr. Vel.	Cal. Cr. Vel. × 0.89 = True Cr. Vel.	True Cr. Vel. × 0.842 = Mean Vel.	No. minutes.	Cubic feet per minute.	Cubic feet per second.	Mean Vel. in 12 ins.	Vel.²	Vel. head = H_v, inches.	Log. of Vel.	Temp., degs. Fahr.	No. readings.	Inches, kerosene.	Inches, kerosene. 60° Fahr.	Temp., degs. Fahr.	No. readings.	Inches, kerosene.	Inches, kerosene. 60° Fahr.	+ 0.064.	Inches, water. 60° Fahr.	Log. inches, water.
37...	.....	.....	.....	.....	.....	20	0.250	0.00417	0.00531	0.00003	0.0	.....	85	31	0.010	0.010	(92)	33	—0.061	—0.064	0.0	.....	.....
38...	.....	.....	.....	.....	.....	14	0.357	0.00595	0.00758	0.00005	0.0	.....	85½	28	0.025	0.026	(92½)	28	—0.027	—0.028	0.036	.....	.....
39...	.....	.....	.....	.....	.....	17	21.000	0.3500	0.4456	0.199	0.0371	—1.6489	85¾	38	0.505	0.522	(92¾)	34	0.500	0.521	0.585	0.1196	—1.0777
40...	.....	.....	.....	.....	.....	17	41.588	0.6931	0.8824	0.778	0.1449	—1.9457	87¼	39	1.886	1.953	(94¼)	37	1.695	1.768	1.832	0.3754	—1.5745
41...	.....	.....	.....	.....	.....	13	68.846	1.14743	1.46084	2.13394	0.3975	0.1647	88	26	4.694	4.865	(95)	27	4.341	4.532	4.596	0.9385	—1.9724
42...	28	1.353	2.698	2.401	2.022	18	89.944	1.46607	1.86651	3.48382	0.6490	0.2710	88⅞	36	7.519	7.800	(95⅞)	35	7.189	7.512	7.576	1.5477	0.1898
43...	25	1.776	3.092	2.752	2.317	7	106.571	1.77618	2.26133	5.11348	0.9526	0.3543	89	15	10.453	10.845	(96)	15	9.957	10.405	10.469	2.1387	0.3302
44...	.....	.....	.....	.....	.....	7	108.429	1.80615	2.29948	5.28770	0.9851	0.3616	89	13	10.569	10.965	(96)	15	10.180	10.638	10.702	2.1863	0.3397
45...	.....	.....	.....	.....	.....	15	0.333	0.00555	0.007066	0.00005	0.0	.....	89¼	21	0.062	0.064	(96¼)	20	0.240	0.251	.....	.....	.....

TABLE No. 34—(*Concluded*).

SIXTH SERIES, JUNE 8TH, 1899.

	6-IN METER No. 116 727.							GAUGE No. 6, SOLVAY GROUNDS, 2-FT. RADIUS.				GAUGE No. 7, SOUTH AND EXPOSITION, 1.08-FT. RADIUS.						
	(112)	(113)	(114)	(115)	(116)	(117)	(118)	(119)	(120)	(121)	(122)	(123)	(124)	(125)	(126)	(127)	(128)	(129)	(130)
	No.	No. minutes.	Cubic feet per minute.	Cubic feet per second.	Mean vel. in 12 ins.	Vel.^{2}	Vel. head $= H_v$, inches.	Log. of vel.	Temp., degs. Fahr.	No. readings.	Inches, kerosene.	Inches, kerosene, 60° Fahr.	Temp., degs. Fahr.	No. readings.	Inches, kerosene.	Inches, kerosene, 60° Fahr.	0.0.	Inches, water, 60° Fahr.	Log. inches, water.
53....	15	0.200	0.00333	0.00424	.....	0.00	.....	87¾	30	—0.283	— 0.293	(86)	30	— 0.187	— 0.193	.....	.....	.....
54....	12	28.667	0.47778	0.60833	0.37003	0.0689	—1.4118	.....	.....	.....	.....	.....	(86)	21	0.700	0.724	.....	.....	.....
55....	13	29.000	0.48333	0.61539	0.37872	0.0705	.....	85⅞	28	0.532	0.550	(86)	28	0.703	0.727	.....	.....	.....
56....	7	0.143	0.00238	0.00303	.....	0.00	.....	.....	.....	.....	.....	.....	(86)	8	— 0.194	— 0.201	.....	.....	.....
57....	10	0.100	0.00167	0.00202	.....	0.00	.....	92½	15	0.020	0.021	(86)	14	0.064	0.066	.....	.....	.....
58....	14	20.286	0.33810	0.43048	0.18533	0.0345	—1.6340	105½	29	0.731	0.771	(86)	28	0.519	0.537	0.537	0.1097	—1.0402
59....	14	40.571	0.67618	0.86094	0.74115	0.1381	—1.9350	94	28	1.889	1.970	86	27	1.661	1.718	1.718	0.3509	—1.5452
60....	14	62.214	1.03690	1.32022	1.74293	0.3247	0.1206	(86)	28	3.721	3.848	85½	30	3.642	3.763	3.763	0.7687	—1.8858
61....	17	90.765	1.51275	1.92609	3.70986	0.6912	0.2847	(86)	34	6.888	7.123	85⅜	32	7.283	7.530	7.530	1.5383	0.1870
62....	8	109.750	1.82917	2.32897	5.42424	1.0105	0.3672	(87)	15	9.793	10.138	87	15	10.430	10.800	10.800	2.2063	0.3436
63....	8	109.250	1.82083	2.31834	5.3747	1.0013	0.3651	(86)	16	9.625	9.953	86	16	10.259	10.609	10.609	2.1673	0.3359
64....	9	1.555	0.02592	0.03300	0.0011	0.0002	.....	(85)	22	0.386	0.399	85	21	0.274	0.283	.....	.....	.....

that the discs would rise enough to give a clear opening, the Pitot tube readings taken here show a very peculiar distribution of velocities, which has already been discussed.

At the piezometer special at Kercheval and South Streets, no Pitot tube observations were taken with the tube placed in the pipe horizontally, owing to the water in the well standing at the top of the 12-in. pipe, and, although it could be lowered somewhat, the hand pumps were unable to keep the well clear. It had been the intention to make Pitot tube observations, both horizontally and vertically, to better determine the distribution of velocities, but, on account of the water, it was impossible to do this.

Gasoline was used in the differential gauges for these series instead of kerosene, which has since been found to be more satisfactory for ordinary use, and one of these gauges was placed on the 13-in. radius curve at Exposition and South Streets; the intention being to have one always at this point, and to move the other from place to place.

TABLE No. 35.—GENERAL PLAN OF EXPERIMENTS ON 12-INCH CAST-IRON PIPE.

Series.	Date.	Artillery and South Curve, 4-ft. Rad.	East 1 000-ft. Tangent.	West 1 000-ft. Tangent.	Entire 2 000-ft. Tangent.	Exposition and South Curve, 13-in. Rad.	Exposition and River Curve, 8-ft. Rad.	River and Solvay Curve, 8-ft. Rad.	Solvay Curve, 2-ft. Rad.	Pitot Tube No. 3 at Kercheval and South.	Pitot Tube No. 3 at Reid and South.	6-in. Meter.
1st..	Nov. 4, 1898..	7 G.	....	....	1 M.	6 G.	....	...	....	*	....	....
2d....	Nov. 16, 1898..	....	....	....	1 M.	7 G.	....	....	....	....	*	....
3d....	June 6, 1899..	....	6 K.	7 K.	....	....	....	....	....	*	....	*
4th..	June 7, 1899..	....	....	....	....	7 K.	6 K.	....	....	*	....	*
5th..	June 7, 1899..	....	....	....	....	7 K.	....	6 K.	....	*	....	*
6th..	June 8, 1899..	....	....	....	....	7 K.	....	....	6 K.	....	....	*

Numbers denote gauges used at points indicated.
K. = Kerosene in gauge.
G. = Gasoline " "
M. = Mercury " "
* Indicates method of observing velocity.

Third, Fourth, Fifth, and Sixth Series.—During the spring of 1899, all final connections were completed, and on June 7th, 8th, and 9th, experimental work was carried on with Gauges Nos. 6 and 7, filled with kerosene. One of the 6-in. meters, No. 116 728, was entirely closed, and all water passed through No. 116 727, which was read to the nearest cubic foot each minute.

Notes of Work.—To properly understand the manner in which the water was wasted, attention is again called to Fig. 78. In the first and second series the water passed through the 12-in. pipe until it reached the 12 x 8-in. tee at the blow-off, thence out of the blow-off gate into a 24-in. sewer. In the third, fourth, fifth and sixth series the water passed through the 12-in. pipe to the meter, through the meter and back in the 10-in. pipe to the 10 x 8-in. blow-off tee, thence out of the same blow-off gate as in the first and second series. In each case the necessary gates to obtain this result were opened and closed at the blow-off, and all regulations of flow were made with the 8-in. blow-off gate.

The observations made in Delray were considered good at the time they were taken; the gauges worked well, no trouble was experienced with leaks, and, all in all, the general results seemed to be very satisfactory.

Before beginning work in 1899, the lines were thoroughly flushed, to remove any sediment which might have collected in the bottom of the pipe.

Six-Inch Meter.—The following general description of the meter used is taken from the manufacturer's catalogue.*

"The displacing or measuring member consists of a plain flat disc having a ball and socket bearing, and is adapted to oscillate in a chamber comprised of two sections joined together, in which each of the inside faces approximates the frustum of a cone; the exterior confining wall assuming the form of a circular zone. The disc has a single slot projecting radially from the ball, which embraces a fixed metallic diaphragm set within and crosswise of one side of the chamber, the discs being thus prevented from rotating; but, when it is caused to oscillate in contact with the cone frustums, the chamber, by these means, is divided into sub-compartments or measuring spaces. Now, if the ports of ingress and egress are properly disposed on opposite sides of the diaphragm, the disc will act as its own valve."

The size used in these experiments, 6-in., has "an advised maximum duty of 100 cu. ft. per minute, but it may be operated at forced flows of 120 cu. ft. per minute." The highest quantity recorded during the work was 108.43 cu. ft. per minute, in the fourth series.

At the time the experiments were made, it was supposed that a rating at different velocities, determined from the result of experiments

* Catalogue of the Thomson Meter, received in 1900.

made to ascertain the accuracy of the various sizes under different conditions of flow, could be obtained from the manufacturer. The writers, however, have not been able to obtain such a rating.

Reductions.

Partial Log of Results.—All observations taken in the field were plotted directly on cross-section paper, 20 ins. wide, ruled ten to 1 in. The abscissas were divided to represent time, and the ordinates to various scales, to graphically show readings of all gauges taken simultaneously. Fig. 81 shows the original plotting of the work of the afternoon of June 8th, sixth series, which is self-explanatory. If there was any difference in time, as shown by the different observer's watches, it became evident on these sheets, and did not interfere with the reductions. Whenever it was apparent from the lines drawn that equilibrium had become established, an observation was averaged and the result was placed on the diagram.

Table of Results, Complete.—In Table No. 34 the results for all series are given, as averaged by aid of the plotted sheets from the field notes, without a change or correction, together with corrections and alterations as made by the writers, and necessary logarithms to properly understand the diagrams which follow.

Diagrams and Explanations.—In Fig. 82 a comparison is made between H_j, in inches of water at 60° Fahr., in the 13-in. radius curve, and H_f, in the 4-ft. radius curve, first series, and the equation found by means of centers of gravity.

In Fig. 83, H_j in the 13-in radius curve, in inches of kerosene at 60° Fahr., is compared with H_f in the 3-ft. radius curve at Exposition and River Streets, which is also in kerosene at 60° Fahr., fourth series, and correction is made for diameter; likewise H_j, in the 13-in. radius curve, kerosene at 60° Fahr., is compared with H_f in the 3-ft. radius curve at River and Solvay, kerosene at 60° Fahr., fifth series; and H_f in the 13-in. radius curve, kerosene at 60° Fahr., with H_f in the 2-ft. radius curve, kerosene at 60° Fahr., sixth series.

As the results obtained in Figs. 82 and 83 are relative, and the addition or subtraction of a constant from a series of any of the observations would not alter the slope of the lines indicated on these diagrams, the points plotted are observations derived directly from the original readings, with no corrections applied except for temperature.

In Fig. 84 the ratio of H_p, in the 2 000-ft. tangent, expressed in inches of water at 60° Fahr., first and second series, with the H_f of the 13-in. radius curve, in inches of water at 60° Fahr., first and second series, are plotted, and the equation determined. There seems to be no evidence that the average line of points could be represented better by a curve than a straight line, as shown on the diagram, and it may therefore be safely predicted that the value of n, in the equation $H_f = m\ V^n$, will be found to be very nearly the same in straight pipes and curves.

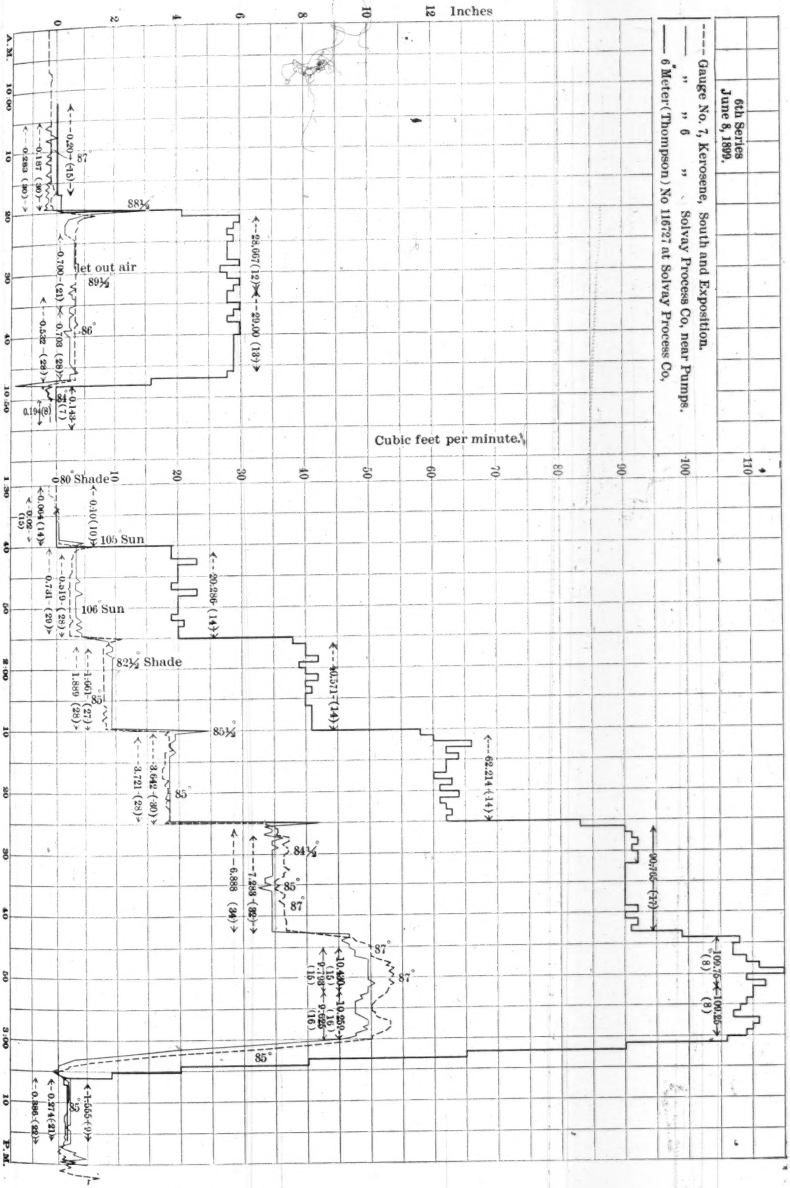
Pitot Tube and Meter.—In Fig. 85 the corrected center velocities, as reduced from observations by Pitot Tube No. 3 at Kercheval and South Streets, third, fourth and fifth series, are plotted with the mean velocity as recorded by the 6-in. meter.

The points are averaged as a straight line, although this line does not pass exactly through the origin. Five of the points plotted were obtained in the third series, and it will be seen by referring to Column 42 that Gauge No. 9 failed to come to about zero when the 12-in. line was very nearly closed off. The correction necessary to be applied, Column 43, —0.148 in., was determined by carefully plotting the observed readings, Column 42, with H_f in the west and east 1 000-ft. tangents, Columns 56 and 62, and estimating the distance necessary to move the line so obtained in order to cause it to pass through the origin.

The slope of the line indicates that the ratio of mean velocity (V_m) by meter, to the true center velocity, as found by the Pitot tube at Kercheval and South Streets, is 0.842. As has been previously demonstrated, under the heading "Pitot Tubes," the ratio of center velocity to mean velocity, determined by the various Pitot tubes, is 0.84, or practically the same as given here.

It will thus be seen that, by two entirely different and dissimilar methods of measurement, the discharge of this pipe coincides within $\frac{1}{4}$ of 1 per cent.

All the results obtained on H_p, in Series 3, 4, 5 and 6, were plotted with $\frac{V^2}{2g} = H_v$ = velocity head, as derived from meter, and all irregularities of points investigated; 1st, for errors in reductions, and 2d, for improper choice of lengths of observations or apparent mistakes in reading the scale on the gauges or meters.


61h Series
June 8, 1899.
----- Gauge No. 7, Kerosene, South and Exposition.
—— " " 6 " Solvay Process Co. near Pumps.
—— 6 Meter (Thompson) No 116727 at Solvay Process Co.
Inches
Cubic feet per minute.
jet out air
80° Shade
105 Sun
106 Sun
82½° Shade

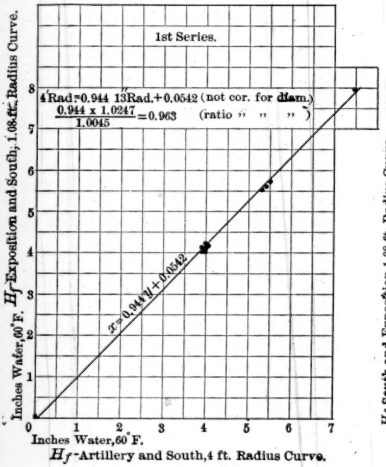


FIG. 82.

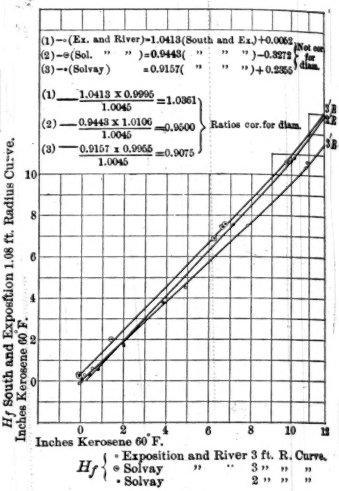


FIG. 83.

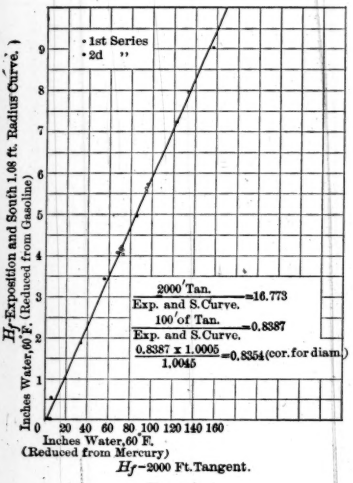


FIG. 84.

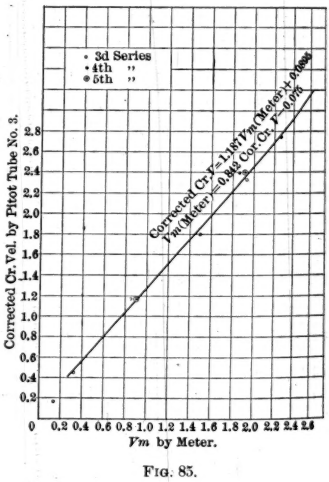


FIG. 85.

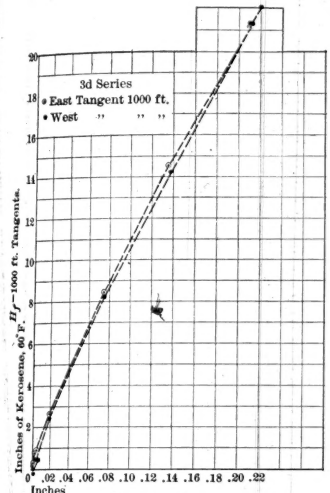


FIG. 86.

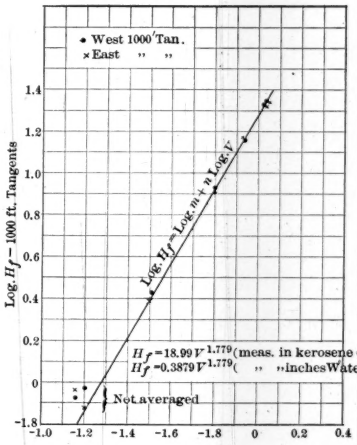


FIG. 87.

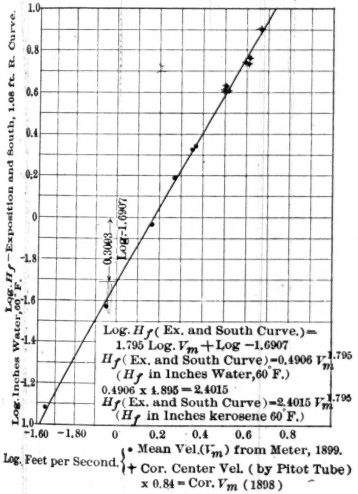


FIG. 88.

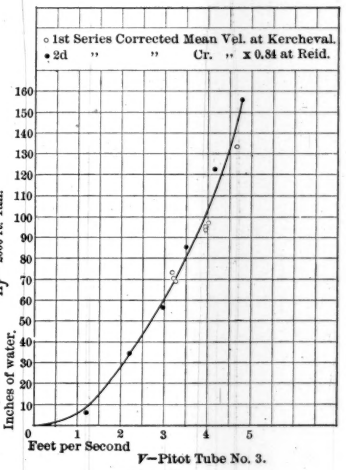


FIG. 89.

The initial error, if any existed, was thus graphically represented, and in Fig. 86 is shown one of these plottings.

In this case, the west 1 000-ft. tangent, is corrected with + 0.159, Column 56, which brings a zero observation (25), when the meter is recording a mean velocity of 0.01 ft. per second, because a careful study of this diagram, when drawn to a large scale, tends to show that Observation 25 is slightly too low, relatively. As the line of points is not straight, it is evident that H_f does not vary as V^2.

Up to the time of making the experiments shown on this diagram, considerable time had been expended in determining whether the size of the hose and lead pipe connecting with the piezometer, or the use of unequal lengths of hose on each side of the differential gauges, would cause a variation in the readings obtained. This diagram proves that the gauge may be set and connected in any manner, without varying the results, provided no other sources of error exist.

Value of n, *in* $H_f = m\ V^n$.—As it was shown by Fig. 86 that H_f in the tangent does not vary as V^2, it is assumed that the equation $H_f = m\ V^n \pm b$, fairly expresses the true relations between H_f and V. As it seems hardly practicable to eliminate $\pm b$ by plotting logarithmically, the assumption is made that it is a constant quantity in any one series, and is first eliminated in the table of results. The equation then becomes $H_f = m\ V^n$, which, for logarithmic plotting, may be written log. H_f = log. m + n log. V, a straight line (see Fig. 87).

Only the logs. in the higher velocities are averaged, and the straight line calculated by the centers of gravity in the manner previously described. In the foregoing equation, n is found to equal 1.779.

The value of n, in the equation $H_f = m\ V^n$, for the 13-in. radius curve, is obtained in a manner similar to that for the tangents in Fig. 87. As H_f was measured with gasoline in the differential gauges in the first series, and kerosene in the fourth, fifth and sixth series, all are brought to the equivalent in inches of water at 60° Fahr. in Fig. 88; and as the velocity in the first series was observed by Pitot Tube No. 3, the log. of its corrected mean velocity is used; while in the fourth, fifth and sixth series, the log. of the mean velocity is plotted as given by the meter.

It is remarkable how closely the points fit the computed straight line and show how closely the velocities coincide when determined at

TABLE No. 36.—Results of 12-Inch Pipe Experiments.

Location.	Radius of curve, in feet, or in diameter of pipe.	Relative frictional loss in 100 ft.		$H_f = m\ V^n$		H_f, in Feet of Water per 1 000 Ft., and Value of c (Chezy Formula), $V = c\sqrt{r\,s}$, at Given Velocity.									
				m for H_f, in feet of water per 1 000 ft.	n	1 ft. per second.		2 ft. per second.		3 ft. per second.		4 ft. per second.		5 ft. per second.	
(1)	(2)	(3)	(4)	(5)	(6)	H_f (7)	c (8)	H_f (9)	c (10)	H_f (11)	c (12)	H_f (13)	c (14)	H_f (15)	c (16)
South Street.....	Tan.	0.843	1.000	0.3233	1.779	0.3233	111.1	1.1092	119.9	2.280	125.7	3.807	129.6	5.662	132.9
Exp. and South.....	1.08	1.0000	1.1970	0.4088	1.795	0.4088	99.0	1.419	106.2	2.9372	112.6	4.9239	114.0	7.3421	116.7
Solvay Grounds.....	2.00	0.9075	1.0863	0.3710	(1.795)	0.3710	103.6	1.2874	111.7	2.6654	116.2	4.4683	119.7	6.6682	122.4
Exp. and River.....	3.00	1.0361	1.2402	0.4236	(1.795)	0.4236	97.2	1.4699	102.7	3.0432	108.8	5.0616	112.4	6.7133	114.6
River and Solvay.....	3.00	0.9500	1.1372	0.3884	(1.795)	0.3884	101.5	1.3478	108.9	2.7905	113.6	4.6781	117.1	6.9812	119.7
Artillery and South..	4.00	0.9630	1.1527	0.3937	(1.795)	0.3937	100.8	1.3660	108.2	2.8283	112.8	4.7413	116.2	7.0756	118.9

various times by Pitot tube and meter. The value of n is found to be 1.795.

As explained before, the readings obtained at Reid and South Streets in the second series, with Pitot Tube No. 3, gave a very unequal distribution of velocities. At Kercheval and South Streets, where the velocities are much more uniformly distributed, it has been shown that $\frac{\text{mean vel.}}{\text{center vel.}} = \frac{V_m}{V_c} = 0.84$. Fig. 89 shows, in a general way, that the ratio 0.84 holds good, within reasonable limits, where applied to a cross-section of unsymmetrical distribution, caused by a partially closed gate, or an ordinary obstruction on one side of the pipe. The average line is drawn by eye, and is only an approximate mean.

Conclusions.—It is evident from Table No. 36 that something is wrong with the results derived from one of the 3-ft. radius curves. As no indication of the cause for this difference can be found in any of the observations, this variation may be due to any of the following reasons: (1) Measurement error in setting a piezometer; (2) one of the piezometers not set at right angles to flow; (3) air in one of the gauge connections which remained in a horizontal section of the hose or lead pipe during a condition of no flow, but which moved into a vertical section as the flow was increased and the reading of the gauge became greater; (4) the effect in the loss of head caused by a change in alignment in front of the first piezometer.

The measurements were very carefully made and checked, and no discrepancies can be found in any of the notes of the work; the piezometers were all set by the same men and with the same tapping machine, and the lead and hose connections were allowed to run for some time before being connected to the gauge.

The only curve on the entire line having a slight swing in the pipe immediately before the curve was at Exposition and South Streets, where a 2° change in alignment was made in the joints about 50 ft. from the first piezometer. How much effect this would have on the gauge readings, the writers are not prepared to say, but it is known from the results obtained on the 30-in. cast-iron pipe that the effect of curvature extends more or less into the following tangent, and it seems probable that the effect in this case has been to increase the value of H_f in the section containing the curve.

Omitting this curve, the length of tangent necessary to produce the

loss of head occasioned by deflecting flowing water through each of the 90° curves is taken from Column 4 and given as follows:

H_f 90° 13-in. radius curve in excess of H_f in equal length of straight pipe is equivalent to H_f for 19.7 ft. of tangent.

H_f 90° 2-ft. radius curve in excess of H_f in equal length of straight pipe is equivalent to H_f for 8.63 ft. of tangent.

H_f 90° 3-ft. radius curve in excess of H_f in equal length of straight pipe is equivalent to H_f for 13.92 ft. of tangent.

H_f 90° 4-ft. radius curve in excess of H_f in equal length of straight pipe is equivalent to H_f for 15.27 ft. of tangent.

From Column 6 it is seen that $H_f \propto V^{1.795}$ for the Exposition and South Streets 1-ft. radius curve. In Columns 7 to 16 the values of H_f and c (Chezy formula) are determined for several velocities. Except those given for the South Street 2 000 ft. tangent, these values are useful principally for comparisons. The equation for flow through 1 000 ft. of the straight pipe is found to be $H_f = 0.3233\ V^{1.779}$, measured in feet of water at 60° Fahr.

Conclusions.

General Discussion.

Curvature and Other Effects.—The foregoing experiments, taken as a whole, prove beyond question or doubt that the hitherto accepted notions of the laws governing curve resistance are wholly in error. As was suggested in the discussion of the distortion of the velocity curve, the resistance due to curvature is proved to be very much less in the short than in the long-radius curves for the same total length of pipe and angle of deflection. From the experiments upon the 30-in. pipe, as well as those on the 12-in., it is apparent that the 90° curve introducing the least resistance in a total length of 80 diameters is one of between 2 and 2½ diameters radius. Fig. 90 shows graphically, more clearly than words can describe them, the net results of the curve investigation. All the curve sections are reduced to a uniform length of 80 diameters by adding on or taking away tangent resistance. The lengths of the axes of the curved pipe, in units of the pipe diameters, are plotted as the abscissas, and the percentages of excess of the observed losses of head, in the curve section 80 diameters long, over those observed in an equal length of preceding tangent, are plotted as

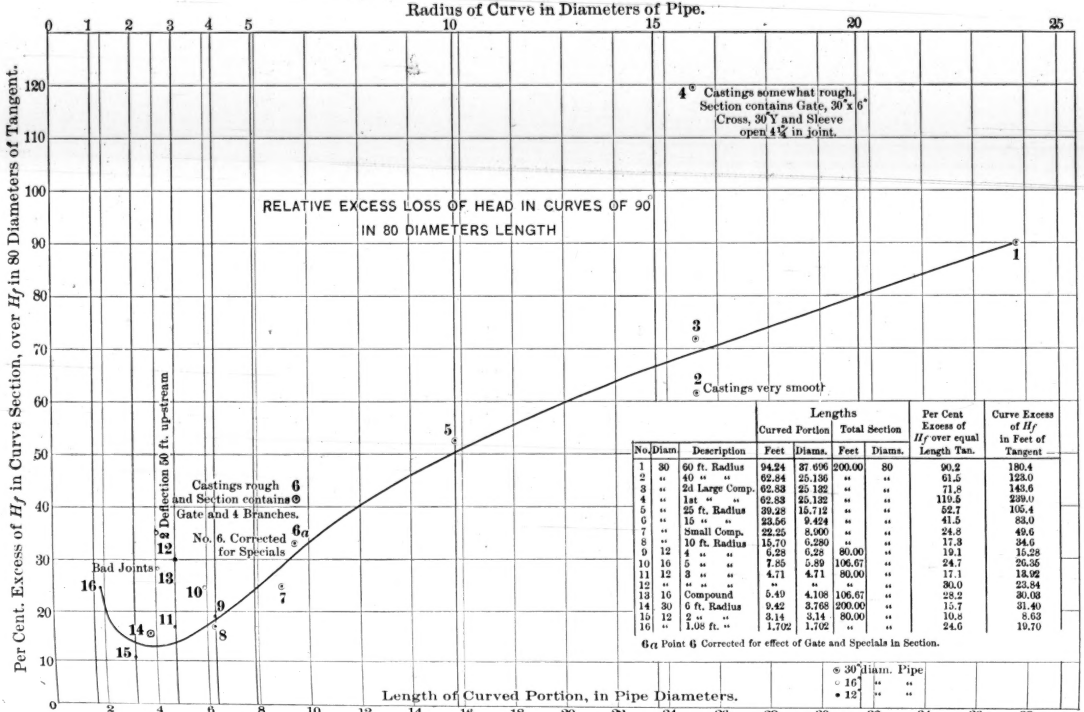
the ordinates. In the cases of the 30-in. curves the resistance in the preceding tangent has been taken as the standard of comparison, except where that tangent was very short, or was affected by abnormal conditions. In the latter cases, those of the 40-ft. radius and small compound curves, the loss of head has been compared to the mean of the second preceding and the following tangent. In the case of the first long compound curve, a value about 3% below that shown by the Orleans tangent has been assumed for its preceding tangent on Scott Street, and the curve section filled out to its full length by adding a section with this resistance. On account of the specials in the Rivard tangents, particularly the upper one, the resistance in the 10-ft. radius curve has been compared to that in the East Alexandrine tangent. The 16-in. curves are compared with the Army tangent, and the 12-in. curves with the long South Street tangent.

The experiments on the 30-in. pipe also show that there is in general a continuous increase in the tangent resistances as each curve is passed, as though each curve added something to the losses in the following tangent. This effect is clearly shown in Table No. 28, and also in Table No. 37 of the values of c in the Chezy formula, which have been computed for the long tangents, and are repeated here for convenience.

TABLE No. 37.—EXPERIMENTAL VALUES OF c, IN $V = c\sqrt{rs}$, IN TANGENTS ON 30-INCH PIPE.

Tangent.	Value of c.	Remarks.
Hale Street	123.4	Contains no specials.
Rowena Street	121.5	" " "
Brady Street	115.9	" " "
South Rivard Street	114.6	" 3 branches distributed.
North " "	109.6	" gate and 3 branches at upper end.
East Alexandrine Avenue	115.4	" no specials.
West " "	115.9	" " "
South Alley	118.2	" gate at lower end
North "	114.4	" 2 branches separated.

It will be seen from Table No. 28 that those tangents in which the resistance is abnormally high are followed by curves in which the resistance is abnormally low, and *vice versa*. From this and the increase of tangent resistance after passing each curve it is evident that the resistance generated by the curve passes beyond the section containing it in these investigations. The same appears to be true with the re-



No.	Diam.	Description	Lengths: Curved Portion Feet	Curved Portion Diams.	Total Section Feet	Total Section Diams.	Per Cent Excess of H_f over equal Length Tan.	Curve Excess of H_f in Feet of Tangent
1	30	60 ft. Radius	94.24	37.696	200.00	80	90.2	180.4
2	"	40 " "	62.84	25.136	"	"	61.5	123.0
3	"	2d Large Comp.	62.83	25.132	"	"	71.8	143.6
4	"	1st " "	62.83	25.132	"	"	110.5	239.0
5	"	25 ft. Radius	39.28	15.712	"	"	52.7	105.4
6	"	15 " "	23.56	9.424	"	"	41.5	83.0
7	"	Small Comp.	22.25	8.900	"	"	24.8	49.6
8	"	10 ft. Radius	15.70	6.280	"	"	17.3	34.6
9	12	4 " "	6.28	6.28	80.00	"	19.1	15.28
10	16	5 " "	7.85	5.89	106.67	"	24.7	26.35
11	12	3 " "	4.71	4.71	80.00	"	17.1	13.92
12	"	" " "	"	"	"	"	30.0	23.84
13	16	Compound	5.49	4.108	106.67	"	28.2	30.08
14	30	6 ft. Radius	9.42	3.768	200.00	"	15.7	31.40
15	12	2 " "	3.14	3.14	80.00	"	10.8	8.63
16	"	1.08 ft. "	1.702	1.702	"	"	24.6	19.70

6a Point 6 Corrected for effect of Gate and Specials in Section.

FIG. 90.

sistances caused by gates, and it will be seen, by the correction applied to the 15-ft. radius curve experiment in Fig. 90, to eliminate the effect of gates and specials, whereby the position of the point *6* is moved to *6a*, that the loss of head through a fully open gate may be half as much as that of the curve of least resistance.

The traverses taken at short distances below gates have shown the water, on the side toward the opening into the bonnet, to be retarded, and when it happens that the setting of a gate in the down-stream portion of a curve section is such as to oppose the action of the bonnet to that of the curve a very high resistance is observed in the curve section, and a very low one in the tangent, because the gate tends to cause the velocities to readjust themselves in passing, with the attendant loss of head there. This is seen in the case of the 15-ft. radius curve at Rivard and Brady Streets, and in the first large compound curve at Scott and Orleans Streets. In the latter the effect of a gate is evidently more than compensated by the 30 × 30-in. Y. In both of these cases the following tangents are seen to have quite low losses of head.

In the table on Fig. 90, in addition to the plotted data, there is shown in the last column the number of feet of tangent that would give a resistance equal to the excess in the curve section. An approximate curve effect locus is sketched through the points that seem to the writers most probable, and memoranda are entered at the several points which do not fit it, stating what reasons are known, to account for the eccentric positions.

The data thus far obtained, however, are insufficient to determine the distance to which the effects of a disturbance travel, and hence the figure cannot be considered as complete, for, if the disturbance extends beyond the outlet piezometer of the section, only a part of the loss due to the curve is shown by the gauge, but, from the traverses, it appears that they may extend farther at low than at high velocities, which is as would be expected, if the friction on the pipe wall increases with a higher power of V than the first.

The values of c already presented appear somewhat high for the 12-in. pipe, and decidedly low for the 30-in. The smoothness of the two lines was the same, as nearly as could be judged, and the conclusion is that the deficiency in carrying capacity of the latter was due to the effects of curvature and specials. The apparent effect of the 2° deflec-

tion of the up-stream tangent, 50 ft. from the 3-ft. radius curve, at River Street and Exposition Avenue, is a startling exhibition of the results of small changes in alignment, as is also the reduction of c, from 123 in the Hale Street tangent to 116 in the Brady Street tangent, of the results of larger deflections. Having shown that the long, so-called easy curves may be productive of a very great increase in the resistances to flow, the importance of careful alignment, both horizontally and vertically, is apparent, and it seems to the writers that this element has in the past been greatly overlooked, while the effects of roughness have been exaggerated. It will be recalled that in riveted and stave-pipe lines the long, easy curve is a prominent feature, and it seems entirely possible that some unexpectedly low coefficients for such lines are due to this cause.

Coefficients.—The values of c, in the Chezy formula, have been already presented for the several investigations, but for convenient reference they are assembled in Table No. 38.

TABLE No. 38.—EXPERIMENTAL VALUES OF c, IN CHEZY FORMULA, FOR STRAIGHT PIPE.

Diameter of pipe, in inches.	VELOCITIES, IN FEET PER SECOND.				
	1.	2.	3.	4.	5.
12	101.5	109.6	114.6	118.3	121.3
16	119.3	125.8	129.4	132.2	134.4
30	109.6 to 123.4	109.6 to 123.4	109.6 to 123.4	109.6 to 123.4	109.6 to 123.4

In Table No. 39 are presented the values of n and m, in the equation of resistance, $H_f = mV^n$.

Table No. 40 presents the calibrations, at 60° Fahr., of the various fluids experimented with in the fluid differential gauges, as obtained by experiments with the gauges.

Accuracy of Results.—Pitot tube measurements of discharge at the same location, made with the same instruments and under similar conditions, but at different times, were found to coincide within $\frac{3}{10}$ of 1 per cent.

Pitot tube measurements of discharge at different locations, with different instruments under similar conditions, but at different times, were found to coincide within from 1 to 1.7 per cent.

TABLE No. 39.—EXPERIMENTAL VALUES OF n AND m FOR TANGENTS, WHERE H_f IS MEASURED IN 1 000 FT., IN WATER.

Diameter of pipe, in inches.	Location.	n.*	m.
12	South Street	1.779	0.3879
16	Porter Street	1.857	0.2141
16	Army Street	1.860	0.2114
30	Hale Street	2.000	0.1051
30	Rowena Street	2.000	0.1084
30	Brady Street	2.000	0.1192
30	S. Rivard Street	2.000	0.1220
30	N. Rivard Street	2.000	0.1330
30	E. Alexandrine Avenue	2.000	0.1201
30	W. Alexandrine Avenue	2.000	0.1192
30	S. Alley	2.000	0.1145
30	N. Alley	2.000	0.1223

* The value of n for the 12-in. pipe is given in Fig. 87. For the Porter tangent it was carefully computed (Figs. 74 and 75.) For the Army tangent it appeared to be slightly greater than for the Porter tangent. On the 30-in. line, n was determined for the 15-ft. radius curve (Fig. 27), and this value was used for all sections, because they all plotted straight lines with the resistance in the 15-ft. radius curve.

TABLE No. 40.—EQUIVALENTS, AT 60° FAHR., OF 1 IN. OF WATER AT SAME TEMPERATURE.

Fluid.	Equivalent in water in differential gauge.
Mercury	0.0795 in.
Water	1.0000 "
Gasoline	3.5200 ins.
Kerosene	4.8950 "
Sperm oil	8.9850 "

Pitot tube ratings in 2-in. brass pipe gave a coefficient, for Tube No. 3, of 0.89 (page 33). Simultaneous observations in the field work in 30 and 16-in. pipe with Tubes Nos. 3 and 5 determined the coefficient of the latter to be 0.75 (page 37). Tube C was rated in 5-in. brass pipe, and its coefficient found to be 0.80. The ratio between the mean and central velocity, when a maximum, was established by traverses as 0.84 (page 34). A traverse was made with Tube A, a duplicate of Tube C, and simultaneously the center velocity was determined by Tube No. 5, at a point less than 1 ft. away, in the 30-in. pipe. Applying the rating coefficient, 0.80, to the observations with Tube A, and the rating coefficient, 0.75, and the ratio $\frac{V_m}{V_c} = 0.84$, to those with Tube No. 5, the two mean velocities obtained agree within 2.7 per cent.

The rating of Tube No. 6 was determined by comparing its reading on center, at a certain position, with the loss of head simultaneously observed in the Brady tangent, and then making a similar comparison between Tube No. 3 and the same tangent, the observations being made in 1899 when the tangent loss was measured in kerosene. In this case, with Tube No. 3, only four velocities were observed. In 1898, Tube No. 3 was observed simultaneously with the loss of head in the Brady tangent for seventeen different velocities, the loss of head being read in sperm oil. The sperm oil and the kerosene were calibrated in terms of water, and the loss of head in 1898 reduced to an equivalent in kerosene. From this a comparison was made with Tube No. 3, and this result compared with the relation of Tube No. 6 to the Brady tangent used above which was determined at 148 different velocities. The coefficient of Tube No. 6, as obtained by the two comparisons, agreed within $\frac{12}{100}$ of 1 per cent.

TABLE No. 41.

Section.	1898: Number of Velocities.	1899: Number of Velocities.	Ratio of Resistances. 1898 ÷ 1899.	Discrepancy: Percentage of Ratio. + = excess in 1899. − = excess in 1898.
I.—1st comp. curve	2	7	0.9619	− 3.81
III.—2d comp. curve	2	10	0.9225	− 7.75
IV.—Hale Tan	13	9	1.0853	+ 8.53
V.—60-ft. rad. curve	2	10	1.0074	+ 0.74
VII.—40-ft. rad. curve	2	10	0.9611	− 3.89
VIII.—Rowena Tan	14	13	1.0081	+ 0.81
IX.—25-ft. rad. curve	17	12	1.0051	+ 0.51
XI.—Small comp. curve	13	7	0.9749	− 2.51
XII.—Brady Tan	38	142	1.0000	0.00
XIII.—15-ft. rad. curve	13	11	0.9568	− 4.32
XV.—N. Rivard Tan	17	10	0.9929	− 0.71
XVI.—10-ft. rad. curve	6	10	0.9426	− 5.74
XII.—E. Alex. Tan	38	14	1.0044	+ 0.44
XX.—6-ft. rad. curve	6	14	0.9668	− 3.32

SUMMARY.

Five sections agree within	less than	1 per cent.
One section agrees	from	2 to 3 per cent.
Three sections agree	" "	3 to 4 "
One section agrees	"	4.32 "
One " "	"	5.74 "
One " "	"	7.75 "
One " "	"	8.53 "

This table shows that, as compared with the Brady tangent in each year, eight sections had increased their carrying capacity and five had decreased it during the year intervening between the two series of experiments. By the tube comparison of the Brady tangent resistances in the two years, it appeared that the carrying capacity of this section had increased $\frac{12}{100}$ of 1 per cent. In the opinion of the writers, these experiments do not warrant any conclusions as to changes in carrying capacity of the pipe line, and it is further their opinion that this was practically the same in the two years.

Tangent and curve resistances were measured between June and October, 1898, on fourteen different sections of 30-in. pipe, and similar observations were made on the same sections in June, 1899. The comparative results are shown in Table No. 41, referred to the Brady tangent resistance, which, as stated in the last paragraph, agreed within $\frac{12}{100}$ of 1% in the two years, by the tube comparisons.

The discharge of the 12-in. main was determined in 1898 by Tube No. 3, for various losses of head in a certain section measured in gasoline, Tube No. 3 having been rated in a 2-in. brass pipe by weighing the water passing by it when on center, the ratio, $\frac{V_m}{V_c}$, being previously determined. In 1899 the resistance in the same section was measured in kerosene, and the discharge measured by a 6-in. disc meter. The kerosene and the gasoline being calibrated in water, for similar losses of head, the discharges, as determined by meter, were found to agree with those determined by the Pitot tube within $\frac{1}{4}$ of 1 per cent.

Relative Reliability of 1898 and 1899 Series.—The experiments of 1899 on the 30-in. pipe were carried on with greater care and accuracy of observation than those of 1898, but the range of velocity was only about one-half that of the earlier investigations. The experiments of 1898 were made at night when the effects of temperature changes, and of the variation of consumption, were less important than in the day-time work of 1899. To reduce the experiments of 1898 to a common basis, it was necessary to go through an extended series of comparisons of the different observation equations, whereby errors in one might be carried into the next, while with those of 1899 the comparisons were direct; as two instruments, the gauge on the Brady tangent and Tube No. 6, were read continuously in the same locations throughout the entire investigation.

In view of all the foregoing conditions the writers believe the results of the two series (except those on the Hale tangent, in which the 1898 observations are considered questionable) to be entitled to equal confidence.

Final Results.

The results obtained and presented in connection with this investigation may be summarized as follows:

A.—The invention of the oil differential gauge, by which it is possible to observe differences of head in closed conduits under pressure,

with as great a degree of precision as is attainable in open conduits with the hook-gauge.

B.—The invention of a form of Pitot tube which may be inserted in a water main without the aid of special devices, other than the tools possessed by every water department; by the aid of which a competent observer may obtain gaugings with as great accuracy as has yet been attained with any other measuring device, except the graduated tank and weighing scale.

C.—The determination of a ratio, 0.84, between the mean and the maximum velocities of water flowing in closed circular conduits, under normal conditions, at ordinary velocities; whereby observations of velocity taken at the center under such conditions, with a properly rated Pitot tube, may be relied upon to give results within 3% of correctness.

D.—The presentation of a series of coefficients for application to the different fluids used in the fluid differential gauges, by which the observations so taken may be conveniently reduced to equivalents in water.

E.—The demonstration of the fact that ratings of Pitot tubes made by dragging those instruments through still water in open troughs do not conform, within any reasonable limits, to those obtained when the instrument is stationary in moving water in a closed conduit.

F.—The demonstration that Pitot tubes must have their coefficients determined, whether they consist of a point opening alone or both point and pressure openings.

G.—The demonstration that under some conditions, in straight pipe, there is a difference of pressure at different points around the circumference of the same cross-section.

H.—The derivation of the ellipse as the approximate form of the normal curve of velocities in straight circular pipes.

I.—The demonstration that the effects of disturbances of the flow of water in pipes are transferred for many diameters beyond the point where the interference occurs.

J.—The inference that careful alignment is as necessary for large carrying capacity in pipe lines as is smoothness of interior surface.

K.—The discovery and demonstration that curves of short radius, down to a limit of about 2½ diameters, offer less resistance to the flow of water than do those of longer radius, and hence that the theories

and practices regarding curve resistance, as set forth in the hydraulic treatises of all nations up to the present time, are absolutely incorrect and the diametric opposite of the true conditions.

Acknowledgments.

In closing, the writers desire to acknowledge their indebtedness:

To the Board of Water Commissioners of the City of Detroit, for the opportunities afforded for making these observations;

To E. A. Fuertes, M. Am. Soc. C. E., Director and Dean of the College of Civil Engineering of Cornell University, for the use of the apparatus utilized in the pipe ratings of the Pitot tubes;

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And last, but far from least, to their ever-cheerful assistants in the Engineering Department of the Detroit Water-Works, who, willingly and without expectation of remuneration or reward, other than the approval and appreciation of their chiefs, stood by their gauges, in lonely streets or badly ventilated gate-wells, through long nights of wind and rain, and performed their tasks with an accuracy and fidelity which the writers believe has seldom been equalled.

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DISCUSSION.

E. C. MURPHY, Assoc. M. Am. Soc. C. E. (by letter).—The results obtained by the two methods of rating the Pitot tubes are very interesting. The wide range of values of the coefficient, $M = \frac{v}{\sqrt{2gh}}$, of any tube when rated in the tank, is surprising. For Tube No. 5 this range is from 10.6% above to 17.1% below the mean of all the ratings used. For the ratings in moving water the variation from the mean is not as large, as a rule, as in the tank, but it is still quite large, being from 6.2% below to 4.7% above the mean, for Tube *E* in the 2-in. brass pipe, and from 12.4% below to 14.6% above the mean, for the relative rating in the 30-in. pipe. It will be instructive to examine these ratings from this standpoint of variation of individual ratings from the mean of all reliable ratings for each tube, and compare these variations with those of other Pitot tubes, and with current meters. Mr. Murphy.

This comparison is not intended in any degree as a criticism of the very valuable and exhaustive investigation described in the paper, or of the investigations of others. There is no reason for thinking that all the results presented were not made with care, and represent correctly what took place during the observations. It is known, also, that the results obtained by experimenters are not strictly comparable, as the conditions (range and constancy of velocity, and duration of an experiment) are not the same. Too much importance, however, cannot be attached to the correct rating of velocity measuring instruments. Errors of observation are in a measure compensating, while errors in a rating table are cumulative, and should be eliminated or reduced to a minimum. It is believed that a study of these variations will throw light on the best method of rating these instruments.

The last column of Table No. 42 gives the greatest and least variation from the mean of all the values of *M* used for each tube for the ratings in still water. The extreme variation from the mean is least for Tube No. 3, and is from — 6.7% to + 10.2%, or a range of about 17 per cent. The largest value of *M*, 0.975, is obtained from a single revolution of the whirler, of about 21 seconds' duration, the smallest value, 0.821, from a single revolution of 15 seconds' duration. The highest velocities observed were 6 to 7 ft. The duration of an observation for them was 6 to 7 seconds, which is only 97% of the smallest watch reading. The extreme range is greatest for Tube No. 5, Gauge No. 9, and is 27.7% of the mean. This appears to be partly due to the locus being a curved instead of a straight line. If the water in the tank remained still, and the motion of the whirler was uniform for

Mr. Murphy. each rating, it is difficult to see why there should be such large variations from the means.

TABLE No. 42.—RATINGS OF THREE PITOT TUBES IN A TANK, BY WILLIAMS, HUBBELL AND FENKELL.

Tube o.	Gauge No.	Number of ratings used in mean.	Mean value of m for all ratings.	Greatest Value of m.		Least Value of m.		Percentage of greatest variation from mean.
				Number of ratings in mean.	Value of mean.	Number of ratings in mean.	Value of mean.	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
3.....	9	85	0.9142	1	0.975	1	0.821	— 6.7* to + 10.9†
5.....	5	133	0.8351	4	0.916	1	0.677	— 12.9 " + 7.3
5.....	9	112	0.8506	3	0.942	8	0.705	— 10.6 " + 17.1
6.....	9	110	0.9658	8	1.091	2	0.895	— 13.0 " + 7.4

* — indicates greater than mean.
† + indicates less than mean.

TABLE No. 43.—RATINGS OF PITOT TUBES IN MOVING WATER IN 2-IN. BRASS PIPE. HYDRAULIC LABORATORY, CORNELL UNIVERSITY. BY WILLIAMS, HUBBELL AND FENKELL.

	Point and Pressure.				Point and Circum. Piezom.			
Tube No.	Number of ratings.	Mean of velocity ratios.	Range of values.	Percentage of variation from mean.	Number of ratings.	Mean of velocity ratios.	Range of values.	Percentage of variation from mean.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
3...	13	0.729	0.740 to 0.718	— 1.5 to + 1.5	13	0.779	0.789 to 0.764	— 1.3 to + 1.9
6...	13	0.683	0.695 " 0.659	— 1.8 " + 3.5	13	0.784	0.799 " 0.746	— 1.9 " + 4.9
C...	12	0.693	0.707 " 0.676	— 2.0 " + 2.5	12	0.821	0.854 " 0.807	— 4.0 " + 1.7
5...	29	0.694	0.727 " 0.647	— 4.7 " + 6.8	....	.....	.....	.....
D...	25	0.684	0.711 " 0.608	— 4.0 " + 11.9	25	0.874	0.943 to 0.853	— 7.9 to + 2.4
E...	....	.....	.....	.....	11	0.824	0.875 " 0.785	— 6.2 " + 4.7
E...	....	.....	.....	.....	12	0.834	0.853 " 0.813	— 2.3 " + 2.5

Columns 5 and 9 of Table No. 43 give the extreme variations from the mean velocity ratio in percentages, for the ratings in the Cornell University 2-in. brass pipe. In these ratings the tube was held at the center of the pipe, at a point about 68 ft. from the inlet, and the mean velocity in the pipe was obtained by weighing the discharge. The duration of an experiment was ordinarily from about 8 to 25 minutes. The extreme range is least for Tube No. 3, and is 3 per cent. The largest value of the velocity ratio, 0.740, is found from an

observation of 10 minutes' duration; the smallest value, 0.718, from an observation of 17 minutes' duration. The extreme range for the ratings in this pipe is greatest for Tube *D*, and is 15% for periods of 14 and 15 minutes' duration. Mr. Murphy.

TABLE No. 44.—RELATIVE RATINGS OF PITOT TUBES IN MOVING WATER AT DETROIT, MICH., BY WILLIAMS, HUBBELL AND FENKELL.

Size of pipes. (1)	Tubes compared. (2)	Number in mean. (3)	Mean of velocity head ratios. (4)	Range of ratios. (5)	Percentage of variation from mean. (6)
30.....	3 and 4	23	0.941	1.004 to 0.832	— 6.7 to + 11.6
16.....	5 " 3	17	1.37	1.419 " 1.293	— 5.7 " + 3.6
30.....	5 " 3	10	1.40	1.502 " 1.263	— 9.8 " + 7.0
16.....	5 " 3	18	1.38	1.420 " 1.333	— 3.4 " + 3.0
30.....	5 " 4	17	1.34	1.535 " 1.173	— 12.4 " + 14.6

Column 6 of Table No. 44 gives the extreme variation from the mean of the ratios, H_v, expressed in percentages of the mean of the relative ratings in the 16-in. and 30-in. pipes. In these ratings the tubes compared were held at the center of the pipe, and simultaneous readings were obtained. The extreme range is least for Tubes Nos. 3 and 5 in the 16-in. pipe, and is 6.4% of the mean. It is greatest for Tubes Nos. 5 and 4 in the 30-in. pipe, and is 27% of the mean. It is seen that the variation from the mean is least for the ratings in the 2-in. pipe, and greatest for those in the tank.

TABLE No. 45.—RATINGS OF A PITOT TUBE, BY DARCY AND BAZIN.

Method. (1)	Number of ratings. (2)	Mean value of *m*. (3)	Greatest Value of *m*.		Least Value of *m*.		Percentage of greatest variation from mean. (8)
			Number of ratings. (4)	*m*. (5)	Number of ratings. (6)	*m*. (7)	
1*.....	92	1.006	4	1.039	8	0.981	— 3.3 to + 2.5
2†.....	87	0.993	1	1.029	4	0.965	— 3.6 " + 2.8
3‡.....	32	1.034	1	1.053	1	1.015	— 1.9 " + 1.9

* Moving water: Velocity obtained from surface floats.
† Moving water: Mean velocity known; tube held at many points in cross-section of small canal.
‡ Still water: Tube held in front of a boat.

Table No. 45 gives some results for three methods of rating a Pitot tube, by Darcy and Bazin.*

* The tube which they used is illustrated, and their methods of rating described, in "Recherches Hydrauliques," Paris, 1865.

Mr. Murphy. In the ratings by Method 1, the tube was held with its point 0.02 m. below the surface of an open channel, and the velocity obtained from surface floats. In Method 2, the tube was held with its point at numerous points in a cross-section of a rectangular canal 2 m. wide, the mean velocity of which was known. In Method 3, the tube was fastened in front of a boat, which was drawn over a straight course with a uniform velocity. Column 8 gives the greatest variation of m from the mean of all the values for these three methods of rating. The range is least for the rating in still water, being 3.8% of the mean in 32 ratings. It is greatest for the second method, and is 6.4% of the

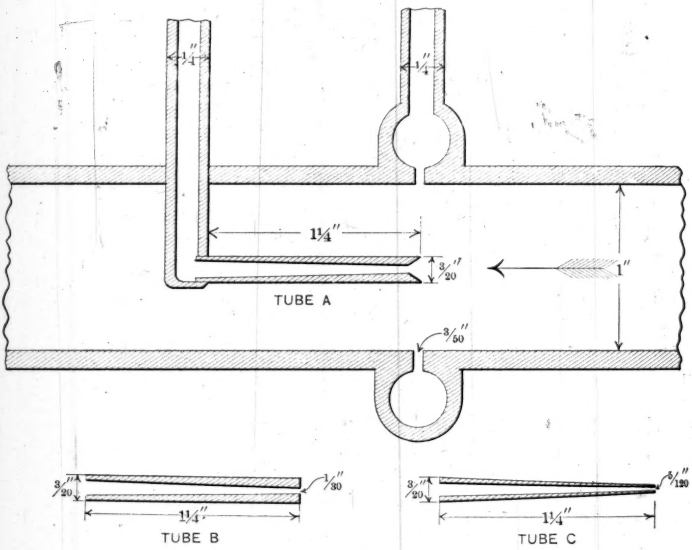

FIG. 91.

mean. The value of m obtained from still-water ratings was rejected, as it contained a constant error due to the point of the tube not being parallel to the water surface. The average of the means obtained by Methods 1 and 2 was used by them.

Table No. 46 gives some results of ratings of three Pitot tubes, made in the Hydraulic Laboratory of Cornell University, by the writer and Mr. C. C. Torrance. These tubes (having three points) are shown in Fig. 91, and are of the point and ring type, made for a pipe 1 in. in diameter. The point screws into the tube, and the tube has a thread on it, so that the point can be set at any place on the

diameter. In these ratings, it was held at the center of the pipe and about 15 ins. from the inlet. The head due to velocity, H_v, was measured with a mercury differential gauge. Each value of m is the mean of two ratings of 50 seconds' duration each. The value of m is seen to increase somewhat as the velocity increases. The greatest variation from the mean is 1.5 per cent. As m increases with the velocity, the variation from the true mean for a given velocity is much less than this amount. Mr. Murphy.

TABLE No. 46.—RATINGS OF THREE PITOT TUBES IN MOVING WATER, 1-IN. PIPE, HYDRAULIC LABORATORY, CORNELL UNIVERSITY, BY E. C. MURPHY AND C. C. TORRANCE.

Tube. (1)	Velocity. (2)	Number of ratings. (3)	Values of m. (4)	Percentage of variation from mean. (5)
A	16.209	2	0.910	— 1.3
A	12.872	2	0.899	— 0.1
A	9.250	2	0.884	+ 1.5
		Mean	0.898	
B	16.081	2	0.896	— 0.5
B	12.844	2	0.902	— 1.1
B	9.211	2	0.878	+ 1.5
		Mean	0.892	
C	15.300	2	0.918	— 0.8
C	13.057	2	0.913	— 0.2
C	9.439	2	0.902	+ 1.0
		Mean	0.911	

It appears that the variation of individual values of m from the mean decreases as the velocity increases, and as the diameter of the pipe decreases.

Table No. 47 gives the results of ratings of "Small Price" current meter No. 351, made at Chevy Chase,* Md., by Mr. E. G. Paul, on December 30th, 1900. These ratings are selected from many because the computations are in shape for ready use. The meter is fastened to a car, which is moved with a uniform speed over a straight course of 100 ft. The car is then moved back over the same course, with the same speed. These two passages over the course, one in each direction, constitute a single rating. Fifteen such ratings were obtained on this date for velocities from 0.35 ft. to 10.8 ft. per second. The equation of the "most probable" linear relation connecting velocity and revolutions is computed and given in this table.

Column 6 gives the residual errors, or variations from this most probable line, and corresponds to the variation from the mean in the

* The rating station at Chevy Chase is illustrated and described in the 19th Annual Report of the Director of the U. S. Geological Survey, Part 4.

Mr. Murphy. other tables. Column 8 gives the percentage of variation in the velocity. It is seen that the greatest variation is less than 1 per cent.

TABLE No. 47.—COMPUTATIONS OF A RATING OF "SMALL PRICE" CURRENT METER No. 351 IN STILL WATER. METER RATED AT CHEVY CHASE, MD., BY MR. E. G. PAUL, ON DECEMBER 30TH, 1900. COMPUTATIONS BY E. C. MURPHY. OBSERVATION EQUATION, $y - a - bx = v$; y BEING VELOCITY, IN FEET PER SECOND, x REVOLUTIONS PER SECOND, v RESIDUAL ERROR, AND a AND b CONSTANTS.

Normal equations $\left\{ \begin{matrix} a\,\Sigma x + b\,\Sigma x^2 = \Sigma\, xy \\ n\,a + b\,\Sigma x = \Sigma\, y \end{matrix} \right\}$, n being the number of observation equations, and Σ meaning summation.

No. of the rating. (1)	Revolutions per second. x (2)	Velocity. y (3)	x^2 (4)	xy (5)	Residual error. v (6)	v^2 (7)	$\frac{v}{y}$ (8)
1.....	4.648	10.811	21.604	50.245	+0.059	0.003481	+0.5%
3.....	2.688	6.250	7.225	16.800	−0.014	0.000196	−0.2%
4.....	1.911	4.440	3.652	8.493	−0.022	0.000484	−0.5%
5.....	1.207	2.857	1.457	3.448	+0.018	0.000324	+0.6%
6.....	0.933	2.222	0.870	2.073	+0.017	0.000289	+0.8%
7.....	0.794	1.887	0.630	1.498	+0.003	0.000009	+0.2%
8.....	0.774	1.895	0.599	1.420	−0.003	0.000009	−0.2%
9.....	0.614	1.471	0.377	0.903	+0.004	0.000016	+0.3%
10.....	0.488	1.170	0.238	0.571	−0.006	0.000036	−0.5%
11.....	0.385	0.939	0.148	0.362	+0.001	0.000001	+0.1%
12.....	0.363	0.885	0.132	0.321	−0.002	0.000004	−0.3%
13.....	0.263	0.649	0.069	0.171	−0.006	0.000036	−0.9%
14.....	0.201	0.508	0.040	0.102	−0.004	0.000016	−0.8%
15.....	0.177	0.455	0.031	0.081	−0.001	0.000001	−0.2%
16.....	0.126	0.341	0.016	0.043	−0.003	0.000009	−0.9%
	15.572	36.724	37.088	86.531		0.004911	

$15.572\,a + 37.088\,b = 86.531$ (1)
$15\,a + 15.572\,b = 36.724$ (2)
Solving Equations (1) and (2) we have
$b = 2.313$ and
$a = 0.047$

Mean error of a single observation $= \sqrt{\frac{\Sigma\, v^2}{n-1}} = 0.019$

Probable error of result $= 0.6745\ \sqrt{\frac{\Sigma\, v^2}{n\,(n-1)}} = 0.003$

Table No. 48 gives the results of ratings of this current meter, and of "Small Price" meter No. 363, in moving water, in the Cornell University experimental canal. The mean velocity in the canal was found from readings of the head on the standard weir. The velocity was obtained simultaneously with the two meters by the ordinary point method. From 6 to 7 observations, each of 50 seconds' duration, were obtained in 8 verticals (48 to 56 in all) in each discharge measurement.

TABLE No. 48.—CURRENT METER AND STANDARD WEIR DISCHARGE COMPARISONS, SERIES E.

Mr. Murphy.

Number of rating. (1)	Date. (2)	Depth, in feet. (3)	Meter velocity. (4)	Weir discharge, in cubic feet (Qw). (5)	Meter discharge, in cubic feet (Q). (6)	$\frac{Qw - Q}{Qw}$ (7)	Name of meter. (8)	Percentage of difference in meter discharge. (9)
	1901.							
21.....	April 6th	4.808	2.985	230.36	230.66	—0.13%	"Small Price" No. 351	
22.....	" 6th	4.808	3.006	230.36	232.30	—0.84%	" " 363	0.71
23.....	" 10th	8.744	1.670	230.75	234.68	—1.70%	" " 351	
24.....	" 10th	8.744	1.657	230.75	232.85	—0.99%	" " 363	0.71
25.....	" 10th	7.874	1.846	229.28	233.61	—1.88%	" " 351	
26.....	" 10th	7.874	1.862	229.28	235.64	—2.77%	" " 363	0.88
27.....	" 10th	6.889	2.108	229.91	233.25	—1.45%	" " 351	
28.....	" 10th	6.889	2.073	229.91	229.38	+0.23%	" " 363	1.68
29.....	" 11th	6.129	2.340	227.16	230.25	—1.36%	" " 351	
30.....	" 11th	6.129	2.320	227.16	228.28	—0.50%	" " 363	0.86
31.....	" 11th	5.483	2.596	225.01	228.59	—1.60%	" " 351	
32.....	" 11th	5.483	2.604	225.01	229.30	—1.91%	" " 363	0.31
33.....	" 11th	5.185	2.725	224.06	226.98	—1.30%	" " 351	
34.....	" 11th	5.185	2.758	224.06	229.73	—2.53%	" " 363	1.23
35.....	" 11th	5.008	2.816	222.60	226.37	—1.69%	" " 351	
36.....	" 11th	5.008	2.884	222.60	227.82	—2.34%	" " 363	0.65
37.....	" 11th	9.256	1.522	223.40	226.38	—1.33%	" " 351	
38.....	" 11th	9.256	1.588	223.40	228.71	—2.38%	" " 363	1.05

The weir discharges in Column 5 are computed from Bazin's standard weir formula, which is not strictly applicable to this weir. They are believed to be in error by not more than 2 per cent. It is seen that the greatest variation of any meter discharge from the corresponding weir discharge is 2.8 per cent. The extreme range for these 18 ratings is only 1.75% for "Small Price" meter No. 351, and 3.00% for "Small Price" meter No. 363.

Column 9 shows the differences between the simultaneous meter discharges. The greatest difference is seen to be less than 1.7 per cent.

Mr. Mills

HIRAM F. MILLS, Esq. (by letter).—The writer, to his great regret, owing to trouble with his eyes, has been unable to read this paper at length, but, upon examining the summary of results, thinks it may be of interest to members of the Society to know that in his studies (made in the year 1875) of the flow of water in a smooth, straight, cast-iron pipe, 1 ft. in diameter, and coated with coal-tar pitch, he found the ratio between the mean and the maximum velocity of the water to differ a little from 0.84, and to vary somewhat with the velocity, as shown by Table No. 49.

The observations upon velocities in the section of 1-ft. pipe were made at a point 327 ft. down stream from the entrance, which was square-edged.

Mr. Mills.

TABLE No. 49.

Mean velocity of water, in feet per second.	Maximum velocity of water, in feet per second.	Ratio.
1.0001	1.206	0.8293
2.090	2.500	0.8360
3.233	3.811	0.8483
4.244	4.990	0.8505
5.164	6.053	0.8531
5.904	6.894	0.8564
6.898	8.091	0.8525
8.087	9.482	0.8529
9.215	10.790	0.8540
10.121	11.857	0.8536
11.088	12.914	0.8589
11.494	13.428	0.8560

The losses of head were read by piezometer, in the lock above the entrance, and in the pipe at points 25, 125, 225 and 325 ft., and in the receiving, basin 356 ft., below the entrance.

With larger pipes, with varying smoothness of surface, the ratio was found to vary from 0.84 in the following manner: In thirteen series of experiments with a mean velocity averaging 2.22 ft. per second, in a wrought-iron penstock, 4.94 ft. in diameter, having plates in and out, with rivets projecting and much of the surface covered with tubercles, the mean ratio was 0.809. In another penstock, 6.5 ft. in diameter, with rivets and plates arranged as before, but with the surface coated with coal-tar pitch and quite smooth, in a series of twelve experiments, with a mean velocity averaging 3.615 ft. per second, the ratio averaged 0.846. Another penstock, 6.5 ft. in diameter, with a surface somewhat rougher than the last, in a series of fourteen experiments, with an average mean velocity of 2.029 ft. per second, gave an average ratio of 0.861.

A still larger penstock, having a diameter of 8.75 ft., with an average mean velocity of 2.731 ft. per second, in a series of nine experiments, gave a ratio of 0.857; and in another series of six experiments, with a velocity of 2.357 ft. per second, gave a ratio of 0.857.

The writer has no reason to question the invention by the authors, in 1897, of "a form of Pitot tube which may be inserted in a water main without the aid of special devices," etc., by which gaugings of great accuracy may be obtained, but it appears to be one of the cases of reinvention, because the measurements of the quantity of water drawn by many of the water wheels of the mills at Lawrence have been made in the feeding penstocks by instruments, which could be thus characterized, devised by the writer in 1877, and used by his assistants, under the direction of John R. Freeman, M. Am. Soc. C. E., during the years 1878 to 1885, and, since the last date, under the direction of Richard A. Hale, M. Am. Soc. C. E., to the present time. These instru-

ments differ in some important particulars from those described in the paper, and indicate the velocity at sixteen or eighteen points in the section at the same moment. Mr. Mills.

Corroborative, in a general way, of the authors' results, but tending to modify at an essential point their curve of excess of loss of head due to bends, the writer gives the results of a long series of experiments, made in 1882, with an experimental line of cast-iron pipe, 1 ft. in diameter and 356 ft. long, when straight and when supplied near its upper end with a 90° bend having a radius of 2 ft.

The bend, with different velocities of flow, caused an increased loss of head, expressed in additional length of straight pipe, as given in Table No. 50.

TABLE No. 50.

Mean velocity in pipe, in feet per second.	Length of curve, in feet.	Additional length of straight pipe that would cause the excess of loss of head in the curve over that of straight pipe, in feet.	Additional length of straight pipe, in terms of the length of curve, to cause the excess of loss of head due to the curve.
1.039	3.142	13.4	4.26 times the length of curve.
2.038	3.142	12.5	3.98 " " "
4.692	3.142	12.6	4.01 " " "
8.811	3.142	11.0	3.50 " " "

The percentage of loss of head in the curved section over the loss of head in 80 diameters of tangent would be 15.5, instead of 10.8 as found by the authors, and this would modify the curve in their diagram of results by raising it a little at its lowest point.

The general result of both these investigations indicates that each 90° curve in a line of pipe adds a loss of head in the line equivalent to the loss in an additional length of straight pipe of from two to five times the length of the curve, nearly all cases being included within four times the length of the curve.

The authors' investigation gives the important additional knowledge that the least loss of head in changing direction in a pipe line obtains when using a curve having a radius of two or three times the diameter of the pipe.

Irving P. Church, Assoc. Am. Soc. C. E. (by letter).—While the authors of this extensive and valuable paper, in their interpretation of the piezometer readings for determining the loss of head between two points of a pipe, have evidently taken into account various causes of error, and have made allowance for some of them as far as practicable, there is one source of error in this connection which they seem to have overlooked (at least there is no reference to it in the paper); and this is the effect of neglecting the possible inequality of sectional areas at the two points of attachment of the piezometer tubes. Mr. Church.

Mr. Church. It is evidently taken for granted throughout the paper that the vertical distance between the summits of the two piezometer columns at the ends of the portion of pipe considered (let this vertical distance be called the "piezometer fall"), is equal to the loss of head, or friction head, H_f, between the points of pipe where the piezometer tubes are attached. Now, as such a relation is true only in case the end sections of this portion of pipe have exactly equal areas (so that the corresponding mean velocities are equal) it may be of interest to note the extent of the error involved by this supposition, under certain circumstances of slightly unequal end areas; and it will be found that for short lengths of pipe between piezometers the error is greater than might at first be supposed. (In the paper there are some comparatively short lengths used in connection with curves: from 24 to 200 ft. for the 30-in. pipe, 100 ft. for the 12-in., and 220 ft. for the 16-in.)

The writer has, therefore, been interested in computing the amount of this error in certain cases suggested by some of the data of the Detroit experiments. In the description of these experiments, however, no specific information is given as to the magnitude of the sectional areas at the points of attachment of piezometer tubes (except that the diameter of each of the special piezometer castings used on the South Street tangent and at the further extremity of an adjacent curve was exactly 12 ins.). We are therefore left to gather the possibilities of inequality of sectional area at these points from statements made of maximum and minimum diameter.

In the 30-in. pipe the maximum and minimum diameters for the different portions considered rarely differ by more than $\frac{1}{2}$ in., though in two cases the difference is slightly greater than 1 in. In the portions of 16-in. pipe this difference of extreme diameters varies from $\frac{5}{8}$ to $\frac{13}{16}$ in.; while in the description of the 12-in. pipe, where some of the lengths tested are as short as 100 ft., no information is given as to variations in diameter along any one section, although the statement is made (in Fig. 79) that the diameter of the piezometer-castings referred to above (South Street tangent) was exactly 12 ins.

The quantity termed "loss of head" (in steady flow of water in a rigid, stationary pipe) owes its origin and meaning, of course, to the well-known Theorem of Bernoulli, viz., the loss of head occurring between any two points in the line of flow is equal to the amount by which the "total head" at the up-stream point exceeds that at the down-stream point, understanding by "total head" the sum (at any point of the flow) of the velocity head, pressure head, and potential head (or height of point above any convenient horizontal datum plane). Referring to Fig. 92, 1 being the up-stream point, and 2 the down-stream point, let v_1 denote the mean velocity, p_1 the internal pressure,

Mr. Church.

z_1 the height above datum, A_1 the sectional area, and d_1 the diameter for point 1; a similar notation with subscript 2 applying to point 2. Also denote by b the height of the water barometer, and by γ the weight of a unit of volume of water; and let h indicate the "piezometer fall" (see Fig. 92), H_f, the loss of head between 1 and 2, and g the acceleration of gravity.

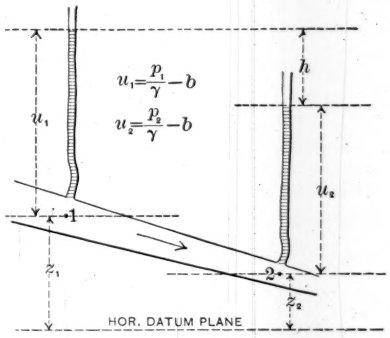

FIG. 92.

We have, then, by Bernoulli's Theorem,

$$H_f=\left[\frac{v_1^2}{2g}+\frac{p_1}{\gamma}+z_1\right]-\left[\frac{v_2^2}{2g}+\frac{p_2}{\gamma}+z_2\right] \quad \ldots\ldots\ldots\ldots (1)$$

Remembering, that, from the "equation of continuity," $A_1 v_1 = A_2 v_2$, and noting the relations between the various heights marked in Fig. 92, where ordinary piezometers are shown attached at the points 1 and 2, we finally obtain

$$H_f = h - \frac{v_2^2}{2g}\left[1-\left(\frac{A_2}{A_1}\right)^2\right] \quad \ldots\ldots\ldots\ldots\ldots (2)$$

If the term $\frac{v_2^2}{2g}\left[1-\left(\frac{A_2}{A_1}\right)^2\right]$ be designated as the "increment of velocity head," we may read Equation (2) as follows:

The loss of head between any two points in the line of flow is equal to the piezometer fall diminished by the increment of velocity head. (In Fig. 92, A_2 has been supposed smaller than A_1; but if the contrary is the case the increment of velocity head becomes negative, and the loss of head will be greater than the piezometer fall instead of less.) We note that the bracket in the increment of velocity head is independent of the length of pipe. Hence, if the length were small and the cause of loss of head were ordinary "skin friction," the loss of head might differ considerably from the "piezometer fall," even when the areas A_1 and A_2 were nearly equal. (As a useful illustration, in this connection, it may be well to recall the fact that in the converging portion of a Venturi meter the loss of head occurring between the entrance and the "throat" is only a small fraction of the piezometer fall.)

To take a rather extreme case, let us first compute the amount of error that would be occasioned by considering the friction head to be equal to the piezometer fall in a 100-ft. length of pipe, nominally 12 ins. in diameter, in case the diameter at Point 1 is $d_1 = 12.5$ ins.,

Mr. Church. while that at 2 is $d_2 = 11.5$ ins., the mean velocity at 2 being $v_2 = 4$ ft. per second.

Here we have $\left(\frac{A_2}{A_1}\right)^2 = \left(\frac{d_2}{d_1}\right)^4 = 0.7165$;

so that the "increment of velocity head" is $\frac{16}{64.4}[1 - 0.7165] = 0.0704$ ft. It will be sufficient for the present purpose if we consider the friction head in the whole pipe to be equal to that in a pipe of equal length and of 12 ins. diameter throughout, in which the same rate of flow is taking place; that is, in which the mean velocity is $4 \times \left(\frac{11.5}{12.0}\right)^2 = 3.673$ ft. per second; and the friction head for this ideal pipe may be taken, by interpolation in the first line of Table No. 36, as $H_f = 0.320$ ft. (This first line refers to the long South Street tangent, on which the sections at piezometer points were exactly 12 ins. in diameter, so that the "observed H_f" may be taken as the friction head.) Now, 0.0704 ft. is 22% of 0.320 ft.; hence, an error of 22% would be made in this case if the whole piezometer fall (which is $0.0704 + 0.32 = 0.3904$ ft.) were looked upon as the loss of head; *i. e.*, the piezometer fall occurring between Points 1 and 2 is in reality 22% in excess of the real friction head.

Calling this 100-ft. length of pipe Division B, let us suppose the next consecutive 100-ft. length (Division C), the up-stream piezometer of which is the down-stream piezometer for Division B, to have at its down-stream end a diameter of 12.5 ins., and to have the same mean diameter (12 ins.) as Division B and the same degree of "roughness" of interior surface. Then, during the same rate of flow as before, the friction head in C would be equal to that in B (viz., = 0.320 ft.), and the increment of velocity head for C would be equal to that for B (= 0.0704 ft.), but of contrary sign, so that the piezometer fall for C would be $(0.320 - 0.0704) = 0.2495$ ft.; as against 0.3904 ft. for B. The latter exceeds the former by 0.1408 ft., which is 56.4% of the former. If, therefore, in this case the principle were adopted that the piezometer fall can be considered as sensibly equal to the friction head, the conclusion would be reached that the friction head in Division B is 56% greater than that in C; when, as a matter of fact, the two friction heads are equal.

In the foregoing example the data as to diameters at piezometer points are rather extreme, as these diameters differ from the mean each way, by $\frac{1}{2}$-in., which is considerable for a 12-in. pipe. If, however, these diameters differ only $\frac{1}{4}$ in. from the mean (one above, the other below), the results of similar computations for two consecutive 100-ft. lengths of 12-in. pipe, B and C, with same rate of flow, etc., as before, are found to show errors of 12 and 25%, respectively, instead of the 22 and 56% of the first example; and again, with $\frac{1}{8}$ in. instead of $\frac{1}{4}$ in., the errors are 5 and 12%, respectively.

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It is to be noted that as the increment of velocity head is exactly, and the friction head approximately, proportional to the square of the mean velocity, the percentage error instanced in each of the foregoing cases would not be materially changed if other velocities were chosen.

As regards the 30-in. pipe, let us first compute a fair estimate for the loss of head due to "skin friction" in a straight length of 200 ft., when the mean velocity is 3.868 ft. per second. (This value will serve, nearly enough, for both the following examples.) For this the Chezy formula will serve, with $c = 123.4$ (from page 184); and $H_f = 0.3142$ ft. is obtained; while for a length of 25 ft. we have one-eighth as much, or 0.0392 ft., as H_f. Considering then, a 25-ft. length of 30-in. pipe, with $d_1 = 30.125$ ins. and $d_2 = 29.875$ ins., so that these diameters differ only by $\frac{1}{4}$ in. (the section of 30-in. pipe called XVIII, in Table No. 21, is only 23.51 ft. long, and its extreme diameters differ by 0.242 in.), with a flow at the rate of $v_2 = 4$ ft. per second, we find the increment of velocity head to be

$$\frac{v_2^2}{2g}\left[1 - \left(\frac{A_2}{A_1}\right)^2\right] = \frac{16}{64.4}\left[1 - \left(\frac{29.875}{30.125}\right)^4\right] = \frac{16}{64.4}\left[1 - 0.966\right] = 0.0085 \text{ ft.};$$

while the value of H_f may be taken (see above) as 0.0392 ft. As 0.0085 ft. is 21.3% of 0.0392 ft., it follows that the piezometer fall, *i. e.*, 0.0477 ft. (= 0.0392 + 0.0085), instead of being equal to the friction head, exceeds it by some 21%, which is the amount of the error that would be made if the slight inequality of end areas were neglected. If the end diameters differed by only $\frac{1}{8}$ in., the error would still be about 10 per cent.

The last example would seem to show in a very striking manner that the precision attained by the oil-and-water differential gauge in the determination of the piezometer fall should be accompanied by an equal precision in the measurement and consideration of end areas, if accurate values of friction head with short lengths of pipe are to be obtained.

As regards the 200-ft. lengths of 30-in. pipe (several of which were used, as described in Table No. 21), in nearly all of these the maximum and minimum diameters differ by about $\frac{1}{2}$ in. (including the Brady tangent), while in two of them (III and XIII) the extreme diameters differ by more than 1 in. By computations similar to the foregoing it can be shown that with ordinary "skin friction" along a 200-ft. length (Division B) of 30-in. pipe, in which the end diameters happen to differ by 1 in., an error of about 10% would be involved in considering the piezometer fall to be equal to the loss of head; and an error of some 20% in comparing the losses of head in Divisions B and C, if in C the end diameters also differ by 1 in., but are oppositely situated with respect to the direction of flow. For $\frac{1}{2}$-in. difference of end diameters these percentages would be reduced about one-half.

Mr. Church. Similar illustrations could also be given in the case of the 220-ft. lengths of 16-in. pipe.

Since the foregoing was written, the writer has had the opportunity of conferring with one of the authors, who has supplied additional data, together with computations based upon them and arranged in tabular form, bearing on the matter of end areas (that is, areas at the points of insertion of piezometers).

From these supplementary statements and computations, the writer is permitted to quote as follows :

"The diameters of the pipe at the exact points of insertion of the piezometer connections were not measured, except in the case of the four special castings used on the 12-in. line, but as eight diameters were measured on every pipe and special, and as the field notes enable the pipes containing the connections to be identified, it is possible to obtain what may be considered a fair criterion of the variation of diameters of end sections by taking the mean of the eight measured diameters of the pipe containing the piezometer connection as the diameter at the connection.

"Upon this assumption Tables Nos. 51, 52 and 53 have been computed. Table No. 51 for the 30-in. line is based upon a velocity of 4 ft. per second in a pipe exactly 30 ins. in diameter. Column 2 gives the mean diameters of the tapped pipes computed as above indicated. Column 3 gives the velocity through a pipe of this diameter corresponding to a velocity of 4 ft. per second in a 30-in. pipe. Column 4 gives the corresponding velocity head, H_v. Column 5 gives the increase or decrease of velocity head between the two ends of the experimental section, which is the correction to be subtracted from or added to the piezometric fall, H_f, for the corresponding velocity. Column 6 gives the piezometric fall that would be observed in the experimental sections computed from the equation, $H_f = 0.133\ v^2$, for the North Rivard tangent by Column 8 of Table No. 28, which section was selected because in it there is no correction for H_v, the diameters being the same at the two ends ; and from the ratios of the several observed piezometric falls to that in this section as determined from Column 4 of the same table, the value of H_f in the above equation being reduced to that of the true length of the section by multiplying by $\left(\frac{184.1}{1000.0}\right)$. Column 7 then gives the percentage correction to be applied to the authors' H_f to account for the change in H_v.

"Table No. 52 for the 16-in. line is based upon a velocity of 4 ft. per second at the outlet of the Army tangent. By the use of the second and third columns of Table No. 32 and the lengths of the several sections as given in Table No. 29, a similar series of computations to those for the 30-in. line give the values of H_f in Column 6, the exponent of V, in $H_f = m\ V^n$, being in this case 1.857, from Fig. 74.

"Table No. 52, for the 12-in. line, is based upon a velocity of 4 ft. per second in the long South Street tangent, which is exactly 12 ins. in diameter at both extremities, and the values of H_f were determined from the equation of the South Street tangent, $H_f = 0.0323\ v^{1.78}$ (Table No. 36, Columns 5 and 6), the coefficient being reduced for a length of 100 ft., and the ratios thereto given in Column 4 of the same table."

By way of comment on the preceding, the writer is of the opinion that the propriety of assuming the mean diameter at a piezometer

point to be equal to the mean of eight measurements made in the same 12-ft. pipe length, all of which measurements were either at the two extremities or 1 ft. from them, is fairly open to question. If the sides of a pipe length were accurately straight, and the piezometer point occurred at the middle of the pipe length, such an assumption would doubtless be justified, but the writer understands that a piezometer point might occur anywhere along the length of its 12-ft. pipe length and not necessarily in the middle. As to the straightness of the sides of ordinary cast-iron pipe (or the lack of it), other contributors to this discussion can probably bear testimony better than the writer. Mr. Church.

It would seem, however, that the above assumption would not be attended with serious error unless experimental sections were very short, like Sections II, VI, X and XVIII of the 30-in. pipe (respectively, 56.42, 47.88, 29.17 and 23.51 ft. long).

Tables Nos. 51, 52 and 53, computed on the foregoing assumption, are seen to show percentage corrections of less than 1% for all the experimental sections except Nos. II, VI, X and XVIII of the 30-in. pipe, for which the corrections called for (without regard to sign) are, respectively, 1.28, 1.41, 3.01 and 6.34 per cent. As might have been expected, the largest correction occurs in the shortest section, XVIII, where, according to Table No. 51, the end diameters differ by a trifle over $\frac{1}{8}$ in., implying sufficient inequality in the end areas to cause a difference of some 10% between loss of head and piezometer fall if the loss of head had been ordinary "skin friction" (see previous illustration by the writer); but the actual loss of head in XVIII was considerably greater than this, on account of the presence of "specials," and hence this difference between loss of head and piezometer fall (being a fixed quantity for a given rate of flow) is only 6.3% of the latter instead of 10.

As to the authors' conclusions relating to the losses of head due to curvature in pipes, in the first perusal of the statement in Conclusion *K* on page 191, the writer understood the view to be advanced that the flatter the curve, the greater would be the resistance; which certainly seemed very paradoxical, as it would imply that a maximum resistance would be found in the flattest of all curves, viz., in a straight pipe. Closer inspection of the context showed, however, that the resistances compared were those occurring (for the same rate of flow) in a number of pipes of the same length (in diameters) in each of which only a fraction of the length was curved, so that a (90°) curve of large radius occupied a much larger percentage of the total length than one of small radius; and that no attempt had been made to distinguish between the effect of angular deviation of flow (or curvature proper) and that of the length of the curved part of pipe. It was also noticeable that in eleven out of the sixteen cases presented the

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TABLE No. 51.—30-INCH LINE.

(1) Experimental section of line.	(2) Diameter of end of section, in inches.	(3) Velocity at end of section corresponding to $V = 4$ ft. in a 30-in. pipe, in feet per second.	(4) Head due to velocity, $\frac{V^2}{2g} = H_v$, in feet.	(5) Increment + or decrement − of H_v from inlet to outlet of section, in feet.	(6) Piezometric fall, H_f, for $V = 4$ ft. in 30-in. pipe by Equation $H_f = m V^2$. m by Table No. 28, in feet.	(7) Percentage of correction to H_f for effect of change of H_v.	(8) Name of experimental section.
I.	30.000	4.0000	0.248756				
	29.891	4.0262	0.252405	+0.003649	0.65381	−0.558	Orleans and Scott compound curve.
II.	29.852	4.0308	0.253725	+0.001320	0.10311	−1.280	" tangent.
III.	29.797	4.0547	0.255605	+0.001880	0.55534	−0.339	" and Hale compound curve.
IV.	29.874	4.0398	0.253080	−0.002525	0.41065	+0.638	Hale tangent.
V.	29.883	4.0314	0.252776	−0.000304	0.64555	+0.0465	Hale and Riopelle 60-ft. radius curve.
VI.	29.922	4.0269	0.251360	−0.001416	0.699582	+1.410	Riopelle tangent.
VII.	29.988	4.0166	0.250833	−0.000527	0.54447	+0.0992	Riopelle and Rowena 40-ft. radius curve.
VIII.	29.946	4.0144	0.250536	−0.000297	0.51914	+0.0520	Rowena tangent.
IX.	29.992	4.0022	0.249999	−0.001536	0.53897	+0.286	" and Russell 25-ft. radius curve.
X.	29.938	4.0166	0.250833	+0.001803	0.658823	−3.009	Russell tangent.
XI.	29.946	4.0144	0.250536	−0.000297	0.45296	+0.0596	Russell and Brady small compound curve.
XII.	29.992	4.0022	0.249999	−0.001536	0.46723	+0.378	Brady tangent.
XIII.	29.922	4.0209	0.251360	+0.001360	0.53965	+0.441	Brady and Rivard 15-ft. radius curve.
XIV.	29.969	4.0083	0.250787	−0.000573	0.99626	+0.173	S. Rivard tangent.
XV.	29.969	4.0083	0.249787	−0.000000	0.39339	0.000	N. Rivard tangent.
XVI.	29.915	4.0228	0.251594	+0.001807	0.43965	−0.412	Rivard and Alexandrine 10-ft. radius curve.
XVII.	29.977	4.0063	0.249574	−0.002674	1.2493	+0.166	E. Alexandrine tangent.
XVIII.	29.844	4.0412	0.254060	+0.004480	0.07069	−6.336	Hastings St. specials.
XIX.	29.852	4.0398	0.253725	−0.000274	0.78421	+0.0344	W. Alexandrine tangent.
XX.	29.946	4.0144	0.253173	−0.001873	0.43598	+0.727	Alexandrine and Alley 6-ft. radius curve.
XXI.	29.844	4.0410	0.253533	+0.000447	0.28352	−1.309	S. Alley tangent.
XXII.	30.008	3.9979	0.248490	−0.005510	1.2935	+0.337	N. Alley tangent.

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TABLE No. 52.—16-INCH LINE.

Experimental section of line.	Diameter of end of section, in inches.	Velocity at end of section corresponding to $V = 4$ ft. at outlet of Army tangent, in feet per second.	Head due to velocity, $\frac{V_2}{2g} = H_v$, in feet.	Increment + or decrement − of H_v from inlet to outlet of section, in feet.	Piezometric fall, H_f, for $V = 4$ ft. at outlet of Army tangent, in feet.	Percentage of correction to H_f for effect of change of H_v.	Name of experimental section.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
I	16.030	4.0240	0.25175	+0.00139	1.4108	−0.09853	Porter tangent.
II	16.008	4.0351	0.25314	−0.00295	0.68706	+0.4294	5-ft. radius curve.
III	16.055	4.0117	0.25019	+0.00050	0.69731	−0.0717	Compound curve.
IV	16.047	4.0155	0.25069	−0.00193	0.61104	+0.3158	Army tangent.
	16.078	4.0000	0.24876				

TABLE No. 53.—12-INCH LINE.

Experimental section of line.	Diameter of end of section, in inches.	Velocity at end of section corresponding to $V = 4$ ft. in a 12-in. pipe, in feet per second.	Head due to velocity, $\frac{V^2}{2g} = H_v$, in feet.	Increment + or decrement − of H_v from inlet to outlet of section, in feet.	Piezometric fall, H_f, for $V = 4$ ft. in a 12-in. pipe, in feet.	Percentage of correction to H_f for effect of change of H_v.	Name of experimental section.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
I	12.046	3.9695	0.24498	+0.00378	0.43912	−0.8608	Artillery 4-ft. radius curve.
	12.000	4.0000	0.24876				
IV	12.039	3.9741	0.24555	+0.00321	0.45600	−0.7040	Exposition and South 13-in. radius curve.
	12.030	4.0000	0.24876				
V	12.023	3.9847	0.24686	+0.00381	0.47246	−0.8064	Exposition and River 3-ft. radius curve.
	11.977	4.0154	0.25067				
VI	12.061	3.9596	0.24376	+0.00310	0.43322	−0.7156	River and Solvay 3-ft. radius curve.
	12.023	3.9847	0.24686				
VII	12.031	3.9794	0.24620	0.00000	0.41383	0.0000	Solvay 2-ft. radius curve.
	12.031	3.9794	0.24620				

Mr. Church. curved portion constituted only a small percentage (2 to 11%) of the total length of pipe, the remainder of the pipe being straight.

Now, the resistance due to a curved pipe (if more than 5 or 6 diameters long) can hardly be considered as having an abrupt, local character, like that due to an elbow or valve gate. Hence the length of the curve, as with a straight pipe, would seem to be of prime importance, aside from the element of angular deviation of the flow. The writer, therefore, would offer the following mode of interpreting the numerical data in the table on Fig. 90.

To obtain a fairly probable value of the loss of head occurring in the curve proper, a necessary preliminary is to make an estimate of the friction head in the adjoining straight portions, which, with the curved part, make up the length between piezometer points. Such an estimate is necessarily based on the results of experiments with other (straight) sections of the same kind and size of pipe; and is such a large fraction (70 to 90%) of the whole loss of head in Cases Nos. 7 to 16 of Fig. 90, that a small error in this estimate would give rise to a large (relative) error in the quantity obtained by deducting the estimate from the whole (measured) loss of head (aside from possible error due to neglect of inequality of end areas). On account of this uncertainty it seems hardly worth while to consider Nos. 7 to 16 so far as intercomparison of pure curve effects is concerned.

The writer, therefore, has selected the cases of Nos. 1, 2, 5 and 6a (each of which is a 200-ft. length of 30-in. pipe and contains a single 90° curve; of radii 60, 40, 25 and 15 ft., respectively) as most favorable for the purpose, and as likely to give the most trustworthy results. The quantities computed by him are placed in Table No. 54, all the values given in which are explained below.

TABLE No. 54.

No.	R	(a)	(b)	(c)	(d)	(e)	(f)
		ft.	ft.				
1.....	60 ft.	94.24	105.76	190.2	52.88	137.32	1.457
2.....	40 "	62.84	137.16	161.5	68.58	92.92	1.478
5.....	25 "	39.28	160.72	152.7	80.36	72.34	1.841
6a.....	15 "	23.56	176.44	133.0	88.22	44.78	1.901
Straight....	.....	0.00	200.00	100.0	100.00	0.00	.....

Five 200-ft. lengths of 30-in. pipe are represented, of which the last is straight throughout its whole length, and 6a is No. 6 "corrected for specials" (see Fig. 90).

Column R gives the radius (in feet) of the 90° curve occurring in the section.

Column (a) gives the length of curved portion (in feet).

Column (b) gives the aggregate length of straight portions in each section (in feet), [by deducting (a) from 200 ft.]. Mr. Church.

Column (c) gives the (relative) loss of head in the whole length.

Column (d) gives the (relative) loss of head in straight parts [computed by taking the same fraction of 100 that the length (b) is of 200 ft.].

Column (e) is obtained by deducting (d) from (c), and hence gives the (relative) loss of head due to the curved portion alone.

Column (f) results from dividing (e) by the length (a) of the curved portion, and hence the numbers in this column are proportional to the loss of head per unit of length along the curve itself.

It is to be noted that these numbers in Column (f) increase (though not rapidly) with increasing sharpness of curvature; which is what would naturally be expected, since they all refer to the same absolute length (1 ft.) of curved pipe (the whole curve in each case being more than five diameters long).

The reason, therefore, why the total resistance for the 200-ft. lengths should increase (from 133 to 190.2) as we read upward in Column (c), meeting with curves of less and less sharpness (but of greater and greater length), would seem to be best expressed by saying that the effect of the increasing length of the curved portion overbalances the contrary, but much less rapid, influence of the decreasing sharpness.

Users of differential gauges will doubtless be interested in the accurate setting forth of the theory of these instruments, as given on pages 78 and 89.

CHARLES H. TUTTON, M. Am. Soc. C. E. (by letter).—On pages 63 and 64 of the paper, the authors present the theory that the curve of velocity in a pipe is an ellipse, rather than, as generally taught in the schools, a curve from the family of parabolas. The writer, having made a number of investigations along this line, believes that the authors are correct in their assumption; and, in the following, which strongly substantiates the same, incidentally presents an entirely new theory and formula for the flow of water in pipes, from a purely theoretic standpoint. As a preliminary step, let it be assumed that uniform motion exists in the pipe, meaning by this that the motion along each fillet is uniform, and that the total resistance to the motion is composed of two entirely different and non-comparable classes of resistance, one being Mr. Tutton.

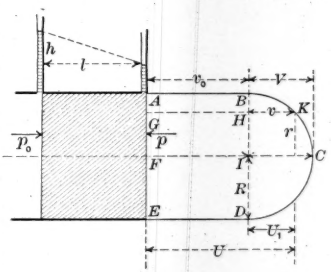

FIG. 93.

Mr. Tutton. the so-called friction along the pipe walls, the other, the internal resistances in the water itself, or the so-called viscosity.

The development of the moving mass of water may be divided into two portions, a cylinder and a "dome" shaped head. The effect of the friction on the pipe walls will be felt only upon the cylinder, and the effect of the viscous resistances will be entirely shown in the shape and size of the "dome," as the authors have named it.

With these ideas in mind, the following general relations may be pointed out.

Referring for the present only to the "dome," and denoting the radius of the pipe by R, the central velocity in the head by v, and the mean velocity in the same by U_1, Boussinesq arrives at the conclusion that the curve of velocity is a cubic parabola. Now, the equation of such a parabola, with origin at C, can be written $y^3 = C\,x$, in which x is parallel to the axis of the pipe. The volume of the generated paraboloid will consequently be $\frac{3}{5}\,\pi\,R^2\,V$, its mean ordinate $\frac{3}{5}\,V = H\,K$, and consequently the distance, $I\,H$, or the radius of the circle of mean velocity, will be

$$R_u = \sqrt[3]{\tfrac{3}{5}}\,R = 0.737\,R.$$

Bazin, in his deductions from experiment, admits that this value is a little too small.

Now, if the supposition of the authors be correct, that the ratio of mean to central velocity be 0.84, that is, that

$$\frac{U}{v_c} = \frac{G\,K}{F\,C} = 0.84,$$

which refers to the total velocity, then, $F\,C$ being 1.0, $F\,I$ must be $0.60\,v_c$, and $I\,C = 0.40\,v_c$.

It may be observed that Boussinesq's supposition as to the effect of viscosity is that it is proportional to $\frac{R}{2}\,U_o\,\frac{R}{r}$ in which r is the radius of any fillet, and U_o is the velocity, $A\,B$, along the pipe wall, with which velocity viscosity has no relation, if the writer's supposition, which will hereafter be presented, be correct.

The authors, without proof, assume that the curve of velocity $B\,K\,C\,D$ is an ellipse. The ellipsoidal volume $C\,B\,D$, consequently, will be $\frac{2}{3}\,\pi\,R^2\,V$, and the mean ordinate $H\,K = \frac{2}{3}\,V$, which, placed in the equation of the ellipse, $\frac{x^2}{v^2} + \frac{y^2}{R^2} = 1$, gives for the radius of mean velocity $x^2 = \frac{5}{9}\,R^2$,

$$\text{or } Ru_1 = 0.74535\,R.$$

The following relations then exist, on the supposition that $\frac{U}{v_c} = 0.84$.

$$F\,I = 0.52\,v_c;\ I\,C = 0.48\,v_c.$$

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It will be seen that this does not vary from the previous result enough to justify the assertion from the results of their experiments alone.

The writer proposes the following theory to justify the assertion that the true curve of velocity is an ellipse. Instead of Boussinesq's assumption that the frictional resistance between filaments per unit of area is proportional to $\varepsilon \frac{d\,v}{r\,d\,r}$, or that of Levy that it is proportional to $\varepsilon \sqrt{r} \frac{v\,d\,v}{d\,r}$, or that of Bazin $\varepsilon \left(\frac{d\,v}{d\,r}\right)^2$, or finally, that of Navier, and upon which the capillary theory and Poiseuille's formula for viscosity is based, that it is simply proportional to $\varepsilon \frac{d\,v}{d\,r}$, let it be assumed that this resistance is proportional to $\varepsilon \frac{v\,d\,v}{d\,r}$, or that what has been called the "rate of distortion" of the liquid varies, not only in direct ratio to the distance of the filament from the center of the tube, but also with the velocity of that filament.

Let the shaded portion in Fig. 93 represent the cylinder of water upon which the pressures act. The light portion, *A B C D E*, will then represent the "velocity solid," or better, the development of the motion per second of and in the actual cylinder. The cylindrical portion, *A B D E*, will represent the development of the total motion along the wall of the pipe, and the dome, *B C D*, will represent the development of the excess of motion of the co-axial cylinders forming the interior of the original cylinder, and which are subjected to the viscous resistances. In that which follows, these fictitious portions will be referred to, but it must be understood that, physically, it is the shaded portion that is considered. The method used is to catch the eye.

If p_o and p be the pressures on the two ends of the cylinder per unit of area, the resultant pressure will be $(p_o - p)\,\pi\,r^2$.

If ε be considered of the same nature as the friction coefficient on the sides of the pipe, in so far that it may be represented as a force, and a factor, $\frac{R^n}{m}$, be introduced to include any possible effect due to the absolute size of the pipe, then the frictional resistance on the surface of the cylinder, the radius of which is r, will be

$$2\,\pi\,r\,l\,\rho\,g\,\varepsilon\,\frac{R^n}{m}\,\frac{v}{2\,g}\,\frac{d\,(v_o + v)}{d\,r}.$$

In uniform motion the sum of these must be 0; or, since $p_o - p = g\,h$, in which ρ is the density of the liquid,

$$\pi\,r^2\,\rho\,g\,h + 2\,\pi\,r\,l\,\rho\,g\,\varepsilon\,\frac{R^n}{m}\,\frac{v}{2\,g}\,\frac{d\,(v_o + v)}{d\,r} = 0.$$

Mr. Tutton. Reducing this expression, and substituting $\frac{h}{l} = s$, the piezometric gradient, or, as usually understood, the slope,

$$\frac{R^n}{m} \cdot \frac{\varepsilon}{2g} \frac{v\,dv}{dr} = -\frac{r\,s}{2}$$

or

$$v\,dv = -\frac{s\,g\,m}{\varepsilon R^n} r\,dr.$$

Integrating this expression between the limits, $v = v$ and $v = V$, corresponding to which are the radii, $r = r$ and $r = 0$, there results

$$V^2 - v^2 = \frac{m\,g\,s}{\varepsilon R^n} r^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

But when $v = 0$, assuming there is no slip along the sides of the pipe as an effect of the viscous resistances, r becomes equal to R.

$$V^2 = \frac{m\,g\,s}{\varepsilon R^n} R^2,$$

and

$$\frac{m\,g\,s}{\varepsilon R^n} = \frac{V^2}{R^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Substitute this in (1), and

$$V^2 - v^2 = \frac{V^2}{R^2} r^2,$$

whence, dividing through by V^2 and transposing

$$\frac{v^2}{V^2} + \frac{r^2}{R^2} = 1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

showing that the curve of velocity is a true ellipse, the semi-axes of which are V and R.

From (1)

$$v = \sqrt{V^2 - \frac{m\,g\,s}{C R^n} r^2},$$

or by the relation in (2)

$$v = \sqrt{\frac{m\,g\,s}{\varepsilon R^n}} \sqrt{R^2 - r^2},$$

and there is obtained for the mean velocity in the "dome,"

$$U_1 = \int \frac{2\pi r v\,dr}{\pi R^2} = \frac{2}{R^2} \sqrt{\frac{m\,g\,s}{\varepsilon R^n}} \int_0^R \sqrt{R^2 - r^2}\, r\,dr,$$

whence

$$U_1 = \frac{2}{3} \sqrt{\frac{m\,g\,s}{\varepsilon R^n}} R \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

and which is of the form

$$U_1 = b\, R^{1 - \frac{1}{2}n} \sqrt{S} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

b being equal to $\frac{2}{3}\sqrt{\frac{m\,g}{\varepsilon}}$, constant for all sizes of pipe, so long as the temperature is constant, and entirely independent of the friction along the sides of the pipe.

Now, for the cylindrical portion, $A\,B$, in which the velocity is v_0, and not only uniform but equal in each fillet, if the ordinary assump-

tion be made that the resistances are proportional to the square of the velocity, the equation for equilibrium will be Mr. Tutton.

$$\pi R^2 \rho g h = 2 \pi R l f \rho g \frac{v_o^2}{2g},$$

or

$$v_o^2 = \frac{g}{f} R S \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

which is of the form

$$v_o = a \sqrt{R S} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

Adding (5) and (7) then obtains the general mean velocity in the pipe

$$U = v_o + U_1 = \left(a + b R^{\frac{1-n}{2}}\right) \sqrt{R S} \ldots\ldots\ldots\ldots (8)$$

in which R is the actual radius of the pipe, a is a factor depending solely upon an absolute coefficient of friction on the sides of the pipe, and b is a factor depending solely upon viscosity and temperature.

The parameter, P, of this ellipse being $\frac{2 v^2}{R^2}$, which by (2) is equal to $\frac{2 m g S}{\varepsilon R^n}$ there results $S = \frac{\varepsilon R^n}{2 m g} P$, or the effect of changing the slope is simply to increase the parameter of the ellipse in a direct ratio, or s varies with V^2, as it should.

That the viscous resistances are a very important factor is shown by the fact that if the authors' ratio of cylinder to dome be correct, 0.52 and 0.48, viscosity would account for 38% of the entire resistance.

In comparing this formula with the Chezy form, it is necessary to change it so that R represents the hydraulic radius. For this, make

$$a = \sqrt{\frac{2g}{f}}; \quad b = \frac{2^{\frac{4-n}{2}}}{3} \sqrt{\frac{m g}{\varepsilon}}$$

and it becomes

$$U = \left\{ \sqrt{\frac{2g}{f}} + \frac{2^{\frac{4-n}{2}}}{3} \sqrt{\frac{m g}{\varepsilon}} R^{\frac{1-n}{2}} \right\} \sqrt{R S} \ldots\ldots (9)$$

The final form will depend upon the values given to m and n. Here it will be assumed that $m = 1$; then, if

$n = -1$	$U = (a + b R) \sqrt{R S}$	Dupuit's form.
$n = 0$	$U = (a + b \sqrt{R}) \sqrt{R S}$	Manning's form.
$n = 1$	$U = (a + b) \sqrt{R S}$	Here adopted.
$n = 2$	$U = \left(a + \frac{b}{\sqrt{R}}\right) \sqrt{R S}$	Etc.

Dupuit obtained his form from a theory somewhat similar to that of Bazin. Manning's was purely empirical, being deduced from a plotted curve.

In adopting the form, $U = (a + b) \sqrt{R S}$, the mistake must not be made of considering it equivalent to the Chezy, $U = C \sqrt{R S}$, as a and b are constants, the properties of which are entirely independent. It

Mr. Tutton. may be written in the following shape, in which it is perfectly homogeneous and applicable to any system of units:

$$U = \sqrt{\frac{2g}{f}} \left\{ 1 + \frac{2}{3} \sqrt{\frac{f}{\varepsilon}} \right\} \sqrt{R\,S}$$

or

$$U = C\sqrt{g} \left\{ 1 + \frac{2}{3} \sqrt{\frac{f}{\varepsilon}} \right\} \sqrt{R\,S}$$

but, to show its character, it is better written

$$U = \sqrt{2g} \left\{ \frac{1}{\sqrt{f}} + \frac{2}{3\sqrt{\varepsilon}} \right\} \sqrt{R\,S}$$

Now, if 0.84 be the constant ratio of mean to maximum velocity, as the value $\frac{1}{\sqrt{f}}$ must account for 0.52 U and $\frac{2}{3\sqrt{\varepsilon}}$ for 0.32 U, then must exist $\frac{32}{\sqrt{f}} = \frac{104}{\sqrt{\varepsilon}}$, whence $f = 0.857\varepsilon$; should the relation be 0.83, then $f = 1.08\varepsilon$; should it be 0.85, then $f = 0.669\varepsilon$; but should it be $\frac{5}{6} = 0.8333$, then $f = \varepsilon$, whence it will be seen that a very slight change in ratio of mean to maximum velocity makes a very great change in the ratio of f to ε, which would indicate that 0.8333 was a more preferable mean ratio than 0.84 assumed by the authors, but one that is likely to change, and is not constant.

The main object of the writer being to present a theory of the ellipse as the true velocity curve, further determination of the constants applicable to this formula will not be attempted. Suffice it to say that a correct determination of ε has yet to be made, as it is not thought that the Poiseuille formula has a correct basis, or that the coefficient of viscosity deduced therefrom is applicable, except within exceedingly narrow limits. Its form would have to be changed for use in the preceding, were it thought applicable.

As a practical formula, experience may show the assumption $n = 0$ preferable, or

$$U = C_1 \sqrt{S}(\sqrt{R} + b\,R)$$

which, if b be given the value $\frac{1}{7}$, becomes almost identical with Professor Robert Manning's formula*, adapted to English measures,

$$U = C_1 \sqrt{S}(\sqrt{R} + \tfrac{1}{7} R - 0.05),$$

the accuracy of which has been too well verified to require farther notice here. This is practically identical with a formula,

$$U = \frac{1.54}{n} R^{\frac{2}{3}} S^{\frac{1}{2}},$$

previously presented by the writer† (the agreement being also noted by Professor Manning), in which n has Kutter's value.

The authors make a rather startling assertion in Conclusion *K*, that

* *Transactions*, Inst. of Civ. Eng. of Ireland, Vol. 20, p. 161.

† *Journal of the Association of Engineering Societies*, October, 1899.

curves of short radius, down to a limit of $2\frac{1}{2}$ diameters, offer less resistance than do those of longer radius, and that all theories hitherto presented are incorrect and diametrically opposite to the true conditions. The writer thinks that the effect of viscosity should be more thoroughly investigated before accepting this statement. As he understands it, both dome and wall losses of head have been included in the exponent n as applied to total mean velocity. It has often been suggested that the assumption upon which Equation (6) is based is in error, and it has again been suggested that the pressure head measured by a circumferential piezometer, or rather the loss of head, is only applicable to v_0 of the present note, and not to the U_1, due to viscosity. These suggestions require further investigation, which may show Conclusion *K* to be in error. Mr. Tutton.

Conclusion *I* is undeniable, but how far *J* is true will depend upon what is above stated in reference to *K*.

J. L. CAMPBELL, M. Am. Soc. C. E. (by letter).—The authors deserve credit for developing the Pitot tube and differential gauge for measuring velocity and pressure, but the fact that the tube registers negatively when the point is oblique to the current appears to limit its application. It would be instructive on its action to present traverses of the pipes on the curves. Mr. Campbell.

From the data given on the increase of resistance in curves of long radius over those of short radius, it is evident that this is due, not to the longer radius, but to the greater length of curve; but the increase of resistance is less than that of length.

Undoubtedly, with a given velocity, the intensity of disturbance, and, consequently, of resistance in a sharp turn is greater than in a gentle turn; but with equal angular deflections it acts through a shorter distance. As this greater intensity of the shorter turn multiplied by its lesser distance is, apparently, smaller than the lesser intensity of the long turn multiplied by its greater distance, the larger resistance of the latter is clearly due to the continued agitation of the water in the long curve.

On the other hand, if this is true, the increase of intensity on going from a gentle to a sharp turn is not proportional, within a wide limit, to the decrease in length of radius, and the experiments in hand appear to indicate that here is a relation that existing formulas fail to express correctly.

Water being instantly and perfectly responsive to disturbance of its equilibrium, it is readily conceivable that the agitation and counter currents set up by the deflections of a curve of 10 ft. radius will be more than half as great as those due to one of 5 ft. radius. As the former is twice as long as the latter, it is obvious that its resistance would be greater. This assumes, of course, that the disturbance of the water continues throughout the length and begins to end at the

Mr. Campbell. termination of the curve. This would probably be shown to be substantially true by tests with the Pitot tube.

It is fairly demonstrated, theoretically and practically, that the resistances of railway curves do not decrease as rapidly as the radii increase. As this is true of the comparatively inelastic movement of a railway train, there is ample warrant to believe it will hold for a perfectly elastic fluid.

The experiments should take a wider range. Smaller diameters and higher velocities would, no doubt, give results differing in degree at least.

After all, the results are not wholly diametrically opposed to all authorities. Above a radius of 5 diameters, Weisbach's formula gives no advantage for longer radii, making the resistance practically constant.

In the discussion of this subject, Mr. Henry gives an illogical interpretation and conclusion in stating that the least resistance to flow would be in a pipe having the greatest number of sharp right-angle bends.

To avoid such confusion, it must be remembered that the total angular deflection in all the experiments was the same (90°) for each case, and, consequently, that the lengths of the curves of different radii varied.

The net result was a less total resistance for the greater intensity of the short turn acting through the shorter distance, and a greater total resistance for the lesser intensity of the long turn acting through the longer distance. This is tersely stated by Mr. Church. But this is no justification for the conclusion that, foot for foot of length, the short turn gives less resistance than the long turn. As a matter of fact, it is known that the short turn would give the greater resistance, and the results of the experiments, taken in connection with the relative lengths of the curved portions of the pipes, prove this.

Suppose, for example, that there are two hollow rings of 12-in. pipe, each full of water, one having a radius of 1 ft. and the other a radius of 2 865 ft. Suppose the water in each is made to flow under a uniform pressure and velocity once around the circle of 360 degrees.

Common sense and a superficial knowledge of the properties of water are sufficient to satisfy us that the resistance to flow in the small ring could not be 2 865 times as great as that in the large ring. It is obvious that the total resistance once around the small ring is very materially less than around the large one, but equally apparent that the intensity of resistance per unit of length in the large ring is very much less than in the small one.

Let a = the length of the short ring, b = the length of the long ring, x = the intensity of resistance in the short ring, and y the intensity of resistance in the long ring per unit of length. Then $y\,b >$

x a. As the writer understands the deductions of the authors, they assert nothing more than the foregoing. Mr. Campbell.

Neither do Mr. Henry's exceptions to the results of the experiments, showing that the locus of maximum velocity may be away from the center of a pipe, appear to be well taken, on the assumption that said locus must be on the center, because the latter point is farthest removed from causes of retardation. Theoretically, that assumption is all right, assuming, in addition, that the perimeter of the pipe gives a uniform resistance all the way around, and that the pressure is likewise uniform. As a matter of fact, these conditions do not always obtain, and, in pipes of large diameter, the pressures at the top and bottom are never equal. Suppose, for example, that the bottom of the pipe is covered with sand and gravel, or that one side is rougher than the other: Under such conditions could it be expected, even theoretically, that the maximum velocity would be found on the center? Certainly not.

Again, the writer does not believe that the theoretical assumption referred to is necessarily logical. That it agrees well with observed facts under perfect practical conditions he does not question, but suppose it is made absolutely theoretical, and it is assumed that there is no resistance to flow in the pipe from the surface of the latter, the viscosity of the water, air, or any cause whatever. Would it not follow that the greatest velocity would be along the bottom of the pipe, due to the greater pressure there? In view of the many things that can and do upset theoretical conditions, it is not at all surprising that the experiments in question found the maximum velocity away from the center in many cases.

In conclusion, a point may be noted which is favorable to long curves, as against very short ones, in laying a line of pipe, viz.: the former will give a shorter total length of line, neutralizing to some extent the advantage of the shorter turns. With two lines, each having ten 90° bends, one laid on radii of 60 ft. and the other on radii of 2½ ft., the former would be 196.7 ft. the shorter.

CHARLES W. SHERMAN, Assoc. M. Am. Soc. C. E. (by letter).—The prior use of oil in a differential gauge, for the purpose of increasing the difference in height between the two water columns, was unknown to the authors of this paper, as stated by them in a foot-note on page 73, and they will doubtless be surprised to learn that this process was patented on March 24th, 1885, by the late Henry Flad, M. Am. Soc. C. E., of St. Louis, the patent being numbered 314 323. The specifications forming part of the application include a number of drawings, and a detailed description of the principle upon which it is based, as well as of some of its applications. The first of the nine claims on which the patent was granted is in part as follows: Mr. Sherman.

"1. The herein-described method of indicating the velocity of running fluid, the same consisting in causing two columns of the fluid

Mr. Sherman. to be respectively acted upon simultaneously by the impact and suction or draft of the stream, and connecting the upper ends of said columns directly by an inclosed fluid medium having a less specific gravity than that of the fluid, the velocity of which is to be ascertained. * * *"

In spite of their conclusion that "under some conditions, in straight pipe, there is a difference of pressure at different points around the circumference of the same cross-section," the authors seem to have generally used as a piezometer connection a single corporation cock screwed vertically into the top of the pipe, and to have accepted the data obtained from these piezometers without question.

The writer is of the opinion that when it is not feasible to tap more than one hole in a pipe for a piezometer connection, this hole should not be higher than 45° above the horizontal diameter. In some experiments on flow of water in a 30-in. pipe, made some years ago, he used three piezometers, each connection consisting of a corporation cock tapped vertically into the top of the pipe; and while, by the exercise of great care and frequently blowing off the air, good results were obtained from two of them, the collection of air at the third was so great that its value was seriously impaired, if not entirely destroyed.

The writer has not been able to give to the paper the attention which it deserves, but it would seem to him from such study as he has given to it that the authors have not sufficiently appreciated how large a proportion of the frictional loss of head in ordinary water pipes is due to the joints. Curves of long radius invariably contain a number of joints, while short-radius curves contain few or none. This would be one reason why curves of short radius should offer less resistance to the flow of water.

Mr. Henry. D. FARRAND HENRY, M. Am. Soc. C. E.—There must be something wrong in the observations or the deductions when the authors reach conclusions on two points so diametrically opposed to accepted theories.

First: That the friction in pipes increases with the radius of curvature. They claim that the least friction was found in a 90° curve with a radius of 2 to 2.5 diameters, and the greatest in a similar curve with a radius of 24 diameters. Can it not be legitimately concluded that a line of pipe through which the water will flow with the least friction must have as many sharp right-angle bends as possible?

Second: That the locus of maximum velocity in a circular pipe may be anywhere except in the center. The heretofore recognized theory in regard to this locus is admirably expressed by M. Dubuat:

* "When the flow of water has become uniform in an inclined pipe the fillet which occupies the axis moves with the greatest velocity, because it is furthest from the cause of retardation, that is, the outer walls."

In other words, the fastest current follows the line of least resist-

* "Principes Hydrauliques," I, p. 65.

ance. This has been the teaching of all velocity measurements, from the small experimental canals to the largest rivers. Mr. Henry.

The velocity observations in small canals by MM. Darcy and Bazin* are very numerous, and are given in such detail that the locus of maximum velocity can be readily found. In rectangular canals the maximum velocity was in or near the center vertical, and where the depth was less than one-fourth of the width it was at the surface; with the depth equal to one-half the width it descended to about one-third of the depth, the friction on the perpendicular sides seeming to force the locus beneath the surface. In triangular canals the surface velocity was greatest until the depth was about one-third of the width, and in semi-circular and trapezoidal canals, the effect of the sides seemed to be between that of the triangular and rectangular forms. MM. Darcy and Bazin remark that the friction of the air has much less influence in retarding the upper layers than this curious influence of the sides. Among their observations were some on a rectangular canal covered at the top and running full,† being the same as a pipe conveying water under a light pressure. The observations were taken at the intersections of imaginary lines crossing the tube at right angles or parallel to the sides of the covered canal. These readings were then plotted and curves drawn through points of equal velocity. Of these they say:

"The curves are extremely regular, and differ very little from rectangles of which the sides are parallel to the sides of the tube. * * * * * The curves of equal velocity evidently tend to reproduce the form of the water, and we notice that in following the contour of the tube they preserve very nearly their respective distances."

Similar observations, but on a much larger scale, were made by M. Racourt one winter on the Neva when it was covered with ice, forming a tube 900 ft. wide and 60 ft. deep. The maximum velocity—about 2 miles an hour—was found a little below the center of the deepest vertical, while in the summer it was at or near the surface. The first Chicago water-works tunnel, a brick tube 2 miles long and 5 ft. in diameter, was gauged with the Telegraphic current meter, and, though the observations were too few to plot more that one curve, it was fairly regular, with the maximum velocity in the center.

All other series of observations to which the writer has access, except one, confirm this law: In pipes the maximum velocity is in the center, in narrow canals it is at more or less depth below the surface, according to the ratio between the depth and width, and in rivers it is at or near the surface, always being in the line of least resistance. The exception is given in the report of Humphreys and Abbott on the gauging of the Mississippi, where it is claimed that the maximum velocity was at about one-third of the depth; but as the writer has

* "Recherches Hydrauliques," p. 187 *et seq.*

† *Idem*, pp. 174 and 175.

Mr. Henry. elsewhere shown * the Mississippi observations were made to conform to the vertical velocity curve observed in a little canal. It may seem foolish to review all this old work, but when an explorer attempts to leave the beaten path it is well to at least look at the records of those who blazed the trail.

The velocity curves plotted in the plates which accompany this paper remind the writer of his despair when he first tried to plot the double float observations made during the first year of the work in determining the outflow of the Great Lakes. The plotted velocity measurements were just about as irregular, and he felt like giving up the work in disgust. The observations of Humphreys and Abbott on the Mississippi—the only other extended series of double float observations of which he has any knowledge—were just as erratic. Of sixty-nine selected vertical curves, which they used in the calculation of their formula, twenty-three gave the velocity at the bottom greater than at the surface, in some cases by more than 1 ft. a second.

Of these observations, Dr. Hagen says:

† "Extraordinary anomalies are shown where these series are graphically represented. In some of them the curves are quite regular, but in others the velocity remains nearly the same from the surface to the bottom, but frequently with increasing depth it increases and decreases considerably. Generally, the velocity decreases with the depth, but a few series have an opposite tendency, that is, a surprising increase near the bottom of the river."

The trouble, then, was not entirely due to errors of observation, but the floats were at fault. Some instrument, therefore, had to be found that would better record the velocities, or the work would have to be abandoned. The best form then known was the "Woltman Mill" or "Tachometer." There were many modifications of this device, the Saxton's meter being perhaps the best. But, as shown by Mr. Baumgarten,‡ while it was unexcelled in work in shoal water, it was impracticable in deep rivers. Fortunately, there came the idea of recording the revolutions of the wheel electrically, and so, for the next two years, the velocities in the rivers connecting the Great Lakes were very satisfactorily measured, and there was very little more trouble from eccentric curves. M. Baumgarten calibrated his little "Mill" by drawing it through still water and by comparing it with floats. The coefficient of the Telegraphic current meter was also obtained by drawing it through still water, and it was also compared with floats at all depths.§ At the surface the agreement was very close, but toward the bottom the floats gave higher velocities than the meter. In 1870 the late E. S. Chesbrough, M. Am. Soc. C. E., Chief Engineer of Public Works, Chicago, found that the formulas for the

* "Flow of Water in Rivers and Canals."
† "Bewegung des Wassers," p. 6.
‡ *Annales des Ponts et Chaussées*, 1847.
§ "Flow of Water in Rivers and Canals," p. 13.

flow of water in pipes gave about 20% less water flowing through the lake tunnel than was recorded by the pumps, and he asked that a Telegraphic current meter might be used to measure the velocity of flow in the tunnel. An assistant was sent with a meter, and observed the velocity at different points in the tunnel, during parts of three days only. The flow calculated from these measurements was 97½% of the amount recorded by the pumps, and as the pumps were estimated to have about 2% slip,* the correctness of the coefficient of the meter was pretty well proven. Mr. Henry.

Thus, by the use of the meter, the writer was saved from the temptation of following Humphreys and Abbott, or of advancing theories utterly at variance with the laws of flowing water. Of course, the floats were at fault, even as now it seems that the instruments used were not fitted for the work, for the patient and efficient work of the authors, and the fullness with which they have presented every detail cannot be too highly praised.

Pitot's tube, when presented to the French Academy by the inventor,† was simply a glass tube, curved 90° at the bottom, with an orifice of considerably less area than the tube. To this was soon added a second tube open at the bottom. In a paper read before the Academy, M. Dubuat speaks of the Pitot tube,‡ and suggests that the tubes be made of tin, containing floats carrying a rod moving in front of a scale from which the heights can be read. But M. Bazin first made it an instrument of precision.§ He constructed the lower portion of the two tubes of brass or copper with a cock at the upper end of each, controlled by a single handle. Above this the tubes were of glass, joined at the top, with a small tube closed by a cock. Between the glass tubes was a scale suitably graduated. In operation, the tubes were placed in the water, and, by means of the small tube, the air was partially exhausted, causing the water to rise to a convenient reading height, when the cock in the small tube was closed, and the water rose and fell according to the action of the current, there being always a partial vacuum over it in the tubes. A cord was fastened to the handle of the lower cock, so that it could be closed quickly when the water in the columns was at maximum or minimum. MM. Darcy and Bazin calibrated these tubes by three different methods:‖

* "Flow of Water in Rivers and Canals," p. 61.

NOTE.—Mr. Chesbrough calculated the flow of water in the tunnel by several formulas, which gave the following percentages: Hawkesley, 84%; Blackwell, 85%; Prony (1), 73%; Prony (2), 79%; Eytelwein, 84%; D'Aubuison (1), 81%; D'Aubuison (2), 81%, and Weisbach 83 per cent. The list was continued by the writer, to embrace the more modern formulas, as follows: Neville, 85%; Jackson, 88%; Saint Venant, 104%; Darcy, 99%; Fanning, 116%; Kutter, 93%, and Sullivan, 109 per cent.

The later formulas contain "a coefficient of roughness, *n*," for which a value has been assumed fulfilling as nearly as possible the conditions of the tunnel.

It is unfortunate that the Kutter formula, which has been praised so much, should have been largely founded on the Mississippi observations of Humphreys and Abbott.

† *Mémoires de l'Académie*, 1732.

‡ *Mémoires de l'Académie*, 1753.

§ *Annales des Ponts et Chaussées*, 1858.

‖ "Recherches Hydrauliques," pp. 64 *et seq.*

Mr. Henry.

First.—By placing the tube in front of a boat and drawing it through still water at different velocities. This gave a coefficient of 1.034.

Second.—By observing the readings in a stream, the velocity being obtained by floats. Coefficient, 1.006.

Third.—By readings taken in different parts of a canal in which the amount of water passing was known by measurement of its volume. Coefficient, 0.993.

The first coefficient was considered to be too large, as the bow of the boat would probably be raised in being drawn through the water, and the tube tilted, the current thus striking the orifice at an angle. These gentlemen also calibrated three other tubes, which show what a marked effect upon the coefficient* is produced by changing the position of the orifice in the second tube.

TABLE No. 55.

Position of the orifices.	Value of coefficient.	Method of gauging.
1. The orifice in the first tube directed against the current and in the second at right angles to it	0.848 0.797	By floats in still water.
2. The ends of both tubes directed against the current, but the orifice in the second stopped at the end and pierced laterally by an opening of 1 mm	0.875 0.864	By floats in still water.
3. The orifice in the first tube directed against the current, and that of the second in the direction of the current, both in the axis of the stream	0.898 0.891	By floats in still water.

Thus, this is a very delicate instrument, and requires careful adjustment and handling. It may be almost perfect in the measurement of velocities in little open canals, but we cannot predicate from that that it will work under pressure. Put the Bazin tube through the side of a water pipe in which the pressure is 50 lbs. or more; the air will be compressed above the water columns and the action may be entirely different from that which obtains when there is a partial vacuum above them. This, the authors themselves discovered. The coefficient found by drawing the tube through still water was nearly unity, while in a 2-in. pipe, through which a known quantity of water was flowing, it was very much less. Had the pipe been of the size of those experimented on, and the pressure equal to that of the water-works, the coefficient might have been still more different from that deduced from the still-water measurements. But this would only be of consequence did we want to know the amount of water passing through the mains. It does not explain the erratic velocities shown in the plates. It only shows that the Pitot tube is not a proper instru-

* "Recherches Hydrauliques," Note B, p. 302.

ment to use in the measurement of velocities under pressure. A more important thing to consider is the effect of the pulsations in flowing water. Even in little canals, MM. Darcy and Bazin found that the velocity of the current was constantly changing, though its effect upon the columns of water in the tubes was minimized by the orifices exposed to the current being very small. The lower cock was closed when the water columns were at maximum and minimum, so that the readings were easily taken. Table No. 56 shows some of these changes.* Mr. Henry.

TABLE No. 56.

Height of water in the first tube, in meters.	Height of water in the second tube, in meters.	Difference, in meters.	Remarks.
0.953	0.822	0.131	Maximum.
0.948	0.830	0.118	Minimum.
0.950	0.822	0.128	Maximum.
0.945	0.825	0.120	Minimum.
0.933	0.803	0.130	Maximum.
0.925	0.808	0.117	Minimum.

The fluctuations given in Table No. 56 varied from 0.008 to 0.013 m., or from 0.3 to 0.4 in. As the area of the orifices was less than one-thousandth of the area of the tubes, the actual range of the pulsations must have been much greater. These pulsations were first noticed by M. Baumgarten in his measurements of the Garonne.† He says:

"The velocity of the water is not permanent or constant at the same point. * * * These oscillations in the velocities are also apparent in the change of level of the surface of the water. If a plumb-bob be so hung that its point will just touch the surface at one moment, in the following moment it will either be plunged into the water or separated from it to a greater or less degree."

The Telegraphic current meter was the first instrument to actually record these pulsations, for by putting a Morse register into the circuit, every revolution of the wheel in the water made a dot on the moving paper. They seemed to follow no regular law, or rather their law could not be ascertained, but every 10 or 15 minutes these waves seemed to increase and then diminish. Near the bottom they were much larger than at the surface, the maximum velocity measured being often double the minimum. These pulsations are found wherever water is in motion, from the toy fountain to the Gulf Stream. They are, apparently, as little subject to known law as the very similar pulsations of air in motion. Some years subsequently, when Chief Engineer of the Detroit Water-Works, the writer arranged a glass mercury gauge at such height that the maximum pressure would be within the compass of the tube. There being then a reservoir connected with the mains, it was the custom of the engineer to stop the

*"Recherches Hydrauliques," p. 50.

†*Annales des Ponts et Chaussées*, 1847.

Mr. Henry. pumps for an hour or two each night. At such times he would watch the gauge, and the mercury would be continually rising and falling, sweeping through 6 to 8 ins., ordinarily, but every few minutes making an excursion of 12 ins. or more, a change of pressure of 5 or 6 lbs. The authors state that these fluctuations often amounted to several inches, and attribute them to the action of the pumps. Doubtless, though pulsations common to all flowing water will be influenced by other water waves, whether due to the pumps or other causes, and may be either increased or diminished.

For some unexplained reason, instead of taking the readings directly from the Bazin tube, the authors used a separate gauge, connected to it by rubber pipes. Had the pressure been constant, possibly the rubber would have merely been distended and the observations but little different from those taken on the tubes directly, as they should have been, but the ever-varying pressures must have had an effect on the elastic connecting pipes, how much can only be conjectured, but it seems probable that the erratic velocities recorded are largely due to these elastic pipes, and that this is the reason why the instrument, which in the hands of MM. Darcy and Bazin did such good work, with the authors, proved so utterly unreliable. Another cause of error may be the alignment of the tube. If the orifice of the first tube does not exactly face the current, the readings may not give the true velocity of flow. MM. Darcy and Bazin rejected the coefficient obtained by drawing the tube through still water, 1.034 m., or $1\frac{1}{4}$ ins. in excess of unity, because they considered that the bow of the boat might have been raised by its swift passage through the water. The tube was placed 4 ft. in advance of the boat; the length over all was probably 20 ft. The rise in the prow was possibly 3 ins. more than would have been noticed, or one-eightieth of the length. The length of the horizontal portion of the tube was about 1 in., so that an inclination of only $\frac{1}{80}$ in. was sufficient to make the coefficient $1\frac{1}{4}$ ins. above unity. The turning of the orifice of the first tube only $\frac{1}{16}$ in. to either side might make an error ten times as great.

Then there may have been small cross-currents in the tube, which would have had the same effect.

In the rivers and in a 700-ft. base, the double floats often ran 100 ft. or more, to one side of the axis of the current, or the path the majority of the floats were following.

It seems a pity that this careful, patient and honest work was not devoted to the observations of some such instrument as the Telegraphic meter, which would never have been affected by differences of pressure, would accommodate itself to cross-currents, and the observers would not have been obliged to stand out in the cold and wet, as the wires could have been carried to any sheltered place; rather than wasted on this ancient tube, which is very well in its place, but uncertain when tried in closed pipes, under pressure.

Mr. Henry.

For the past 150 years, tube, mill and every other instrument except the double floats have recorded the maximum velocity in the line of least resistance, the latter placing it anywhere from the surface to the bottom of a river; and it is scarcely probable that these observations will be accepted and all previous work discarded, any more than the Humphreys and Abbott formula, founded on the double float observations, placing the maximum velocity at three-tenths of the depth, displaced other formulas based on more accurate work.

When engineer of Palmer Park, near Detroit, where water-works and other improvements were being made, the writer had an example of the retention of gas by water, with which it is commingled, which may be of interest in this connection. There being a vein of water in the gravel just above the rock, half a dozen wells were bored through the clay to a depth of about 120 ft. and cased with 6-in. iron pipe. Considerable natural gas escaped from the wells, and as the water stood at 26 ft. below the surface, a central pumping well was constructed and connections made with the cased wells by a tube at the same depth, the pipe in each case rising to the surface to allow the gas a free escape. As long as everything was still there was no perceptible escape of gas, but, as soon as the pump was started, gas would bubble up the escape pipes, and water drawn in a glass would look like milk for some time, or until the entrained gas escaped. This continued for several months, the gas commingled with the water not escaping until it was pumped out with the water, though there was a perfectly free vent. This makes one rather skeptical as to entirely expelling the air from a water pipe, especially in Detroit, where one often sees a glass of water look turbid for several seconds until the air bubbles rise through it, leaving it clear and bright.

The long piezometric pipes of rubber—instead of lead as used by most observers—are, therefore, not only subject to the constantly varying pressures, tending to deform them, but also to the action of the entrained air from which the water in those pipes can never be entirely free. The varying pressure would also have much more effect on these long pipes than upon the connections of the Bazin tube, because the area of the piezometer openings is far greater than that of the orifices of the tube.

These observations, therefore, will hardly tempt us to cast aside all established theories, or stultify our common sense.

If they are correct, how foolish the estimate of engineers of the extra power required to force air around corners.

Mr. Wisner.

GEORGE Y. WISNER, M. Am. Soc. C. E. (by letter).—Hydraulic engineers should appreciate the radical advance made by the authors of this paper in their investigations, in basing their arguments and conclusions upon careful and extensive experiments made under conditions existing in large distributing systems for water supply.

Mr. Wisner. The writer is familiar with the nature of the experiments made by the authors, the difficulties encountered, the immense amount of gratuitous work involved, and the intricate analytical investigation necessary to harmonize results, where possible to do so, and to show why differences from heretofore accepted conclusions ought to exist; and he thoroughly realizes that few have had the personal experience which will warrant questioning the results of these experiments, if disposed to do so, which, fortunately, is not likely to be the case.

It should be remembered that at the time the experiments were first undertaken, suitable instruments for obtaining satisfactory results as to the law governing the flow of water in pipes did not exist, and that the formula for such flow was based largely on observations made under conditions widely different from those found in actual practice.

The difficulty in correctly rating meters and other devices for measuring the velocity of flowing water is the principal source of error in determining the volume of flow in pipes and open channels, and although great improvements in methods and instruments have been made there is still room for further refinement.

The recent remeasurement, by the Government, of the discharge of three large rivers, which were carefully measured about thirty years ago, shows a decrease of about 20% in the computed discharge for similar stages, which, so far as can now be determined, is largely due to erroneous rating of the meters in the earlier work.

If errors of 20% can exist in what was believed to be good work where the observations were made in open channels in full view of the observers, it is evident that the difficulties encountered by the authors in properly interpreting the conditions and results of observations in closed pipes were such as to call for the highest type of analytical ability and good judgment.

The determination of coefficients for meters and Pitot tubes is unsatisfactory, for the reason that the conditions under which ratings can be made are generally entirely different from those in the pipe or channel where the velocity of flow is to be determined. For instance, if a meter or Pitot tube be moved at different velocities through still water the coefficients obtained would be practically correct for flowing water where all the threads of the current are parallel, but as such a condition never exists in either open channels or closed pipes, there are grave doubts whether coefficients obtained by moving the gauging instruments through still water are correct when the instrument is used in a fixed position in flowing water.

In the ratings of meters for some recent government work, an attempt was made to check the accuracy of the coefficients of the instruments by rating each meter by movement through still water and afterward placing them side by side in the stream being gauged,

and from the respective readings obtaining a relative rating between the meters under actual conditions in the current to be measured. This proceeding is all very good to detect any change of coefficient which may occur in either meter during work, but so far as detecting any error in the absolute ratings of the meters due to errors in methods, it is of no use as long as each meter is rated in a like manner under similar conditions. Mr. Wisner.

For instance, if the meters be both rated in a canal so small that the movement of the instruments through the water should cause a current such that the actual movement was 10% greater than that through the water, the coefficients obtained would be in error 10%, yet if these instruments be compared with each other by simultaneous readings in the main current of the stream being gauged, the results obtained would agree with each other.

The failure of the authors to obtain ratings of Pitot tubes in still water which would agree with those obtained in flowing water indicates clearly that relative ratings may agree closely and yet be largely in error.

It is the writer's opinion that absolute ratings, to be reliable, must be made under conditions nearly similar to those of the stream or pipe to be gauged. That is, for open-stream work the coefficient of the meter to be used should be determined by float observations in close proximity to the meter in different locations in the stream, and for Pitot tubes in closed pipes the coefficient should be determined by an actual measurement of the volume of flow at the time the tubes are being read.

The distribution of velocities of flow in pipes and channels has an important bearing on the loss of head which will occur. If the maximum velocity be at the center, the distribution will be that which will produce the greatest volume of flow for any given head, and the more the velocity curve be distorted by curves and deflections in the pipe line, the less will be the discharge for any fixed head.

This being the case, the loss of head due to curves in a pipe line will be a function of the amount of distortion in the resulting velocity curve and of the length of pipe through which the disturbance tends to diminish the mean velocity of flow. With curves of long radii it is probable that the position of the maximum velocity is not shifted as far from the center as with curves of short radii, yet in the latter case the length of pipe through which the disturbance takes place is shorter, and therefore with pipes of different diameters, there should always be some certain curve which will produce less total resistance to flow in each case than any other curve of greater or less radius.

The authors' result, K, is apparently in harmony with theory, and their determination of the ratio between the diameter of pipe and the radius of curve which will produce the minimum resistance to flow is a long step in advance, in the matter of obtaining the best practical results in the design of distribution systems for water supply.

Mr. Wisner. The failure of pipes and conduits to pass the volume of water often expected may in part be traced to the causes enunciated by the authors in their final results, *I*, *J* and *K*.

The volume of water which can be passed through a pipe line with a given head is controlled largely by the conditions in the sections of greatest resistance, and where imperfect alignment and frequent curves and deflections exist the mean flow will be greatly decreased.

The coefficient of roughness, n, is often made to vary to cover variations in flow arising from distortion of the velocity curve, when, in fact, it should remain practically constant.

From Figs. 1 and 2 it will be seen that the glass tubes of the differential gauges had internal diameters of $\frac{1}{8}$ and $\frac{5}{16}$ in. It is a well-known principle that capillarity will cause water in small tubes to stand higher than in the reservoir with which connected. This being the case, at what diameter of tube does this effect become sufficient to affect materially the results of the observations?

In such tubes, when of dimensions to be so affected, does the effect vary with the head—that is, does a difference of head of 1 ft. indicated by a tube $\frac{1}{8}$ in. in internal diameter indicate an actual difference of 1 ft. of head in the reservoir?

Slow movement of water in small tubes is likely to produce a concave surface in the tube when moving downward, and a convex surface when rising in the tube. Are the errors from capillarity and direction of movement appreciable for the size of tubes used, and if so, are the errors so arising taken care of in the coefficients of the Pitot tubes, as determined in connection with such gauges?

Mr. Thrupp. EDGAR C. THRUPP, A. M. Inst. C. E. (by letter).—The authors deserve the cordial thanks of their professional brethren all over the world for the scrupulous care with which they have carried out, and recorded the results of, their experiments.

They have demonstrated very clearly how useless it is to attempt the solution of difficult problems in hydraulics by purely mathematical reasoning, without frequently resorting to experimental proof of the assumptions involved. Methods of treating these problems after the manner of Weisbach will in future be less and less trusted by engineers, and a wider distinction will be drawn between "pure mechanics" and "hydromechanics."

The subject is one of peculiar interest to the writer, because he has been experimenting, at intervals, for many years past, to ascertain the laws of movement of water under conditions which have not hitherto been satisfactorily studied, and he has been rewarded by the discovery of a remarkable phenomenon with regard to the "critical" velocities at which the movement of water changes from stream-like to sinuous or turbulent motion, and a change in the law of resistance also occurs.

Mr. Thrupp.

An outline of the writer's work was indicated in a paper presented at the International Engineering Congress at Glasgow on September 4th, 1901, and the full details will be set out in the writer's book on "Hydraulic Experiments and Engineering Problems" now preparing for publication.

The writer has found that when the hydraulic radii of pipes or open channels exceed 2 ins., the critical velocities increase with the hydraulic radius; whereas, from experiments on pipes less than 1 in. in diameter, it is well known that the critical velocity bears an inverse relation to the hydraulic radius. He has also found that the velocity required to disturb fine silt in channels of various depths is apparently identical with the "critical" velocity, and the whole question of the scouring power and silt-carrying capacity of rivers is intimately bound up with the critical velocity laws, which are the main factors determining the "degree of turbulence" of the motion and the consequent power of the water to disturb and transport solid matter.

The "degree of turbulence" and the nature of the turbulence depend first, upon the proximity or remoteness of the hydraulic conditions to those of the critical velocity, and are undoubtedly much influenced by the curvature of the channel. On the effect of curvature on the "degree of turbulence" the authors' experiments throw considerable light. One point, to which attention may be drawn, is that the traverses obtained with Pitot tubes show very irregular curves at low velocities and more regular curves at higher velocities, when the motion may be expected to be more of a quivering, rushing nature. The authors have very properly explained how the diagonal currents could give a zero or negative reading on the Pitot tube, but, although these readings are of no value as gaugings, they show how a diagonal eddy can remain long enough in one spot to enable several readings to be taken. The same thing occurs in swirling eddies in open channels, when the velocities are not much above the critical point, but cannot occur at very high velocities. It would be interesting if the authors would add a few traverse diagrams based on a single reading at each point, to bring out the irregularities more clearly.

The potency of slight curvature to produce eddying disturbances and consequent scour in rivers has long been known, and its importance recognized in selecting sites for landing stages, and for ports in tidal estuaries, but the efforts hitherto made to bring the data on the subject usefully into line with scientific hydromechanics have not been very successful. The subject is a difficult one, and has been rendered more difficult by the absence of any established theory of scour in relation to the three main hydraulic factors: Hydraulic radius, surface slope, and mean velocity.

The writer hopes to remove much of the difficulty by establishing, with a fair degree of accuracy, the correct locality of the critical

Mr. Thrupp. region and by tracing lines representing equal degrees of turbulence in relation to depths and velocities.

The modification of those lines by curvature of channels will follow in course of time as data are obtained on the new basis, and the work of the authors in developing the differential gauge, with oil in place of air, is welcomed as a valuable contribution to scientific methods of studying small losses of head. The writer has used hook-gauges, and long connections of ⅝-in. rubber hose, immersed in rivers, for leveling from station to station, in preference to relying on spirit leveling, to observe very small surface falls, and he is satisfied that valuable information may be derived in that way, and perhaps the oil differential gauge may give better results in some cases.

The great advantage of such appliances is that they enable results to be obtained in short distances, and therefore suitable experimental sites can be more easily found.

The Pitot tube, with the oil gauge, should also lead to further data being collected, at a comparatively small cost, from very large pipes at very low velocities.

The authors' main conclusion, as to pipe curves of 2½ diameters radius giving the least resistance, shows how great a disturbance can be caused by a slight curve, but it is evident that the sharper bends give a higher resistance, for each foot of their length, than the easy bends; and the fact of their total resistance being less is only due to their ratio of lengths being greater than their ratio of resistance per foot run.

It is pleasing to find among the authors' results some clear indications of the critical velocity phenomena closely agreeing with the writer's data.

The 12-in. pipe experiments show the critical velocity at about 0.14 ft. per second, the 16-in. at about 0.30 and the 30-in. at about 0.70 ft. per second.

The 12-in. and 16-in. indications are visible in the results of the measured loss of head at low velocities, and the 30-in. by the rapid falling off of the ratio of mean velocity to the center velocity with low discharge. (See Table No. 8.)

American engineers should be particularly interested in this critical velocity question when it is pointed out to them that the phenomenon is indicated clearly in the experiments of the Mississippi River Commission at Carrollton in 1882. The writer hoped to have met Mr. Ockerson at Glasgow, and to have raised a discussion about it there, in connection with his paper on the Lower Mississippi, but, as he was absent, the subject was not opened. The present communication is an appropriate opportunity for announcing the fact.

Hitherto, engineers have been in the habit of assuming that the resistance in large rivers is proportional to V^2 or some higher exponent of V, occasionally up to V^5.

Humphreys and Abbott, Hagen, Gauckler and Kutter took this view. The latter's complicated formula is framed so as to represent a higher power of V as the slopes become flatter. The writer now claims to have discovered the limit to which this may be considered as correct, and to be the first to realize that the true interpretation of the Carrollton experiments is that the conditions existing there closely approximate to the critical velocity conditions, and show the resistance varying much more nearly in simple proportion to V than to V^5 or even V^2. Mr. Thrupp.

It would prolong this communication unduly to go fully into the details, and, moreover, they will be published in due course in the writer's book, but this brief indication of the main issue will suffice to explain the following critical remarks on the authors' methods of treating their data.

First, let us try to learn more about the deviations from the formula $H_f = m\ V^n$ and not obliterate them by "straight line" analysis, after fixing the position of the line by "centers of gravity of groups," or by the "method of least squares," or otherwise.

Secondly, let us abandon the theory that the discharge of pipes is ever proportional to $\sqrt{d^5}$ (*i. e.*, $d^{2.5}$ or $r^{2.5}$), for all the best experimental data show that it is rarely less than $d^{2.65}$.

It would also facilitate the study of the results if they were finally reduced to terms of the observed velocities, slopes and diameters.

Finally, it is suggested to engineers entrusted with the treatment of large rivers, that the most promising field for experimental research is to be found in observations of surface slopes by hydrostatic tube leveling in conjunction with velocities, cross-sections, scour or silting, and notes on the "degree of turbulence" as made evident by visible eddies or by fluctuations of velocity at given stations and by irregularities in the shapes of vertical velocity curves.

RUDOLPH HERING, M. Am. Soc. C. E.—The speaker thinks that this paper is destined to mark an important point in the study of the flow of water through pipes. It describes a very carefully conducted series of experiments, and draws conclusions, some of which vary substantially from commonly accepted beliefs held for years by eminent hydraulicians. Mr. Hering.

A number of years ago, in Philadelphia, when the street railways were first introduced, the curves, or rather compound curves, from one street to another, were made with as long a radius as possible. A few years' experience seemed to point to the conclusion that a single curve uniting the two tangents would answer all purposes and cause much less work for the horses. The speaker remembers the chief engineer's remark: "Make the shortest curve and be done with it." It would seem, from some of the authors' statements, that herein may partly lie an explanation of their results.

Mr. Hering. At the same time it must not be forgotten that friction of solids against solids is a totally different thing from friction of fluids against solids. In the former case, the friction, as is known, increases with the pressure, due to the fact that the little protuberances or slight elevations on the surfaces must get over or pass each other by a compression of the material composing them. With a flowing liquid the case is entirely different, because the liquid particles roll over the protuberances; friction, therefore, has nothing at all to do with the pressure of the liquid, but only with the resistances met in its flow. Herein may lie the explanation of the authors' results. With flowing water it is more a question of the extent of surface against which the liquid flows and by which it is worried into contortions by the little irregularities on the surface, and also by the longer continuous change of direction of flow, than it is a question of the length of the radius of curvature.

Mr. Seddon. JAMES A. SEDDON, M. Am. Soc. C. E. (by letter).—This very interesting series of observations was so fully discussed by the authors that the writer did not see at the time anything that he was called on to add to it. But Mr. Tutton has taken the matter beyond the question of facts and into the forces that made them, and in this has given an illustration of what the writer thinks is a misunderstanding of these forces, and not an uncommon one.

All that is known, directly, of the energy consumed in a flow is observed in the difference of head or level between two points in it. The discharge, by the head, is just so many foot-pounds a second, whether it goes over the dam, goes through the wheels, or goes down the river. Calling this the gravity potential, the problem in resistance to flow is: How does its gravity potential reach its limit of acceleration?

In the drop over a dam it is simple enough. We need not go into the forces to see that the flow is accelerated all through the fall, and beaten into foam and eddies at the foot of it. In the case considered by Mr. Tutton it is necessary first to analyze the forces. Taking his concentric cylinder of water in the pipe (Fig. 93), the forces in the direction of flow, as a whole, are equal to $(p_o - p) \times \pi r^2$, or, the difference in the pressures at the two ends of the cylinder, of radius r, multiplied by its end area. This is just as Mr. Tutton gives it, and is the total of what may be called the gravity forces.

But while this is a true summation of the gravity forces, in the case considered, the question of where these forces are located is quite another matter; it is plain that they are in some way distributed through the cylinder, and that this is merely an expression for a volume integral of them. Certainly, also, in balanced flow, counterforces will be found equal to them; but that counterforces equal to their sum should lie on the circumferential boundary of this cylinder

does not follow. If this boundary was an imaginary plane without thickness the writer would say that the counterforce on it was an absolute zero. Mr. Seddon.

Navier's expression for internal resistance, $\mu \frac{d v}{d r}$, or this counterforce, the writer takes to be the coefficient of a volume integral, just as an expression for a pressure is the coefficient of a surface integral. To take it as a force on an imaginary plane in the flow is then the equivalent of starting out to calculate the strains in a dam with the assumption that the water pressure is 62.4 lbs. on a line 1 ft. long and 1 ft. below the surface. The counterforce in the bounding film, $d r$ in thickness, of Mr. Tutton's cylinder the writer would put as

$$2 \pi r l d r \left(\mu \frac{d v}{d r} \right),$$

where l is the length of the cylinder and μ the coefficient of viscosity. This has no relation to the total of the gravity forces $(p_o - p) \times \pi r^2$ distributed through the cylinder; at most, it could only balance that element of them which lies in this film.

It should be said, however, that the writer has not found in general that others think as he does on this matter. He was surprised a few years ago to find that the whole faculty of a leading university in the West took essentially Mr. Tutton's view of it; and, so far as he commits himself in his article in the Britannica, Unwin seems to take the same position. Those great mathematicians, from Navier to Stokes, the writer would credit with having fully developed the character of fluid friction; but he does not claim to have studied their works; he has simply looked into them enough to get for himself a definite idea of the process through which the energy of motion in water passes into heat.

But the counterforces in this process are wholly insufficient to balance the gravity forces. Calling the sum of these counterforces F, and assuming a form for the system of velocities in the pipe, the expression

$$d F = 2 \pi r l d r \left(\mu \frac{d v}{d r} \right)$$

is easily integrated.

With the velocity forms

(I.) A cone $F = \mu \pi l R V$

(II.) An ellipsoid $F = \mu \frac{\pi^2}{2} l R V$

} Where R is the radius of the pipe, and V the maximum velocity at the center.

Taking Helmholz's value of μ, in foot-pound units, and the length, L, in thousands of feet, in the place of l in feet, these values are

(I.) $F = 0.0060\ L\ R\ V$

(II.) $F = 0.0094\ L\ R\ V$

Mr. Seddon. Probably every actual case of flow in a pipe would lie between these values; but, at the most, with $L = 1$, $R = 1$, and $V = 8$,

$$F = 0.075 \text{ lbs.},$$

while 1 000 ft. of pipe, 2 ft. in diameter, with a maximum velocity of 8 ft. a second in it, could hardly have less than 1 ft. of head on it, and $(p_o - p)\ \pi\ R^2 = 196$ lbs. $= G$, the gravity forces.

Some years ago the writer had occasion to calculate this relation of F to G for rivers, and he stated at the time that the resistance of the counterforces found in their flow velocities was but little over 0.01 of 1% of the gravity potential consumed in their flow. Here, in pipes, it is found to be more than twice as large as it should be from the greater curvature of the velocity forms in them. But it may be said, for flow in general, that F is from 0.01 to 0.04 of 1% of G; or that the counterforces in the velocities of flow are an utterly insignificant fraction of the gravity forces in it.

Of course, it is understood that there are also forces of fluid friction in the circulation, but if the velocities of flow are taken out of the case the remaining motions have no resultant; altogether, the forces in them are equal in all directions, and can in no part be counterforces to the gravity potential. It may be stated, then, from the work of these mathematicians, that fluid friction and resistance to flow have practically nothing to do with each other; understanding "fluid friction" to be that process through which the energy in the motion of the water passes into heat, and "resistance to flow" that process through which the energy in the gravity potential reaches its limit of acceleration.

But fluid friction is not at all mysterious. It must be understood that on one side of it there is mechanical motion and on the other molecular motion; and we may go at once into the mechanical particle, $d\,x\ d\,y\ d\,z$, to look for it. In this, taking the axis of x in the direction of the velocity, the friction force is

$$\mu \left(\frac{d\,v}{d\,y} + \frac{d\,v}{d\,z} \right) d\,x\ \ d\,y\ \ d\,z.$$

Thus, in a river, $\frac{d\,v}{d\,y}$ may be taken for the slant of its flow velocities from top to bottom, and $\frac{d\,v}{d\,z}$ from side to center, while $\mu \left(\frac{d\,v}{d\,y} + \frac{d\,v}{d\,z} \right)$ is the coefficient of a volume integral for the total of the friction forces in these motions. In water, this is the most general expression for this friction force in the particle, $d\,x\ d\,y\ d\,z$; two motions of shifting on planes at right angles is all that it can have in it; gases may have in addition a motion of dilation. That the sum of the first differentials of these motions expresses the force is an assumption, but the viscosity of water is determined on this assumption, and its coefficient applies to the form given.

Though μ is a very small quantity with the water beaten into foam at the foot of Niagara, and the values of $\frac{dv}{dy}$ and $\frac{dv}{dz}$ running up into infinities on the boundary of every internal whirl and eddy, this friction will account for the conversion of a great amount of energy into heat very rapidly. Again, following the remnant of a circulation in a reservoir, days after it has been filled, the loss of motion in this form is almost imperceptible, as the writer has had occasion to note in tracing its effect on sedimentation. Mr. Seddon.

All the energy was, at one time, a gravity potential, and it passed through a condition of flow into a system of internal circulation; and again 99.96% of it passed through the circulation into heat. Energy is not destroyed, it is only transformed; from a linear acceleration to an internal eddy, it is still motion; but one has direction and the other has not. This transformation can limit the acceleration of a gravity potential; after it is in the eddy, fluid friction will change it into heat, though it may be days before it is altogether through with it; but whether it is days or seconds makes no difference in the relations of the two processes; the transformation into heat is not a part of the resistance to flow, but a product of it.

With the eddy practically in all cases the end of the acceleration—and the eddy a very effective end to it, even if it is a drop over Niagara—we may now go back to the gravity forces, which we left at the beginning with the simple statement that they were in some way distributed through the cylinder.

It now seems plain that this distribution can have no fixed character, either in time or space; where a gravity force lies on a line, that is practically a line of uniform acceleration, and it runs itself into an eddy, and that is the end of it; the action is essentially shifting and intermittent in its nature. The writer does not see how this can be the case in a pipe unless $\triangle h$ itself is intermittent, or there is a pulse in the hydraulic grade line. He spoke of this to one of the authors, while their observations were in progress, but concluded, with him, that a service-pipe line with the beat of a pump through it was not a place where the question could be tested.

However, in cases where the flow has a free surface, the matter is not so intricate. Probably everyone has noted the standing waves on the face of the rapids, where the acceleration reaches its limits and throws up a ridge of water in its course against which it checks itself. Where the channel is symmetrical this may be taken as a resistance furnished by the flow itself, in contrast with the energy that is thrown into eddies by the rough or irregular form of the boundary; there is a pulse, also, in the flow of rivers, which seems to mark a resistance of the same character.

The case of flow with a free surface is not under discussion here,

Mr. Seddon. but it may be stated, generally, that if anyone will undertake to estimate the energy expressed in these surface forms he will be struck at once with the fact that a large part of the gravity potential may be taken up in them. Mr. Tutton's division of the resistance, into an element that belongs to the boundary, and an element that lies in the body of the flow itself, is a conclusion that the writer regards very highly, as, indeed, he does the whole spirit of this paper, with its effort to get at the inside facts in the matter, and, from them, to deduce the forces.

Mr. Frizell. J. P. FRIZELL, M. Am. Soc. C. E. (by letter).—The writer has read this paper with great interest, and though he has not familiarized himself with all the details of the work, he will venture to offer a few comments on certain specific points.

In considering the relation between the height of water in a Pitot tube and the relative velocity of the water at the point thereof, the authors cite the well-known principle that the pressure of a jet of water against a plane surface, normal to its direction, is double that of the head due the velocity of the jet; but they appear to be in some doubt as to how this principle applies to the case in question. It appears to the writer that it has really no application. The best way to realize this is to call to mind the reasoning on which this statement is founded, viz.:

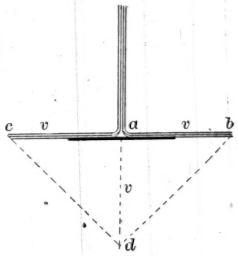


FIG. 94.

A jet of water is moving in the direction $a\ d$ (see Fig. 94) with a velocity of v feet per second, making $a\ d = v$. A particle of water, having reached the point a, would be at the point b at the end of one second, if its motion were not controlled. By the interposition of the plane surface a, it finds itself, at the end of one second, at a point b or c, at a distance v from its line of motion. The result is the same as if its motion of v feet per second were combined with a second motion of $d\ b$ or $d\ c$ feet per second. The pressure of the plane on the jet, which is the same as the pressure of the jet on the plane, is such as to communicate a velocity of $d\ b$ feet in one second, and is equivalent to a head of

$$\frac{d\ b^2}{2\ g} = \frac{v^2 + v^2}{2\ g} = 2\left(\frac{v^2}{2\ g}\right).$$

Now, when we come to consider the case of a plane immersed in an indefinite current, we find a phenomenon akin to what is represented in Fig. 95. The water does not change direction at right angles and glance off to right and left; as

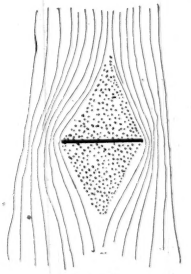


FIG. 95.

such action would be incompatible with the general movement of the mass of water. It is probable that a mass of "dead water" accumulates on the vane, both above and below, and the moving fillets are not deflected uniformly at a right angle, but in a variable manner, some more, some less, constituting a movement on which no such reasoning as the above can be predicted. In the experiments of Darcy, the rise of water in the tube approximated closely to the head due the relative velocity of the water at the point of the same. In these experiments it varies widely; the ratio of the computed to the true velocity ranging from 0.63 to 1.24, or, nearly, as 1 to 2. The authors apparently met with difficulty in reconciling these discrepancies, which led them to conclude that the "method of tank rating is not the proper one to apply." Mr. Frizell.

This method has been used by the writer, in rating current meters, with very satisfactory results. In that case the rotation was accomplished by a weight attached to a string, which passed over a fixed pulley and was led to a pulley on the central shaft above the head of the observer, who sat at the center of rotation, as in the present case. Each turn of the meter, in that case, was marked by a slight click, which was transmitted to the observer through a tense wire passing near his ear. In that case the meter traveled exactly 100 ft. at each revolution, and it was found advisable to use only one revolution at a time, on account of the movement communicated to the water by the passage of the meter.

The present case appears to involve a source of error from which the case referred to was free. It is entirely presumable that the error was perceived and allowed for, but as no mention of it is to be found in the paper, the writer ventures to refer to it. The Pitot tube appears to be connected with a glass tube at the center of rotation by means of a flexible pipe, the glass tube resting against a gauge and a partial vacuum formed over the water to raise it to the range of observation. The rotation of such a system would develop centrifugal force in the connecting pipe which would entirely mask the indication of the Pitot. A pipe filled with water and rotating around a vertical intersecting axis will develop a centrifugal force at any point exactly equal to the head due the velocity. If this source of error was considered and allowed for, we would naturally expect to find some reference to it in the paper.

There is strong reason to think that the maximum velocity in a pipe could never be measured by a Pitot tube, *i. e.*, by a single point fixed in position.

In an open channel, the thread of maximum velocity is not stationary, but oscillates through a wide range of movement about its normal position. This fact is very distinctly discernible by means of the form of meter just referred to, viz., a meter which makes a slight

Mr. Frizell. click at each turn, said click being transmitted to the observer's ear through the metallic rod on which the instrument is mounted. The first peculiarity to strike the observer, on applying his ear to such a rod, is the varying rate of revolution of the current wheel. There is no reason to doubt a similar oscillation of the thread of maximum velocity in the closed pipe. This feature of the movement cannot be detected by the Pitot tube as ordinarily constructed. The point is necessarily small, compared with the caliber of the tube, and the maximum velocity does not act long enough to disclose itself through the elevation of water in the latter. The apparent maximum, as determined by the tube, is an average of the true maximum combined with slower velocities.

M. Bazin. M. HENRY BAZIN* (by letter).—The writer has read this memoir with much interest, especially as he has made, quite recently, some experiments to determine the distribution of velocities in pipes, the results of which are discussed in a memoir to the Academy of Sciences of Paris, presented herewith, and from which it will appear that his determinations correspond very well with those obtained by the authors under normal conditions of flow.

Concerning the principal object of the investigations, the Resistance of Curves, the results of the authors' work will surprise many persons, but though they are contradicted apparently by admitted ideas, is it not possible to explain away the contradiction in the following manner?

The experiments upon the resistance introduced by curves that have been heretofore presented are few, and are confined to pipes of small diameter and mainly abrupt turns, in which the phenomena accompanying the sudden change of direction, are supposed to be analogous to the effects of a contraction, and the resistances are assumed to be proportional to the square of the velocity and inversely to the radius of curvature. In reality, very little has been known of what actually takes place, and, in practice, the subject is dismissed as involving unnecessary calculation. The conditions obtaining in the experiments of the authors are far from those indicated above, their pipes being of considerable diameter, and the curves laid out upon long radii. When the water enters these curves, it receives no sudden change of direction, as in the case of sharp bends, but the distribution of the velocities is modified as the stream progresses, and ultimately becomes quite different from the condition of straight pipe. On leaving the curve, again, the fluid requires some time for the resistances to re-establish the conditions of straight pipe, or the normal distribution of velocities. Hence, the work of the internal resistances has a greater and greater effect the more the arc increases, as is shown by these experiments.

* Inspecteur-Général des Ponts et Chaussées. This discussion has been translated by Gardner S. Williams, M. Am. Soc. C. E., and has been revised by M. Bazin.

Whatever else may be said, these investigations are of great importance, and it is hoped they may be extended further. The differential gauge of two liquids permitting a more accurate study of the losses of head, if several of them should be connected to a series of points along the curve, it should be possible to get a fair idea of what takes place, especially if the work could be done upon an experimental conduit undisturbed by the exigencies of the maintenance of a municipal supply. M. Bazin.

"EXPERIMENTS UPON THE DISTRIBUTION OF VELOCITIES IN PIPES."* BY M. HENRY BAZIN.

"When water flows through closed circular conduits it is known that the velocities are not the same throughout the whole extent of the transverse area, but that they decrease regularly from the center to the circumference. The first experiments undertaken to determine the law of this decrease were those of M. Darcy, made in 1850. They were made at the Chaillot Water-Works Station, in Paris, and are 22 in number, giving 19 for pipes 0.19, 0.24 and 0.30 m. diameter (old caliber 7, 9 and 11 pouces, or, in English measure, 7.48, 9.45 and 11.81 ins.), and 3 for one of 0.50 m. diameter, or 18.69 ins.† The velocity was measured at five points on the vertical diameter of the pipe, to wit: At the center, and at four points symmetrically situated above and below it at distances of one-third and two-thirds of the radius. From these experiments M. Darcy deduced the relation:

$$V - v = 11.3 \frac{r^{\frac{3}{2}}}{R} \sqrt{I}$$

Where V = the maximum velocity, at the center; v the velocity at a distance r from the center; R the radius of the pipe; and I the loss of head per meter. If, to consider ratios only, this equation be divided by U, the mean velocity, and the coefficient $\frac{R\,I}{U^2}$ be designated by b, then

"(1) $$\frac{V - v}{U} = 11.3 \sqrt{b} \left(\frac{r}{R}\right)^{\frac{3}{2}}$$

"Some years later M. Darcy began an extended series of new experiments upon open channels, but his premature death, which snatched him away from science at the beginning of 1858, prevented his finishing them. It was, however, the rare good fortune of the writer to have been associated with M. Darcy almost from the beginning, and it fell to him to pursue and fulfill the programme as originally planned. That programme comprehended the study of the distribution of velocities in canals of various sections and especially in the semi-circular canal.

"By regulating the discharge of the latter canal, so that the surface of the flowing water was exactly at its horizontal diameter, its conditions might be likened to those of a closed circular pipe of the same radius. These experiments were made in 1857 and 1858, when there were built four canals ranging in diameter from 1.20 to 1.40 m.,

* *Mémoires de l'Académie des Sciences de Paris, Recueil des Savants Étrangers*, Tome xxxii.

† M. Darcy considered that, on account of the disposition of the apparatus, the last three are not strictly comparable with the others, and do not meet the conditions of a normal regimen.

M. Bazin. with a uniform slope of 0.0015 m. per meter, and having various surfaces (boards, smooth cement, cement and fine sand, and cement mixed with fine gravel). The velocities were measured, not alone at five points on the central vertical, but at a great number of points distributed upon concentric circles and approaching the walls as closely as possible.* The variation of the velocities was thus determined throughout the entire extent of the cross-section, and showed immediately that, beyond two-thirds of the radius, *i. e.*, in the region not investigated in 1850, the velocities diminished more rapidly than was indicated by the formula. The discussion of those experiments led the writer to the substitution of a new expression:

"(2) $$\frac{V-v}{U} = 21\sqrt{b}\left(\frac{r}{R}\right)^3.$$

"The curves, $O\,A$ and $O\,B$, Fig. 103, represented by Equations (1) and (2), using for ordinates the ratio $\frac{r}{R}$, and for abscissas $\frac{V-v}{U\sqrt{b}}$, intersect at a point the abscissa of which is $\frac{r}{R} = 0.662$, or a little less than two-thirds of the radius. Between that point and the origin their greatest divergence is at an abscissa of 0.417, and has a value of 1.52. Beyond their point of intersection the curves diverge rapidly. The new results appeared contradictory to those of M. Darcy, obtained in 1850. He had then, indeed, only experimented upon pipes of small diameter, for, excluding the three observations on the largest pipe, considered as hardly comparable with the others, the largest diameter he used was 0.30 m. (less than 1 ft.), wherein the measurement of velocities near the walls would have been exceedingly difficult. On the other hand, the question may be raised whether there is a complete analogy between a semi-circular open canal and a circular closed pipe.

"Some further experiments seemed necessary, which should be made upon a closed conduit of such increased diameter as to permit the multiplication of the points of observation, especially throughout the region beyond two-thirds of the radius. Such experiments require a considerable time, and, consequently, a large quantity of water, and, therefore, it was not possible to attempt them, under the same conditions as those of 1850, since the dependence for water upon one of the laterals of the Paris distribution system rendered their execution incompatible with the demands of the supply to the city. It has, however, at last been possible for the writer to carry out such experiments by utilizing the canal at Dijon where the investigations of the flow of water over weirs were made.†

"A cement pipe, 0.80 m. in diameter and 80 m. long, was constructed upon the bed of the canal. This pipe was straight, perfect in bore, and opened at its extremities into two basins, 2 m. wide and 15 m. long, having vertical walls (Figs. 96, 98 and 99). The upper basin was supplied with water from the Canal de Bourgogne; the lower basin received the discharge of the pipe, and delivered it through a sluiceway 0.80 m. wide, which was furnished with stop planks, thus enabling the surface of the water in the basin to be kept at any desired height. After passing the sluiceway the water was measured over a weir 50 m. below. The elevation of the water surface in each basin, which was kept uniform during an experiment, was in-

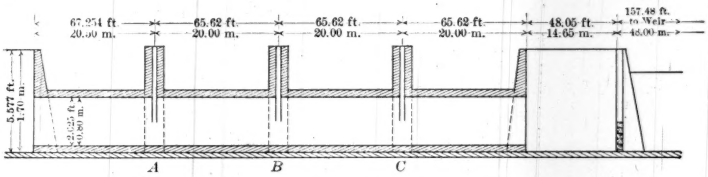
* The number of points amounted to 80 for the canal in smooth cement, 63 for that in cement and sand, 82 for that in planks, and 92 for the canal in gravel.

† *Annales des Ponts et Chaussées*, October, 1888.

dicated by the apparatus used for a similar purpose in the weir investigation.* M. Bazin.

"In order to measure the velocities throughout the cross-section of the pipe, it was necessary to introduce a rigid instrument of as little volume as possible; for the purpose of admitting the instrument, there were constructed three shafts opening the full width of the pipe, 0.80 m., and 0.08 m. in length, along the axis of the pipe (Figs. 96, 98 and 99). These divided the length of the pipe into four equal parts, the one at the middle being 40 m. from the extremities of the pipe and the others each 20 m. On the right of each shaft was installed a manometer, consisting of a glass tube with a scale, the former being connected to the interior of the pipe by an orifice 0.002 m. in diameter, pierced in the wall, without any protuberance.

"The apparatus used to measure the velocities was a modification of the gauging tube of M. Darcy; but this tube, constructed at the time of the experiments of 1857 to operate in a shallow current, was not immediately applicable to a pipe 0.80 m. in diameter, on account of the great distances to be passed through between the ajutages and the end of the stem, because, the support of the latter being 1.60 m. above the ajutages, the instrument would not present sufficient rigidity. It became necessary, therefore, to support the instrument at both its extremities, just as M. Darcy had done with the one in the iron pipes in 1850; but as he was operating only within the central vertical, it was unnecessary to displace the stem horizontally in his case, while in the 0.80 m. diameter conduit it was desired to explore the whole section. The difficulty was very satisfactorily overcome by M. Hegly;† the apparatus which he devised, and which will be next described, permitting the ajutages to be placed at any point whatever in the cross-section of the pipe.



LONGITUDINAL SECTION OF EXPERIMENTAL CONDUIT

FIG. 96.

"The ajutages of the old Darcy tube were soldered to the end of a stem sliding by a collar along a blade of brass 0.004 m. thick, beveled on its front edge (Fig. 100). The blade formed the lower part of a vertical bar carried by a collar, through which it could slide, and which could itself be moved along a horizontal bar placed across the pipe and bearing a scale. By sliding the collar along the horizontal bar, and at the same time moving the vertical bar in its collar, the latter was brought to the desired vertical, when, the bars being fixed, the ajutages were made to run over this vertical by sliding the stem on the blade. In order to secure the foot of the brass blade serving as a guide to the ajutages, it extended to the bottom of the pipe, where it was connected by a knuckle joint comprising two cups terminating the blade which imprisoned a ball that was fastened to a steel tape run-

* For description and use of this apparatus, dial floats and movable point gauge, see the memoir already cited, *Annales des Ponts et Chaussées*, October, 1888.

† Conducteur des Ponts et Chaussées.

M. Bazin. ning in an annular groove (Fig. 99); this groove, cut in the wall of the pipe, was closed by two copper bands, leaving a very small space between them; the steel tape at its exit from the groove arose vertically to the top of the shaft where it passed over a dial pulley and fell finally outside, being held in place by a counterweight.

"Four scales, conveniently placed, measured the displacements of the bar, the ajutages and the tape. To explain the use of this apparatus, suppose the bar placed so that the extreme or point orifice is at the center of the pipe; the exploration of that vertical requires simply the maneuvering of the stem carrying the ajutages. To pass to another vertical, the collar on the horizontal bar is moved horizontally, and, at the same time, the pulleys are turned so that the tape describes the necessary arc to keep the blade plumb, the vertical bar sliding in the collar on the horizontal bar and carrying the ajutages up to an amount equal to the versed sine of the arc, which must be compensated for by sliding them on the blade an equal distance downward by means of the stem, which places them always, when in the horizontal diameter, at a constant distance below the cross-bar, measured on the stem. The location being thus checked, the measuring of velocities follows as in the center vertical.

"In order to lighten the instrument it was separated from the glass gauge, which latter was mounted upon a post at the side of the pipe (Fig. 98), and was connected to the ajutages by rubber hose of 0.002 m. interior diameter and 0.0025 m. in thickness, sufficient to prevent them from being kinked over the top of the shaft. The gauge board carried, in addition to the two glass tubes connected to the adjutages, a U-tube gauge, designed to measure the partial vacuum determined by the suction on the apparatus. When the partial vacuum had been made, with the apparatus for that purpose, a valve communicating with the manometer was opened and communication established between the two parts of the apparatus. The inequality of level between the two columns of the U-tube gauge showed the height by which the water columns had been raised by the suction, above the level they would have reached in free communication with the atmosphere.*

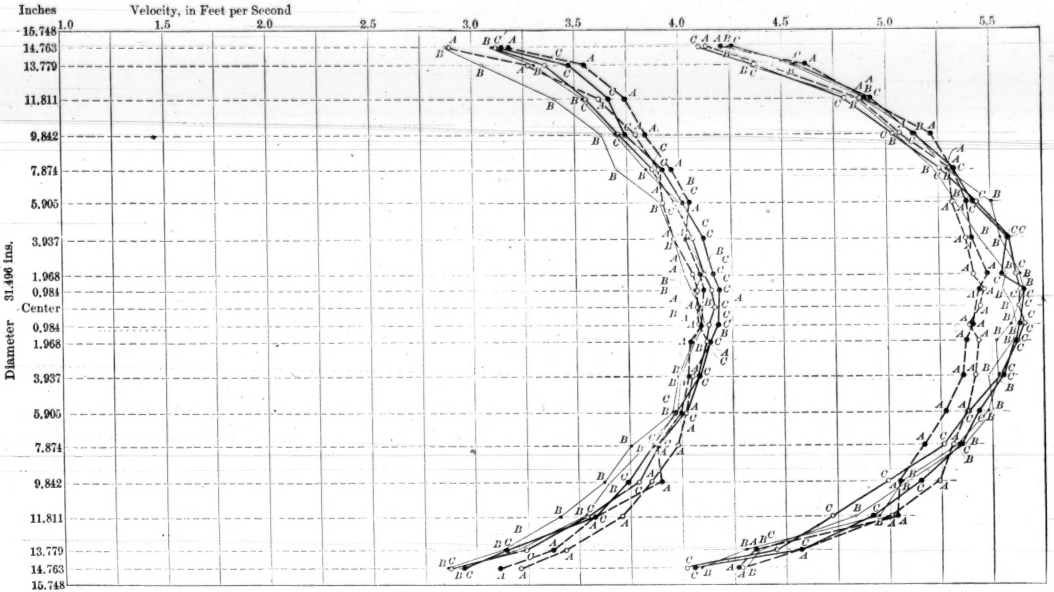
"*Determination of the Coefficient b.*—A series of preliminary experiments was made to determine the coefficient *b*. Discharges varying from 400 liters to 1 cu. m. per second were caused to flow through the pipe, the volume delivered being measured over a weir 2.01 m. wide, having a sharp edge, without end contractions, located 50 m. beyond the lower basin, of which, the coefficient of discharge *m*, had been perfectly established in the previous experiments upon weirs.† The data from which *b* has been deduced is presented in Table No. 57. The water levels of the piezometric columns in the three manometers and that of the lower basin are referred to the surface of the upper basin as a datum. The slope *I*, per running meter, has been obtained by dividing the difference between the heights of the piezometric columns, *A* and *C*, by their distance apart, 40 m.

"The coefficient *b* is practically constant, and the mean value for the ten experiments is 0.000332.

"The total loss of head *H* through the upper and lower basin is not the result of the resistances through the pipe alone, but also those due to the sudden changes of section at entry and exit. An approxi-

*The inequality of the two columns of the U-tube gauge enabled the obtaining of the absolute pressures existing at each point in the section of the pipe, and it was found that these pressures varied hydrostatically.

†For a table of these coefficients, see *Annales des Ponts et Chaussées*, October, 1888.



• = POINTS IN HORIZONTAL TRAVERSES:
○ = POINTS IN VERTICAL TRAVERSES;
LETTERS SHOW LOCATION OF TRAVERSE.

FIG. 97.

M. Bazin.

"TABLE No. 57.—DETERMINATION OF b.

Number of Experiment.	Head upon the weir, h.	Coefficient of discharge for weir, m.	Discharge per second, $Q = m\,l\,h\sqrt{2\,g\,h}$.	Corresponding mean velocity through pipe, $U = Q \div \pi R^2$.	Elevation of water surface in Upper Basin, above axis of pipe.	Differences of Head between Upper Basin and: Piezometer A.	Piezometer B.	Piezometer C.	Lower Basin H.	Difference i between the heads at Piezometers A and C.	Loss of head per running meter through the pipe, $I = \frac{i}{40}$.	Coefficient, $b = \frac{R\,I}{U^2}$.
	Meters.		Cubic meters.	Meters.	Meters.	Meters.	Meters.	Meters.	Meters.	Meters.		
1	0.468	0.4432	1.0994	2.008	0.998	0.342	0.404	0.473	0.469	0.131	0.003275	0.003325
2	0.372	0.4413	0.8914	1.773	1.003	0.262	0.305	0.365	0.365	0.103	0.002575	0.003328
3	0.334	0.4392	0.7548	1.501	0.800	0.185	0.219	0.259	0.260	0.074	0.001850	0.003328
4	0.328	0.4380	0.7340	1.460	0.802	0.178	0.207	0.248	0.246	0.070	0.001750	0.003328
5	0.325	0.4387	0.7236	1.439	1.001	0.172	0.202	0.241	0.244	0.069	0.001725	0.003328
6	0.301	0.4374	0.6431	1.279	1.003	0.135	0.159	0.190	0.193	0.055	0.001375	0.003336
7	0.275	0.4362	0.5600	1.114	0.802	0.103	0.122	0.145	0.148	0.042	0.001050	0.003338
8	0.265	0.4357	0.5201	1.053	1.000	0.090	0.105	0.127	0.129	0.037	0.000925	0.003334
9	0.242	0.4346	0.4606	0.916	0.798	0.071	0.081	0.098	0.100	0.027	0.000675	0.003322
10	0.230	0.4340	0.4261	0.848	1.000	0.057	0.067	0.082	0.084	0.025	0.000625	0.003348

TRANSVERSE SECTION THROUGH GAUGING SHAFT

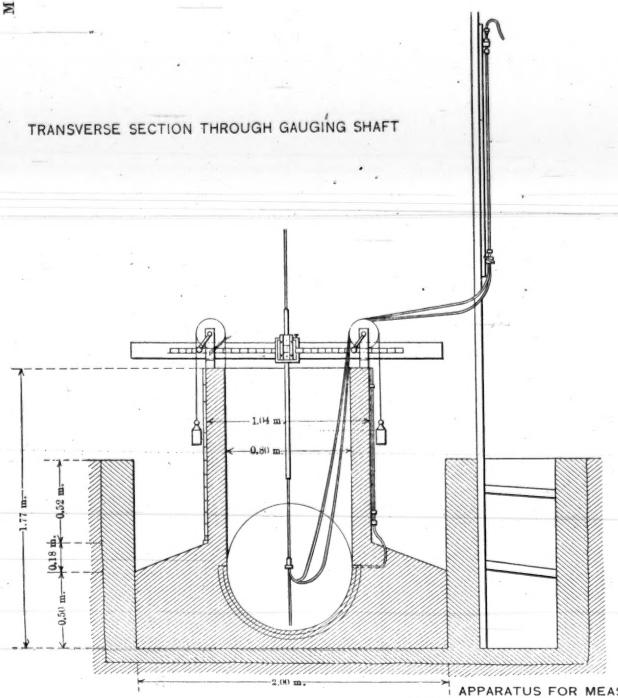


FIG. 98.

LONGITUDINAL SECTION THROUGH GAUGING SHAFT.

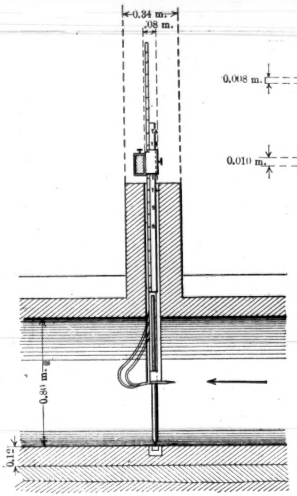
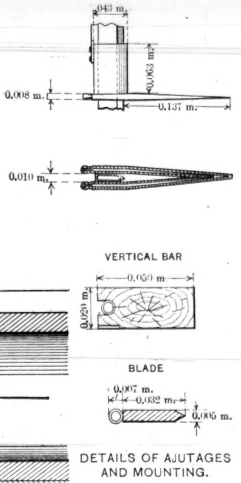


FIG. 99.



DETAILS OF AJUTAGES AND MOUNTING.

FIG. 100.

APPARATUS FOR MEASURING VELOCITY
IN EXPERIMENTS OF H. BAZIN, ALL DIMENSIONS IN METERS.

M. Bazin. mate computation of the relative importance of these elements may be made, upon the assumption that the loss of head per meter for the whole conduit coincides with that for the section of 40 m. between A and C. For example: In Experiment No. 1, the slope from A to C was 0.131 m., and on this basis, from the entrance to A, and from C to the outlet, a distance of 20 m. each, it would be 0.0655 m. each. The fall from the upper basin to Manometer A was 0.342 m., and the portion chargeable to resistances at entrance is therefore 0.342 m. — 0.0655 m. = 0.2765 m. Similarly at the outlet, the loss from C to the outlet being 0.0655 m., added to the loss to C gives 0.0655 m. + 0.473 m. = 0.5385 m. The total fall, H, is only 0.469 m., and therefore there has resulted a recovery of head, between C and the lower basin, of 0.5385 m. — 0.469 m. = 0.0695 m. Similar computations for all the experiments are presented in Table No. 58.

"TABLE No. 58.

Number of Experiment.	Mean velocity through the Pipe, U.	$\frac{U^2}{2g}$	Loss of Head: At entrance, h_1	Loss of Head: Through pipe, h_2	Recovery of head at efflux h_3	Total fall between the two basins, $H = h_1 + h_2 - h_3$	Ratios: $\frac{h_1}{H}$	Ratios: $\frac{h_2}{H}$	Ratios: $\frac{h_3}{H}$	Ratios: $h_1 : \frac{U^2}{2g}$	Ratios: $h_2 : \frac{U^2}{2g}$	Ratios: $\frac{U}{\sqrt{2gH}}$
	Meters per second.		Meters.	Meters.	Meters.	Meters.						
1..	2.008	0.2055	0.2765	0.262	0.0695	0.469	0.590	0.559	0.148	1.345	0.338	0.662
2..	1.773	0.1602	0.2105	0.206	0.0515	0.365	0.577	0.564	0.141	1.314	0.321	0.663
3..	1.501	0.1148	0.1480	0.148	0.0360	0.260	0.569	0.569	0.138	1.289	0.313	0.665
4..	1.460	0.1087	0.1430	0.140	0.0370	0.246	0.581	0.569	0.150	1.316	0.341	0.665
5..	1.439	0.1056	0.1375	0.138	0.0315	0.244	0.564	0.566	0.129	1.303	0.298	0.658
6..	1.279	0.0834	0.1075	0.110	0.0245	0.193	0.557	0.570	0.127	1.289	0.294	0.657
7..	1.114	0.0633	0.0820	0.084	0.0180	0.148	0.554	0.568	0.122	1.296	0.285	0.654
8..	1.053	0.0565	0.0715	0.074	0.0165	0.129	0.554	0.574	0.128	1.265	0.292	0.662
9..	0.916	0.0428	0.0575	0.054	0.0115	0.100	0.575	0.540	0.115	1.344	0.269	0.654
10..	0.848	0.0367	0.0445	0.050	0.0105	0.084	0.530	0.595	0.125	1.214	0.286	0.661
						Means.	0.565	0.567	0.132	1.298	0.304	0.660

"The proportion of the head, H, in h_1, h_2, h_3, is practically the same in all the experiments, and it may therefore be considered that the three heads vary but little from proportionality to U^2. The loss at entry is equal to $1.30 \frac{U^2}{2g}$ (neglecting the effect of velocity in the channel of approach).* The discharge through the pipe, which is only

*The velocity through the channel of approach, in which the water flowed regularly and without disturbance, did not exceed 0.33 m.; this was not the case, however, in the lower basin, where the water was considerably agitated and flowed out of the pipe and through the sluiceway at the lower end of the basin with quite a high velocity; therefore, the coefficient 0.30, of recovery obtained in the table is not capable of a theoretically precise interpretation.

The coefficient 1.30 applied at entry would certainly be a little higher if the contraction were complete, which was not the case at the bottom where the pipe was only 0.12 m. above the bed of the channel, Fig. 99. The side services, which were distant only 0.40 m. from the edge of the pipe, could have exerted a similar influence, though in a proportion much less important.

dependent upon the difference of head, H, between the two basins, may be represented by $U = m^1 \sqrt{2\,g\,H}$, and from Table No. 58 the coefficient, m^1, is practically constant, with a mean value of 0.660. This has, therefore, been used as a means of determining the discharge through the pipe during the experiments upon the distribution of velocities. The area of a normal section, 0.80 m. in diameter, is 0.5027 sq. m., whence the discharge, for a difference of surface height, H, is $Q = 0.660 \times 0.5027 \sqrt{2\,g\,H} = 0.3317 \sqrt{2\,g\,H}$. M. Bazin.

"*Experiments upon the Distribution of Velocities.*—Passing now to the experiments upon the distribution of velocities throughout the interior of the same section of pipe, observations were made at three sections preceding the manometers, A, B and C, and at each of these with two different discharges. The velocities were measured upon two diameters at right angles to each other, one vertical and the other horizontal, and for each the velocity was measured at points 0.025, 0.05, 0.10, 0.15, 0.20, 0.25, 0.30, 0.35 and 0.375 m. from the center, and at the center, making 37 points of observation. That number was augmented at B, for the higher discharge, to 113, the points being distributed along concentric circles.* In order to determine as nearly as possible the areas of the three sections of the pipe, they were measured at each location on a number of diameters; from these measurements mean diameters were obtained: of 0.798 m. at A, 0.802 m. at B, and 0.800 m. at C. The discharges deduced from the difference of level in the two basins H for the six experiments are shown in Table No. 59.

"TABLE No. 59.—MEAN VELOCITIES.

Section.	H.	$Q = 0.3317 \sqrt{2\,g\,H}$.	Section of pipe, F.	Mean velocity through the section, $\frac{Q}{F}$
	Meters.	Cubic meters.	Square meters.	Meters.
Section A.....	0.148	0.5652	0.5001	1.130
	0.260	0.7491	0.5001	1.498
Section B.....	0.136	0.5418	0.5052	1.072
	0.258	0.7462	0.5052	1.477
Section C.....	0.143	0.5556	0.5027	1.105
	0.259	0.7477	0.5027	1.487

"Table No. 68 shows, for each point in the horizontal and vertical diameters, the velocities deduced from the difference of level, δ, of the two columns of the tube gauge, by assuming that the coefficient of the instrument is unity, when there results:

$$V = \sqrt{2\,g\,\delta}\,\dagger$$

* A similar experiment was made at Station B with the smaller discharge, and, similarly, 113 points were observed. It will be seen further on why they have been rejected: the loss of head between the two basins was $H = 0.148$ m., the discharge $Q = 0.3317 \times \sqrt{2\,g\,H} = 0.5652$, and, consequently the mean velocity through the section was $U = \frac{0.5652}{0.5052} = 1.119$.

† Fig. 97 shows the observations on the horizontal and vertical diameters plotted in a manner similar to that adopted by the authors of the paper.—G. S. W.

M. Bazin. "Fig. 101 shows the velocities for the experiment at B with the higher discharge, when 113 points were observed.

"Calculating, by means of these data, the mean velocity for each of the concentric circles, Table No. 60, is obtained.

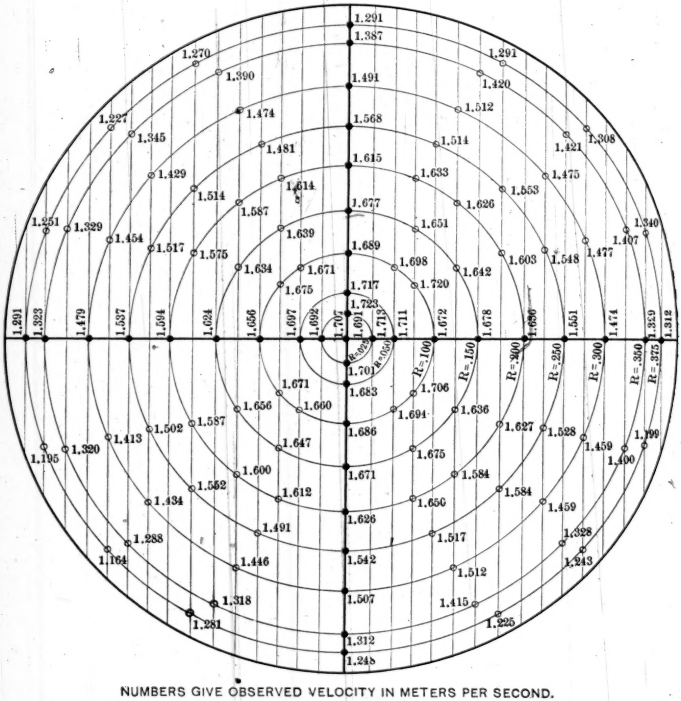


FIG. 101.

"Before proceeding further, it is necessary, by way of verification, to determine, by the aid of the local velocities, the value of the mean velocity of the whole section, and compare it with that which has been determined directly from H. This determination is made by multiplying the area $\pi (r_2^2 - r_1^2)$ of the ring between two successive circles, r_1 and r_2, by the mean, $\frac{u_1 + u_2}{2}$, of the velocities for each of the two circles and dividing by the area, πR^2, of the whole section, the sum of the partial discharges.

"TABLE No. 60.—MEAN VELOCITIES OBSERVED FOR EACH CONCENTRIC CIRCLE. M. Bazin.

Radius of Circle.	SECTION *A*.		SECTION *B*.		SECTION *C*.	
	1st discharge.	2d discharge.	1st discharge.	2d discharge.	1st discharge.	2d discharge.
	Meters.	Meters.	Meters.	Meters.	Meters.	Meters.
Center*.....	1.249	1.665	1.246	1.706	1.271	1.721
0.05.....	1.246	1.665	1.239	1.702	1.261	1.708
0.10.....	1.227	1.644	1.219	1.683	1.247	1.696
0.15.....	1.218	1.626	1.208	1.652	1.219	1.652
0.20.....	1.199	1.609	1.153	1.611	1.185	1.616
0.25.....	1.173	1.568	1.113	1.531	1.140	1.548
0.30.....	1.117	1.510	1.060	1.468	1.092	1.477
0.35.....	1.039	1.368	0.966	1.358	1.007	1.369
0.375.....	0.945	1.288	0.896	1.255	0.912	1.254
0.40.....	0.820	1.130	0.805	1.080	0.795	1.095
Perimeter.						

*The velocity adopted for the center is the mean of five points observed, one at the center and four at a distance of 0.025 m. from it. The velocity at the perimeter is determined approximately by the graphical process of drawing upon a sufficient scale the curve with radii as abscissas and velocities as ordinates and prolonging it by judgment to an abscissa of $r = 0.40$ m.

"TABLE No. 61.—COMPARISON OF TOTAL MEAN VELOCITIES.

Designation.	MEAN VELOCITIES.		Ratio, $\frac{U}{U^1}$.
	By local velocities, U.	By total slope, H, U^1.	
A. 1st discharge.....	1.108	1.130	0.981
2d "	1.486	1.498	0.992
B. 1st "	1.060	1.072	0.989
2d "	1.469	1.477	0.995
C. 1st "	1.087	1.105	0.984
2d "	1.478	1.487	0.984

"These new velocities are all a little less than the former ones, the mean ratio being, for the lower discharge 0.985, and for the higher 0.994.*

*The same calculation applied to the experiment at *B*, which it has been thought should be rejected, gave $U = 1.065$ m. and $\frac{U}{U^1} = \frac{1.065}{1.119} = 0.952$. This ratio seemed to indicate that, from some accidental cause, the observed velocities must have been about 4% below the recognized values. Concerning this, it must be acknowledged that this experiment was made at the beginning of the work and, the rubber tubes being very flexible, their section was contracted in passing out of the shaft, so that they transmitted imperfectly the pressures from the ajutages; these were replaced by thicker tubes for the execution of the remaining experiments, thus avoiding the difficulty. It is to be remarked, in passing, that the values obtained from this first experiment were very little different from the rest, as if the true velocities had been altered in a nearly constant ratio. The values are as follows:

r.....	0	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.375	0.40
v.....	1.251	1.243	1.225	1.196	1.166	1.122	1.063	0.973	0.901	0.805
$\frac{v}{U}$....	1.175	1.167	1.150	1.123	1.095	1.054	0.998	0.914	0.846	0.756

M. Bazin. "The method of calculation used tends, it is true, to reduce the velocities a very little, but this cause of error is quite insignificant, and it seemed wise to search for the explanations of the difference which is established, either in the indeterminateness of the coefficient 0.660, or rather through the indeterminateness of the coefficient used for the gauging tube. It has been supposed, in effect, that the coefficient by which to multiply $\sqrt{2\,g\,\delta}$ to obtain the velocity is unity; but it is possible, by reason of the oblique direction in which the fluid fillet may have encountered the lateral orifice placed in the surface parallel to the current, that the value of this coefficient may exceed unity by a few thousandths.

"Whatever may be the cause, as the difference is fortunately of little importance, a comparison of the velocities may more safely be made among themselves, for which they are divided by the mean velocity deduced from the observation of the same velocities, and not from those results established by formulas for application to other conditions. Dividing through all the values of Table No. 60 by U there are obtained the values of the ratio $\frac{v}{U}$, shown in Table No. 62.

"TABLE No. 62.—VALUES OF THE RATIOS, $\frac{v}{U}$.

Radius of Circle, r.	Ratio, $\frac{r}{R}$	Section A.		Section B.		Section C.		Mean for each section.		
		1st Discharge.	2d Discharge.	1st Discharge.	2d Discharge.	1st Discharge.	2d Discharge.	Section A.	Section B.	Section C.
Meters.										
Center	0	1.127	1.124	1.175	1.161	1.169	1.165	1.1205	1.1680	1.1670
0.05	0.125	1.124	1.114	1.169	1.158	1.160	1.155	1.1190	1.1635	1.1575
0.10	0.250	1.107	1.106	1.150	1.146	1.147	1.147	1.1065	1.1480	1.1470
0.15	0.375	1.100	1.094	1.140	1.124	1.121	1.118	1.0970	1.1320	1.1195
0.20	0.500	1.082	1.083	1.088	1.097	1.090	1.094	1.0825	1.0925	1.0920
0.25	0.625	1.059	1.055	1.050	1.042	1.049	1.048	1.0570	1.0460	1.0485
0.30	0.750	1.008	1.016	1.000	0.999	1.005	0.999	1.0120	0.9995	1.0020
0.35	0.875	0.938	0.921	0.911	0.925	0.926	0.926	0.9295	0.9180	0.9260
0.375	0.937	0.853	0.867	0.845	0.854	0.839	0.848	0.8600	0.8495	0.8435
Perimeter	1.000	0.740	0.760	0.759	0.735	0.731	0.741	0.7500	0.7470	0.7360

"On inspection of Table No. 62 it is seen that the values of the ratio $\frac{v}{U}$ are quite concordant for the four experiments made at Sections B and C, but, for Section A, they are appreciably different; being less, to the extent of 0.05, for the central region, and more, by about 0.01, for the region of the perimeter. The reason of this difference is apparent at once, for, because of the contraction which takes place at the entry of the pipe, the ratios of the velocities do not have the same values that exist further on when a uniform regimen is established; this readjustment is progressive, and is but imperfectly realized when the fillets reach Section A, only a distance of 20 m. from the entrance; the equality of the ratios for Sections B and C shows that the velocities are readjusted upon reaching Section B. It is therefore Sections B

and C that furnish data of the true distribution of velocities for the conditions of regular flow. Taking the means of the ratio $\frac{v}{U}$ of the four experiments at these two sections, there results, in the first place, for the central velocity, $\frac{V}{U} = 1.1675$; dividing the differences, $\frac{V-v}{U}$, by $\sqrt{b} = 0.0182$ and comparing the values thus obtained with those of Formulas (1) and (2) there results Table No. 63. M. Bazin.

"TABLE No. 63.—COMPARISON OF EXPERIMENTAL VALUES OF $\frac{V-v}{U\sqrt{b}}$ WITH FORMULAS (1) AND (2).

$\frac{r}{R}$	Experimental Values of $\frac{v}{U}$.	$\frac{V-v}{U}$	$\frac{V-v}{U\sqrt{b}}$. by Experiment.	by Formula (1)	by Formula (2)	Differences. Formula (1)	Formula (2)
0	1.1675	0	0	0	0	0	0
0.125	1.1605	0.0070	0.38	0.50	0.04	+ 0.12	−0.34
0.250	1.1475	0.0200	1.10	1.41	0.33	+ 0.31	−0.77
0.375	1.1358	0.0417	2.29	2.59	1.11	+ 0.30	−1.18
0.500	1.0923	0.0752	4.13	4.00	2.63	− 0.13	−1.50
0.625	1.0473	0.1202	6.60	5.58	5.13	− 1.02	−1.47
0.750	1.0008	0.1667	9.16	7.34	8.86	− 1.82	−0.30
0.875	0.9220	0.2455	13.49	9.25	14.07	− 4.24	+0.58
0.937	0.8465	0.3210	17.64	10.26	17.30	− 7.38	−0.34
1.000	0.7415	0.4260	23.41	11.30	21.00	−12.11	−2.41

"Table No. 63 shows that neither formula represents, through the entire series, the results of experiment. That of M. Darcy applies well to the central portion, but diverges rapidly when r exceeds $\frac{3}{4} R$; Formula (2), on the other hand, fits well the outer region and gives too low values for the center. To render the results easy of comprehension, the experimental values are represented graphically, with the values of $\frac{r}{R}$ as abscissas and those of $\frac{V-v}{U\sqrt{b}}$ for ordinates, in Fig. 103, with the two curves $O\ A$ and $O\ B$, representing Formulas (1) and (2). The figure shows well that, for values of $\frac{r}{R}$ below 0.6, Formula (1) is applicable, while above that limit the divergence is rapid, and then Formula (2) may be better substituted; the latter shows better the character of the general phenomena, except for the central region, where it furnishes values of $\frac{V-v}{U\sqrt{b}}$ a little too small. It is now proposed to see if a corrective term can be found to apply to the second member to bring the results more closely in harmony with experiment. To obtain a first approximation, a term

$$\lambda = \alpha\sqrt{b}\left(\frac{r}{R}\right)^2\left(1-\beta\frac{r}{R}\right)^2$$

Mr. Bazin. is introduced, and there results:

"(3) $$\frac{V-v}{U} = \sqrt{b}\left\{21\left(\frac{r}{R}\right)^3 + \alpha\left(\frac{r}{R}\right)^2\left(1-\beta\frac{r}{R}\right)^2\right\}.$$

"The values of α and β, by reason of the smallness of λ, can vary quite appreciably without notably affecting that of $\frac{v}{U}$. Those which appear the most accordant are $\alpha = 27$, and $\beta = 1.1$; but results are obtained but little different with $\alpha = 21$ and $\beta = 1$. The formula, being thus considerably simplified, becomes, on reduction:

$$\frac{V-u}{U} = 21\sqrt{b}\left\{\left(\frac{r}{R}\right)^3 + \left(\frac{r}{R}\right)^2\left(1-\frac{r}{R}\right)^2\right\}$$
$$= 21\sqrt{b}\left\{\left(\frac{r}{R}\right)^2 - \left(\frac{r}{R}\right)^3 + \left(\frac{r}{R}\right)^4\right\}$$

"Whatever values are adopted for α and β, Formula (3) does not well fit the points near the perimeter, where it furnishes always for $\frac{V-v}{U\sqrt{b}}$ a value a little above 21, while that from experiment is 23.41; this last quantity, it is true, does not result directly from experiment, but simply from prolonging the curve by judgment; nevertheless, it is clear from inspection of the figure that the true value is above 23. Useful results may now be obtained either by adopting a value of γ very complex in form, or abandoning the expression $21\left(\frac{r}{R}\right)^3$ as the first approximation, and looking for another totally different empiric formula. The expression

"(4) $$\frac{V-v}{U} = \alpha\sqrt{b}\left[1-\sqrt{1-\beta\left(\frac{r}{R}\right)^2}\right],$$

which corresponds graphically to the arc of an ellipse, applies better near the perimeter, and making $\alpha = 29.5$ and $\beta = 0.95$, gives $\frac{V-v}{U\sqrt{b}} = 22.90$ at the wall. On the outside of this special region, these different formulas follow very closely, as far as can be judged from the graphical representation adopted with Formulas (1) and (2); dividing by $\sqrt{b}$ and replacing α and β by their numerical values, the formulas can be written:

"(5) $$\frac{V-v}{U\sqrt{b}} = 21\left(\frac{r}{R}\right)^3 + 27\left(\frac{r}{R}\right)^2\left(1-1.10\frac{r}{R}\right)^2.$$

"(6) $$\frac{V-v}{U\sqrt{b}} = 29.5\left[1-\sqrt{1-0.95\left(\frac{r}{R}\right)^2}\right].$$

"The two curves, $O\,C$ and $O\,D$, corresponding to these equations, are shown in Fig. 103, in which each may be merged with that of Formula (2) between abscissas of $\frac{r}{R} = 0.75$ and $\frac{r}{R} = 0.95$. From there, Formula (6) separates from the two others.

"Continuing to examine the preceding expressions regarding the mean velocity, U, and the ratio, $\frac{V}{U}$, and considering at first Formula (3), in which we note that $U\sqrt{b} = \sqrt{RI}$:

FIG. 102.

EXPERIMENTS OF JOHN R. FREEMAN, M.AM. SOC. C.E., MADE UPON A PIPE 0.03 M. DIAMETER.

$$\text{Curve} = \frac{V-v}{U\sqrt{b}} = 22.5\left[1-\sqrt{1-0.95\left(\frac{r}{R}\right)^2}\right]$$

EXPERIMENTS OF M. DARCY

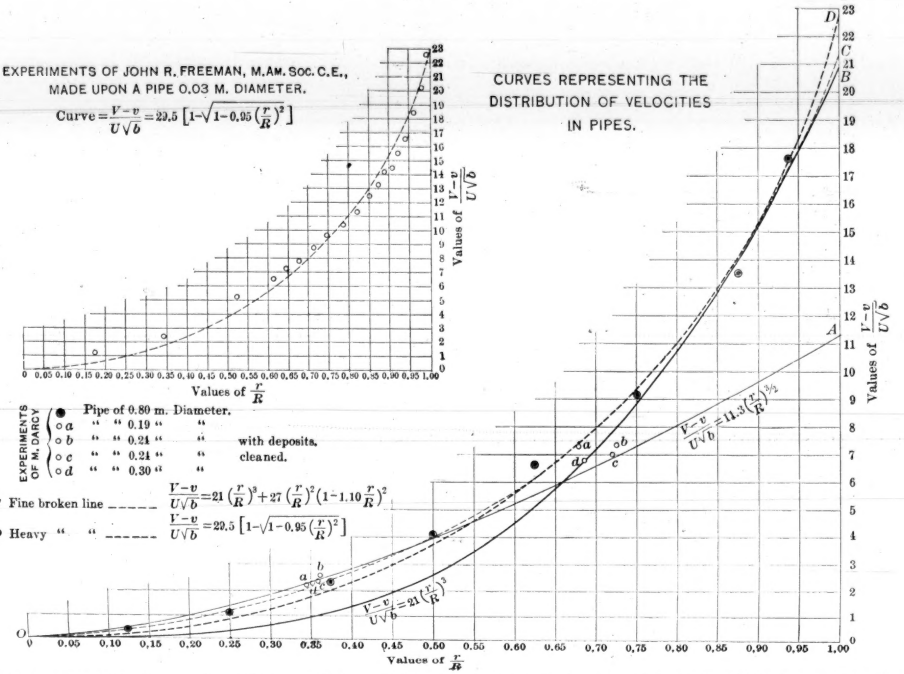
- ● Pipe of 0.80 m. Diameter.
- ○ a " " 0.19 " "
- ○ b " " 0.24 " " with deposits.
- ○ c " " 0.24 " " cleaned.
- ○ d " " 0.30 " "

O C Fine broken line _____ $\frac{V-v}{U\sqrt{b}} = 21\left(\frac{r}{R}\right)^3 + 27\left(\frac{r}{R}\right)^2\left(1-1.10\,\frac{r}{R}\right)^2$

O D Heavy " " _____ $\frac{V-v}{U\sqrt{b}} = 22.5\left[1-\sqrt{1-0.95\left(\frac{r}{R}\right)^2}\right]$

FIG. 103.

CURVES REPRESENTING THE DISTRIBUTION OF VELOCITIES IN PIPES.



M. Bazin.

$$U = \frac{1}{\pi R^2}\int_o^R v\, 2\pi r\, dr = \frac{1}{R^2}\int_o^R \left[V - \sqrt{RI}\left\{21\left(\frac{r}{R}\right)^3 + \alpha\left(\frac{r}{R}\right)^2 - 2\alpha\beta\left(\frac{r}{R}\right)^3 + \alpha\beta^2\left(\frac{r}{R}\right)^4\right\}\right] 2\, r\, dr,$$

and integrating,

$$U = V - \sqrt{RI}\left(8.40 + \frac{1}{2}\alpha - \frac{4}{5}\alpha\beta + \frac{1}{3}\alpha\beta^2\right)$$

which, when divided by U, and $\frac{\sqrt{RI}}{U}$ is replaced by $\sqrt{b}$, becomes:

"(7) $$\frac{V}{U} = 1 + 8.40\sqrt{b} + \frac{\alpha\sqrt{b}}{30}(15 - 24\beta + 10\beta^2) = 1 + 9.03\sqrt{b},$$

and, making $\sqrt{b} = 0.0182$, there results $\frac{V}{U} = 1.1643$, and experiment shows 1.1675. There follows, in applying Formula (2), $\frac{V}{U} = 1 + 8.40\sqrt{b} = 1.1529$. With Formula (1) there results a value decidedly too small, viz.:

$$\frac{V}{U} = 1 + \frac{4}{7} \times 11.3\sqrt{b} = 1 + 6.46\sqrt{b} = 1.1175.$$

"Effecting the same integration with Formula (4):

$$U = \frac{1}{R^2}\int_o^R \left[V - \alpha\sqrt{RI} + \alpha\sqrt{RI}\sqrt{1 - \beta\frac{r^2}{R^2}}\right] 2\, r\, dr;$$

there is deduced

"(8) $$\frac{V}{U} = 1 + \alpha\sqrt{b}\left\{1 - \frac{2}{3\beta} + \frac{2(1-\beta)^{\frac{3}{2}}}{3\beta}\right\} = 1 + 9.03\sqrt{b},$$

a value identical with that of the last formula.

"The position of the mean velocity is easily determined from Formulas (5) and (6) by making $v = U$ and replacing $\frac{V}{U}$ by its value from (7) or (8). From these, to determine the distance, r, of the fillet of mean velocity, we have,

$$21\left(\frac{r}{R}\right)^3 + 27\left(\frac{r}{R}\right)^2\left(1 - 1.10\frac{r}{R}\right)^2 = 9.03,$$

$$\text{and } 29.5\left[1 - \sqrt{1 - 0.95\frac{r^2}{R^2}}\right] = 9.03,$$

from which $\frac{r}{R} = 0.74$, approximately; for both cases. Formula (2) gives a value but little different: $\frac{r}{R} = \sqrt[3]{\frac{2}{5}} = 0.737$, but the formula of M. Darcy gives the much lower value of $\frac{r}{R} = 0.689$.

"Making comparisons of the two empiric formulas presented, the values of $\frac{V-v}{U}$ which they furnish are calculated. Table No. 64 gives these different values compared with the experimental results, by which one may judge of the degree of approximation. M. Bazin.

"TABLE No. 64.—COMPARISON OF EXPERIMENTAL VALUES OF $\frac{V-v}{U}$ WITH FORMULAS (5) AND (6).

$\frac{r}{R}$	Values of $\frac{V-v}{U}$. By experiment.	By Formula (5).	By Formula (6).	Differences. Formula (5).	Formula (6).	Remarks.
0	0	0	0	0	0	
0.125	0.0070	0.0064	0.0040	−0.0006	−0.0003	
0.250	0.0200	0.0221	0.0162	+0.0021	−0.0038	
0.375	0.0417	0.0440	0.0371	+0.0023	−0.0046	
0.500	0.0752	0.0727	0.0681	−0.0025	−0.0071	
0.625	0.1202	0.1122	0.1111	−0.0080	−0.0091	
0.750	0.1667	0.1697	0.1705	+0.0030	+0.0038	
0.875	0.2455	0.2566	0.2565	+0.0110	+0.0110	
0.937	0.3210	0.3153	0.3188	−0.0057	−0.0022	
1.000	0.4260	0.3871	0.4169	−0.0389	−0.0091	

Remarks.	Mean Errors. Taking account of sign.	Not taking account of sign
Formula (5)..	−0.0037	0.0074
" (6)..	−0.0024	0.0054

"One sees that the differences between the observed and calculated values are of small importance. For a similar comparison with the experiments of M. Darcy, it must be remarked, at the beginning, that the distances r_1 and r_2, at which the velocities v_1 and v_2 are measured, are not exactly at one-third and two-thirds of the radius. The true values $r_1 : R$ and $r_2 : R$ are as shown in Table No. 65.

"TABLE No. 65.—CONSTANTS OF M. DARCY'S EXPERIMENTS.

Designation of Series.	R	r_1	r_2	$\frac{r_1}{R}$	$\frac{r_2}{R}$
	Meters.	Meters.	Meters.		
1. Pipe of 0.19 m. diam.	0.0940	0.0325	0.0637	0.346	0.678
2. Pipe of 0.24 m. diam., with deposits...	0.1216	0.0440	0.0880	0.362	0.724
3. Pipe of 0.24 m. diam., cleaned.	0.1223	0.0440	0.0880	0.360	0.720
4. Pipe of 0.30 m. diam.	0.1485	0.0520	0.1020	0.350	0.687

"In the discussion of these results, M. Darcy, to correct the velocities V, v_1 and v_2, multiplied them by a coefficient which varied somewhat from one experiment to another; in order not to complicate the subject by the introduction of foreign elements, this coefficient has not been taken into account in the formation of Table No. 66; but its value is given, for reference, in Column 6.

M. Bazin.

"TABLE No. 66.—EXPERIMENTS OF M. DARCY.

Mean Velocity, U, Meters.	$\sqrt{b}$	Ratio of Velocities by Experiment. $\frac{V}{U}$.	$\frac{v_1}{U}$.	$\frac{v_2}{U}$.	Coefficient of Correction adopted by M. Darcy.	Remarks.
(1)	(2)	(3)	(4)	(5)	(6)	(7)
Series 1. Pipe 0.19 m. Diameter.						
0.758	0.0245	1.245	1.189	1.069	0.938	The velocities v_1 and v_2 are generally equal above and below the center. When they are not, the mean of the values of each has been taken.
1.128	0.0244	1.249	1.192	1.084	0.925	
1.488	0.0239	1.216	1.164	1.039	0.966	
1.933	0.0238	1.223	1.167	1.041	0.964	
2.506	0.0239	1.203	1.152	1.036	0.969	
4.323	0.0235	1.206	1.157	1.030	0.975	
Means.	0.0240	1.224	1.170	1.050	0.956	
Calculated Values by Formula (6)..		1.217	1.175	1.040	.....	
Series 2. Pipe 0.24 m. Diameter; with deposits.						
0.452	0.0347	1.239	1.159	0.956	1.027	The value of $\sqrt{b}$ is notably higher for this second series than for the other three; this series was made upon a pipe, the surface of which was covered with deposits; it was the same pipe cleaned which served for Series 3.
0.707	0.0339	1.173	1.085	0.929	1.057	
1.547	0.0341	1.239	1.149	0.999	0.988	
1.833	0.0340	1.249	1.164	0.990	0.998	
3.833	0.0340	1.242	1.153	1.019	0.965	
Means.	0.0341	1.228	1.142	0.979	1.005	
Calculated Values by Formula (6)..		1.308	1.243	1.015	.....	
Series 3. Pipe 0.24 m. Diameter; cleaned.						
0.537	0.0265	1.277	1.220	1.076	0.918	
0.949	0.0260	1.123	1.064	0.950	1.035	
1.904	0.0262	1.156	1.085	0.973	1.013	
4.497	0.0262	1.144	1.080	0.973	1.013	
Means.	0.0262	1.175	1.112	0.993	0.995	
Calculated Values by Formula (6)..		1.237	1.187	1.014	.....	
Series 4. Pipe 0.30 m. Diameter.						
0.355	0.0287	1.183	1.115	0.966	1.035	
1.236	0.0245	1.226	1.175	1.069	0.936	
1.665	0.0245	1.234	1.175	1.064	0.940	
2.865	0.0244	1.199	1.147	1.047	0.955	
Means.	0.0255	1.210	1.153	1.086	0.967	
Calculated Values by Formula (6)..		1.230	1.185	1.037	.....	

"The ratios of velocities as they are given in Table No. 66 are calculated by the aid of the experimental results, without correction.

"The mean velocity, U, was the result of a direct measurement, the velocities V, v_1 and v_2, are deduced from the formula $v = \sqrt{2\,g\,\delta}$, where δ is the difference of elevation between the fluid surfaces in the two columns of the instrument. At the end of each series have been introduced, for purposes of comparison, the mean values of $\frac{V}{U}$, $\frac{v_1}{U}$ and $\frac{v_2}{U}$, from experiment, with their values computed from Equation (6) by introducing the mean value of the coefficient, $\sqrt{b}$, applicable to the series considered.

"There are similarly presented in Fig. 103, the eight points, of which the co-ordinates are the mean values of $\frac{r}{R}$ and $\frac{V-v}{U\sqrt{b}}$ for each series, which values were calculated from the foregoing experimental data, without making use of the coefficient of correction. M. Bazin.

"The results obtained for the pipes of 0.19 and 0.30 m. diameter concord well with those of Formula (6). The mean ratio of experimental to calculated values being, for the means of the former, 1.004, and of the latter, 0.985; this last mean is raised to unity if the first experiment in the series, which seems anomalous, is rejected. It is not thus with the series on the pipe of 0.24 m. diameter, where, for the first series (pipe covered with deposits), the mean value for the ratio is 0.941, and for the second (pipe cleaned) is 0.955. It appears that the values of the velocities have decreased about 5% in these cases, from some unexplained cause; the introduction of foreign bodies to the orifices of the instrument may have perhaps caused the variation; such accidents occurred quite frequently, as M. Darcy himself has stated; and he has had to reject a series of experiments, made upon an iron pipe lined with bitumen, which was affected in this manner. It has been seen that a somewhat similar accident has prevented the use of one of the experiments at Section *B* on the writer's pipe of 0.80 m. diameter.

"*Experiments of John R. Freeman, M. Am. Soc. C. E., with a Pipe of Small Diameter.**—These experiments were made upon the jet discharging from a straight piece of brazed brass tubing, $1\frac{1}{8}$ ins. in diameter and 5 ft. long, which was screwed to the $1\frac{1}{8}$-in. diameter tip of a fire-nozzle fed by $2\frac{1}{2}$-in. hose, under fire pressure. The point of the instrument used for measuring the velocity was made from that of a fine stylographic pen mounted upon a knife-edge, and connecting to a Bourdon gauge, from which the pressure was read. The observations were made by sliding this point across the jet at about $\frac{1}{8}$ in. from the end of the pipe.

"Table No. 67 shows the results obtained by Mr. Freeman, and arranged by him graphically on a large scale. The velocities are not exactly alike above and below the axis, and that engineer attributes this to the method of manufacture of the pipe. However, the difference is of small consequence, and the mean of the two values for symmetrical points has been used.

"Columns 7 and 8 of Table No. 67 are calculated from Formula (6); as the value of b is not determined in the experiment, it has been necessary for applying that formula, to adopt one which is best adapted to the experimental results. By making $29.5\sqrt{b} = 0.625$, the agreement between the experimental and computed values of $\frac{V-v}{U}$ is quite satisfactory. This accordance appears on inspection of Fig. 103, where the experiments of Mr. Freeman are represented graphically in the same manner as those previously described. If one adopts, for the very special case which we are considering, the coefficient 29.5, there results for $\sqrt{b}$ a value, 0.0212, somewhat less than the coefficients obtained by M. Darcy for the pipes of 0.027 and

* As Mr. Freeman's own description of the apparatus used in these experiments is to be found in the *Transactions* of this Society, Vol. xxi, p. 411, *et seq.*, the translator has at this point taken the liberty of abridging M. Bazin's memoir, so far as the description extends.—G. S. W.

M. Bazin. 0.40 m. diameter, which coefficients were all comprised between 0.025 and 0.029. Nevertheless, it is considered that the observations represented exceptional conditions, the velocities exceeding 20 m. per second; it seems that on arriving at the terminal orifice, after a flow of 1.50 m., only (about 50 diameters), the velocities of the fillets have not yet again assumed the normal distribution of velocities conforming to Formula (6); but the position of the mean velocity notable, which is at the distance $r = 0.74\ R$, agrees very well with the case of the writer's pipe of 0.80 m. diameter.

"TABLE No. 67.—Experiments of Mr. Freeman.

Distance from center r.	Ratios, $\frac{r}{R}$	Velocities, v.	Ratios, $\frac{v}{U}$.	Values of $\frac{V-v}{U}$. By Experiment.	Values of $\frac{V-v}{U}$. By Formula (6)	Differences: Formula minus Experiment.	$\frac{V-v}{U\sqrt{b}}$	Remarks.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Inches.		Feet.						
0	0	74.8	1.197	0	0	0	0	
0.10	0.174	73.3	1.173	0.024	0.009	−0.015	1.13	The mean velocity U is obtained by dividing the total section into concentric rings, and multiplying the area of each ring by the velocity corresponding to it, and summing these products. This gives for U the value 62.5 ft., or 19 m. per second. $R = \frac{1.15}{2} = 0.575$ m. $= 0.01461$ in.
0.20	0.348	71.6	1.146	0.051	0.037	−0.014	2.41	
0.30	0.522	68.0	1.088	0.109	0.087	−0.022	5.14	
0.35	0.609	66.1	1.058	0.139	0.122	−0.017	6.56	
0.37	0.643	65.4	1.046	0.151	0.138	−0.013	7.12	
0.39	0.678	64.5	1.032	0.165	0.156	−0.009	7.78	
0.41	0.713	63.3	1.013	0.184	0.176	−0.008	8.68	
0.43	0.748	62.0	0.992	0.205	0.197	−0.008	9.67	
0.45	0.783	61.0	0.976	0.221	0.221	0	10.42	
0.47	0.817	59.7	0.955	0.242	0.247	+0.005	11.42	
0.49	0.852	58.2	0.931	0.266	0.277	+0.011	12.55	
0.50	0.870	57.3	0.917	0.280	0.294	+0.014	13.21	
0.51	0.887	56.2	0.899	0.298	0.311	+0.013	14.06	
0.52	0.904	55.6	0.890	0.307	0.329	+0.022	14.48	
0.53	0.922	54.2	0.867	0.330	0.351	+0.021	15.57	
0.54	0.939	52.8	0.845	0.352	0.373	+0.021	16.60	
0.55	0.957	50.4	0.806	0.391	0.400	+0.009	18.44	
0.56	0.974	48.0	0.768	0.429	0.429	0	20.24	
0.57	0.991	45.0	0.720	0.477	0.463	−0.014	22.50	

" *Résumé.*—The preceding discussion may be condensed into a few words. The first experiments of M. Darcy led him to consider as the definite law of decrease of velocity away from the center, Formula (1); but he did not work at all with a pipe of large diameter, and he did not measure the velocities beyond a point at two-thirds of the radius. The new experiments, made a few years later under his direction, were extended to semi-circular canals of much larger dimensions, which permitted the multiplication of the points of observation in the divisions of concentric circles; these experiments produced Formula (2). According to the new expression, the decrease was less rapid in the central region than was supposed from Formula (1), but it was more pronounced beyond two-thirds of the radius. A question was then raised as to whether it were possible always to compare a full, closed pipe, in which the flow took place under pressure, with an open, semi-circular canal where the absence of pressure along the free surface favored the production of irregular motions. If such a comparison were legitimate for the outer region not explored in 1850, it was not as much so for the central region. It was necessary, to solve the ques-

M. Bazin.

"TABLE No. 68.—Observations of Velocity in a Pipe 0.80 M. Diameter.

Position from Center of Pipe.			On Vertical Diameter.						On Horizontal Diameter.						Position.
			At Section A.		At Section B.		At Section C.		At Section A.		At Section B.		At Section C.		
Meters.	Feet.	Inches.	Meters.	Feet.	Meters.	Feet.	Meters.	Feet.	Meters.	Feet.	Meters.	Feet.	Meters.	Feet.	$\frac{r}{R}$
			First Discharge.						First Discharge.						
0.375	1.230	14.76	0.970	3.182	0.948	3.110	0.958	3.143	0.878	2.880	0.877	2.877	0.918	3.012	0.937
0.350	1.148	13.78	1.081	3.546	1.008	3.291	1.058	3.471	0.998	3.274	0.929	3.048	1.023	3.356	0.875
0.300	0.984	11.81	1.142	3.747	1.081	3.546	1.116	3.661	1.103	3.619	1.042	3.419	1.083	3.553	0.750
0.250	0.820	9.84	1.170	3.838	1.125	3.691	1.138	3.733	1.157	3.796	1.106	3.629	1.129	3.704	0.625
0.200	0.656	7.87	1.208	3.963	1.172	3.845	1.195	3.920	1.189	3.901	1.127	3.697	1.180	3.871	0.500
0.150	0.492	5.90	1.234	4.048	1.217	3.993	1.223	4.012	1.195	3.920	1.194	3.917	1.213	3.980	0.375
0.100	0.328	3.94	1.229	4.032	1.232	4.042	1.255	4.117	1.211	3.973	1.211	3.973	1.239	4.065	0.250
0.050	0.164	1.97	1.251	4.104	1.253	4.111	1.268	4.160	1.239	4.065	1.227	4.025	1.255	4.117	0.125
0.025	0.082	0.98	1.255	4.117	1.243	4.078	1.278	4.193	1.245	4.085	1.239	4.065	1.268	4.160	0.062
C	.....	.....	1.245	4.085	1.251	4.104	1.270	4.167	1.245	4.085	1.251	4.104	1.270	4.167	0.000
0.025	0.082	0.98	1.251	4.104	1.245	4.085	1.274	4.180	1.247	4.091	1.253	4.111	1.263	4.144	0.062
0.050	0.164	1.94	1.234	4.048	1.235	4.052	1.261	4.137	1.261	4.137	1.239	4.065	1.259	4.130	0.125
0.100	0.328	3.94	1.232	4.042	1 215	3.986	1.247	4.091	1.237	4.058	1.219	3.999	1.245	4.085	0.250
0.150	0.492	5.90	1.218	3.996	1.207	3.960	1 211	3.973	1.226	4.022	1.215	3.986	1.227	4.025	0.375
0.200	0.656	7.87	1.185	3.888	1.145	3.756	1.180	3.871	1.215	3.986	1.167	3.829	1.183	3.881	0.500
0.250	0.820	9.84	1.189	3.901	1.106	3.629	1.141	3.743	1.174	3.852	1.114	3.655	1.153	3.783	0.625
0.300	0.984	11.81	1.090	3.576	1.042	3.419	1.090	3.576	1.132	3.714	1.076	3.530	1.079	3.540	0.750
0.350	1.148	13.78	1.029	3.376	0.953	3.127	0.958	3.143	1.046	3.432	0.978	3.209	0.990	3.248	0.875
0.375	1.230	14.76	0.950	3.117	0.873	2.864	0.895	2.936	0.980	3.215	0.886	2.907	0.875	2.871	0.937
Mean velocity			1.108	3.635	1.060	3.478	1.087	3.566	1.108	3.635	1.060	3.478	1.087	3.566	.....
			Second Discharge.						Second Discharge.						
0.375	1.230	14.76	1.284	4.212	1.291	4.235	1.301	4.268	1.261	4.137	1.291	4.235	1.250	4.101	0.937
0.350	1.148	13.78	1.407	4.616	1.387	4.550	1.393	4.570	1.343	4.406	1.323	4.340	1.333	4.373	0.875
0.300	0.984	11.81	1.491	4.892	1.491	4.892	1.501	4.924	1.486	4.875	1.479	4.852	1.464	4.803	0.750
0.250	0.820	9.84	1.589	5.213	1.568	5.144	1.565	5.134	1.544	5.065	1.537	5.042	1.534	5.033	0.625
0.200	0.656	7.87	1.621	5.318	1.615	5.298	1.623	5.325	1.617	5.305	1.594	5.229	1.602	5.256	0.500
0.150	0.492	5.90	1.642	5.387	1.677	5.502	1.651	5.416	1.623	5.325	1.624	5.328	1.657	5.436	0.375
0.100	0.328	3.94	1.648	5.407	1.689	5.541	1.700	5.577	1.639	5.377	1.656	5.433	1.700	5.577	0.250
0.050	0.164	1.97	1.671	5.482	1.717	5.633	1.692	5.551	1.651	5.416	1.697	5.567	1.712	5.617	0.125
0.025	0.082	0.98	1.659	5.443	1.723	5.653	1.723	5.653	1.663	5.456	1.692	5.551	1.721	5.646	0.062
C	.....	.....	1.661	5.449	1.707	5.600	1.715	5.626	1.661	5.449	1.707	5.600	1.715	5.626	0.000
0.025	0.082	0.98	1.648	5.407	1.707	5.600	1.720	5.643	1.645	5.397	1.713	5.620	1.724	5.656	0.062
0.050	0.164	1.97	1.640	5.380	1.683	5.521	1.712	5.617	1.658	5.439	1.711	5.613	1.714	5.623	0.125
0.100	0.328	3.94	1.635	5.364	1.686	5.531	1.691	5.548	1.653	5.423	1.672	5.485	1.691	5.548	0.250
0.150	0.492	5.90	1.601	5.252	1.671	5.482	1.657	5.436	1.640	5.380	1.678	5.505	1.643	5.390	0.375
0.200	0.656	7.87	1.577	5.174	1.626	5.334	1.636	5.367	1.620	5.315	1.636	5.367	1.605	5.266	0.500
0.250	0.820	9.84	1.541	5.056	1.542	5.059	1.571	5.154	1.598	5.243	1.551	5.088	1.523	5.497	0.625
0.300	0.984	11.81	1.535	5.036	1.507	4.944	1.501	4.924	1.527	5.010	1.474	4.836	1.441	4.728	0.750
0.350	1.148	13.78	1.327	4.354	1.312	4.304	1.394	4.573	1.394	4.573	1.329	4.360	1.358	4.455	0.875
0.375	1.230	14.76	1.303	4.275	1.248	4.094	1.240	4.068	1.305	4.281	1.812	4.304	1.225	4.019	0.937
Mean velocity			1.486	4.876	1.469	4.820	1.478	4.849	1.486	4.876	1.469	4.820	1.478	4.849	.....

M. Bazin. tion, to work with a pipe of large diameter for conditions more favorable than those which M. Darcy had, and to multiply the points of observation, as in the open canal. For this purpose has been utilized the canal in which were carried on, with the support of the Minister of Public Works, the study of flow over weirs. This canal, built at Dijon and furnished with an intake from the Canal de Bourgogne, afforded an excellent opportunity for these experiments, and their execution has been entrusted to M. Hegly, who has been at the writer's command for several years in the study of weirs. The assistance of that sagacious and experienced co-laborer is considered very valuable, and has enabled the determination with precision of the distribution of velocities in a pipe of 0.80 m. diameter.

"This distribution is more complicated than indicated by either Formulas (1) or (2), each of which is only applicable to one part of the section, Formula (1) to the central and Formula (2) to the outer region. The latter, however, is that which conforms most closely to the facts as a whole; it requires a corrective term to adapt it to the central region, when the divergence of the two formulas beyond this is not very great. This correction is not necessary in an open canal where the motions which occur near the free surface check the regular decrease of velocities; under such circumstances Formula (2) gives a more accurate representation of the phenomena than Formula (1) which was derived from the case of a pipe. The difference of the two modes of motion appears thus in the central region, but in that region only; for the distribution of velocities becomes the same for the two cases when they depart from the center of the section."

Mr. Kuichling. E. KUICHLING, M. Am. Soc. C. E.—This paper is a most valuable addition to the literature of hydraulics, as it brings to the notice of engineers and experimenters an extremely ingenious and useful instrument for the accurate measurement of small differences of hydraulic head or pressure, as well as a new application of the Pitot tube. A vast amount of careful thought, patience and labor on the part of the authors is also plainly evident in the various experiments described and the interesting results reached, and it therefore becomes very difficult to prepare an adequate discussion of the latter, especially as the investigations on curved pipes are limited to deflections of 90° only. There are, however, a few points which seem to the speaker capable of being made somewhat clearer by the authors, and it is hoped that the same will receive consideration in their closure of the discussion.

The first is with reference to the means by which the pressure opening of the Pitot tube is always kept truly normal to the direction of the axis of the pipe, and the effect of slight divergences from the proper position at different points in the "traverse." It is inferred from the description that a cross-arm on the upper part of the instrument indicated approximately whether the pressure opening was normal to the pipe's axis, also that a slight divergence did not produce an appreciably different result. If this be true, the usefulness of the instrument is materially increased by avoiding loss of time in delicate adjustment at each point of observation. In using such tubes in open streams or

jets, vanes may serve to keep the opening normal to the current; but as they cannot be applied in the case of a pipe under pressure, it is important to know what effect a little obliquity produces. Mr. Kuichling.

The next point relates to the marked irregularity of many of the velocity curves for currents of less than 2 ft. per second in the several pipes. It is very probable that these irregularities are not due to lack of sensitiveness or accuracy of the instruments, but to a constant shifting of the position of the water filament having the greatest velocity, or to a spiral motion of the same; and the question arises whether simultaneous observations at a number of consecutive cross-sections, 1 or 2 ft. apart, would not demonstrate such a spiral motion. In this event it would follow that for each instant of time the true velocity diagram would be represented by a regular curve having its vertex at some distance to one side of the axis of the pipe, instead of by the irregular figures shown in the drawings, which indicate that the successive thin cylindrical shells of water from the axis outward have alternately greater and smaller velocities.

The consideration of this question naturally involves the accuracy of all gaugings made by isolated observations of the velocity at different points in the cross-section of a body of moving water, and the method that should be pursued in making such observations in order to obtain the most correct value of the computed discharge. In a pipe of limited diameter, it is reasonable to presume that only a single filament of water acquires the maximum velocity, and that all contiguous or surrounding filaments or cylindrical shells have a smaller velocity which gradually decreases the nearer they approach to the sides of the pipe. Furthermore, observation of small immersed and floating objects in open channels of uniform section and moderate size shows that when the current is not rapid their motion is usually curvilinear or spiral; hence, analogously, a similar motion may be presumed to exist in water flowing under pressure through a pipe.

On this basis, it is obvious that a true velocity diagram can be obtained only by making a relatively large number of observations simultaneously in the same cross-section; but as such a procedure is impracticable, it follows that a "traverse" with the Pitot tube should be made as quickly as possible. It is of interest, therefore, to know the best attainable speed with this instrument, the least number of observations consistent with accuracy, and the probable error in the discharge. To determine these matters, other experiments will doubtless be necessary, and reference thereto has been made in the hope of eliciting from the authors further particulars concerning the manipulation of an instrument which has heretofore found only a limited application in hydraulic operations.

It also follows, from the foregoing, that if the low mean velocities, v, deduced from the observations with the Pitot tube in straight lines

Mr. Kuichling. of pipe do not possess a high degree of accuracy, the corresponding values of the coefficient, c, in the Chezy formula, $v = c\sqrt{RS}$, are equally of doubtful value; and hence the law of variation in the value of c for different values of v, exhibited in Table No. 38 for straight cast-iron pipe, is not firmly established. For the higher velocities, on the other hand, the data are much more satisfactory.

With respect to the resistance or loss of head in curved lines of pipe, the experiments indicate either that the teachings of our standard books on hydraulics are erroneous, or else that the results of former observations in this direction have not been rightly interpreted, as the conclusion reached by the authors is directly opposed to what is set forth in the most recent treatises. It must be remembered, however, that the authors' experiments are limited to curves of 90° deflection, and that similar observations on other degrees of curvature are necessary before a new theory of curve resistance can be established. The problem is complicated, furthermore, by the use of compound curves of varying radii to make the given deflection, with the direction of flow always from the flatter to the sharper curvature. This leads at once to the question whether the resistance would be the same if the direction of the current were reversed. The influence of the joints in the compound curves also requires careful study, before the whole of the observed loss of head can be charged to curvature, as it is possible that a break in the continuity of the interior surface may induce a considerable resistance.

The foregoing remarks are submitted without the intention of detracting in the slightest degree from the great value of the authors' work, but rather in the hope of stimulating their interest in the investigations which they have undertaken, and evoking further contributions from them. With their skill, experience and ingeniously devised appliances, they are particularly well fitted for prosecuting practical research in this branch of hydraulics, wherein there is evidently room for much advancement; and, hence, the speaker earnestly looks forward to the presentation of future papers in which their interesting work will be continued.

Mr. Ferris. WALTER FERRIS, Esq. (by letter).—Aside from the remarkable conclusions reached by the authors, one of the most striking things about this paper is the immense amount of painstaking labor expended in making, verifying, and reducing the experiments. In this case, the expenditure was justified by the importance of the investigation and the results obtained. There are, however, many problems in measurement of water occurring continually in the practice of engineering, which might be solved by the methods so fully illustrated in this paper, but that the labor and expense involved are too great to be incurred.

A glance over the pages of the paper shows that a large part of the

work consisted in calibrating the instruments used, especially the Pitot tubes. It is the writer's object, in a brief discussion, to call attention to some facts relative to Pitot tubes, which seem to indicate that the instrument may be made in such a manner as to require no calibration for all ordinary work—that is, for work where the difference between a 3% error and a 2% error is not worth a great deal of labor and money. Mr. Ferris.

A paper on the Pitot tube,* by Mr. William Monroe White, of New Orleans, should be read by all who are interested in this subject. From his experiments, Mr. White draws two very important conclusions: (1) The head shown by the impact opening of a pair of Pitot tubes is not affected by the size or shape of the impact nozzle, provided that it be a shape of revolution, such as a cylindrical tube or a converging or diverging cone. (2) The head due to the impact tube is always exactly equal to the velocity head, $\frac{v^2}{2g}$. If the instrument as a whole shows a different result, it is due to suction or impact at the static opening.

These results are qualified by the fact that they were not obtained under pressure in a closed pipe, and may possibly not exactly apply under those conditions. In the discussion of Mr. White's paper, the writer cited a series of experiments made by himself which seemed to show results at variance with these conclusions; Mr. White, however, has pointed out a possible source of error; and, including data in the paper now under discussion, the weight of evidence seems to be that Mr. White's conclusions do apply to closed pipes under pressure.

If the impact opening of a pair of Pitot tubes shows the exact velocity head plus the static head, it is only necessary to construct the static opening in such a manner that it will show the exact static head. The reading of the pair of tubes connected to a differential gauge will then give the exact velocity head. It is a well-known fact, especially through experiments with the Venturi meter, that a simple drilled hole in the side of a pipe, normal to the axis, does show the static head correctly within 4 or 5 per cent. That is, the head shown by such a tube is equal to the total head at that cross-section, less the head due to the average velocity of flow through the section. As this static opening does not come in contact with the average velocity, but with a much smaller velocity, while a velocity higher than the average exists at the center of the pipe, it is especially noteworthy that it indicates the average, and not the velocity adjacent to itself.

It has usually been thought by experimenters that a vital point in making an accurate Pitot instrument is to have the two openings exposed to the same velocity. Hence the openings were both included

* *Journal of the Association of Engineering Societies*, for August, 1901.

Mr. Ferris. in the instrument inserted into the flow. The result has been that almost every pair of tubes has a coefficient different from every other pair, and rarely, if ever, equal to unity. The variation has been supposed to be mainly due to variations in the impact opening.

Turning now to the paper under discussion, there are given, in Table No. 10, the mean results of five sets of experiments with five different Pitot tubes, inserted in a 2-in. pipe under a head of about 65 ft. In all these cases, the static pressure was taken from a chamber surrounding the pipe, and communicating with the interior by four holes, $\frac{1}{16}$ in. in diameter. The impact nozzles of four of the tubes tested were shapes of revolution, and all these tubes have coefficients within 3% of unity. The fifth tube had an impact opening of peculiar shape—a round hole in a knife-edge, giving a notched side view—and hence the head shown by this tube for a given velocity is much lower; that is, its coefficient is higher.

Assuming that the velocity figure is an ellipsoid of revolution, as believed by the authors, the locus of mean velocity is at $0.745\,r$, or about three-fourths of the distance from the center to the circumference of a circular pipe. If inserted in a long stretch of straight pipe, a Pitot tube constructed as indicated (the nozzle a shape of revolution and the static opening in the side of the pipe) may be expected to give results in accordance with the formula $v = \sqrt{2gh}$ when the impact nozzle is on this mean-velocity circle. Or, if the nozzle is on the center, the true mean velocity will be 0.84 of the velocity by the formula, or the true velocity head will be 0.707 of the observed head. The safest way is to put the nozzle on center if no traverse is to be made, as a slight shifting of the velocity curve will cause less error.

The writer does not think it essential to have the circumferential chamber shown in Fig. 4. A single tube, $\frac{1}{8}$ to $\frac{1}{4}$ in. in diameter, screwed into the side of the pipe and filed off flush inside, is probably nearly as accurate.

From the foregoing facts, taken in connection with his own experience in making such observations, the writer thinks that good results may be obtained with a simple piece of $\frac{1}{8}$ or $\frac{1}{4}$-in. brass tube, bent to face the flow at the center of the pipe, and a simple piezometer opening in the side of the same cross-section. These should be connected to an air-gauge (*i. e.*, an inverted U-tube with compressed air in the top) as described in the paper, for mean velocities between 1 and 9 ft. per second, if the gauge is made of tubes 30 ins. long. For mean velocities from 4 to 36 ft. per second, the same gauge may be turned upside down and filled with mercury. In this case the connection between the tubes, if not a continuous bend in the glass, should be of steel, cast iron, or copper, to resist the action of the mercury. If a long stretch of straight pipe is not obtainable, the impact nozzle should be made so as to permit traversing.

In working up all observations for measurement of flow, it is well to remember that a slide rule is more accurate than the data themselves, and saves a great deal of labor with little or no loss of accuracy. For checking results, it is well to emphasize still further the benefit and easy application of the straight-line diagram so extensively used by the authors. Mr. Ferris.

E. SHERMAN GOULD, M. Am. Soc. C. E. (by letter).—This paper and the subsequent discussions thereon are so voluminous and elaborate that the writer is frank to admit that he has failed to give to them that critical study which their importance merits. He was certainly startled at learning that actual experiments have shown resistance to diminish in proportion to the diminution of the radius of curvature, but, if his somewhat superficial examination has not deceived him as to the conditions of the experiments, he thinks that the foregoing is hardly a fair way of putting it. His understanding of the case, which, perhaps, is an erroneous one, is this: It was found that if a certain length of pipe, say, for instance, 2 000 ft. (although the experiments were made on much shorter lengths), was made to change its direction through 90°, its resistance to flow is less if the pipe be divided into two straight lengths of, say, 990 ft. each, connected by 20 ft. of pipe curved to a radius of 10 ft., than if the whole 2 000 ft. were thrown into a single curve, of which the radius would be 1 274 ft. Mr. Gould.

Now, it must be borne in mind that these two different dispositions of pipe could not be used to join the same two points, although the length of pipe would be the same in both cases. In the case of the abrupt bend, the distance between the two ends of the pipe in a straight line would be about 1 400 ft.; and in the case of the easy curve, it would be about 1 800 ft.

It seems to the writer that the practical presentation of the problem would be this: Given, two points, distant from each other on a straight line 1 400 ft., would a greater delivery be secured by joining them with 2 000 ft. of pipe turned in the middle abruptly through 90°, or by 1 571 ft. of pipe of the same diameter, laid on a uniform curve of 1 000 ft. radius, it being understood that the two tangents in both cases should be at right angles to each other? The water-supply engineer would not be long in making his decision. But, again: should he, preferably, if at liberty to follow any line he pleased, lay his pipe along the straight line 1 400 ft. long, joining his two fixed points, by using a sharp bend through 45° at each end? Probably he would consider this the best arrangement of all.

The writer must apologize for contributing only these hastily written lines to a discussion that has called forth so much mature thought, his object being to suggest that, from a practical point of view, the question is not so much how best to bend a given length of pipe, regardless of what becomes of the two ends, as to determine the

Mr. Gould. best way to join two given points from which are laid two pipe lines having different directions. He thinks that, other things being equal, the practical decision will always be in favor of the shortest line.

Mr. Brinckerhoff. H. W. BRINCKERHOFF, M. Am. Soc. C. E. (by letter).—The point of greatest interest, to most persons at least, is to be found in Conclusion *K*, where it is stated that "curves of short radius, down to a limit of about $2\frac{1}{2}$ diameters, offer less resistance to the flow of water than do those of longer radius." This, as stated by the authors, is directly opposed to all received theory and practice, and naturally demands an explanation, which the authors do not pretend to give.

Let us, therefore, consider, first, why a curved pipe should offer any greater resistance to the flow of water than does a straight pipe. The answer is to be found in the effect of the centrifugal force which is inseparable from the passage of water through a curved pipe. By this, it is not meant that the pressure due to the centrifugal force increases the friction on the side of the pipe furthest from the center of curvature (which, for convenience, will hereafter be called the convex side), because, as is well known, the friction of water against the inside of a pipe is entirely independent of the pressure. The effect of the centrifugal force is to divide the flowing water into two parts having opposite spiral motions. This absorbs energy and also increases the velocity of the water in contact with the pipe, resulting in greater friction and resistance. To understand how this effect is produced, let it be conceived that the curve is in a vertical plane with the convex side up, and that a view is taken at a transverse section in the curve a short distance from its commencement. When the water enters the curve from the straight pipe, its greatest velocity is at the center; when deflected by the curvature, a centrifugal force is developed in all parts of the water in proportion to the square of the velocity of each part. As the mean velocity is only 84% of the maximum, it follows that the centrifugal force of the water at the center will exceed that of the remainder by about 42%, on the average, and it will, by virtue of that superior force, be driven toward the convex side of the pipe, displacing the water already there, until, in approaching the pipe surface, the friction there existing will diminish its velocity and, in even more rapid ratio, its centrifugal force, and it will in turn give place to the more rapidly moving water from the center. Looking now at the transverse section which has been conceived, the water in the center will be seen moving continuously upward, until, reaching the inner surface of the pipe, it separates right and left into two portions which flow out, down, and in around the main body of water to meet at the bottom and flow up again in continuous circuits. If the pipe were transparent and the course of the water, as viewed from one side, could be traced by some floating particles, the water at the center would be seen to be rising,

and proceeding with ever increasing velocity as it rose, as shown in Figs. 11 and 12, until it neared the top of the pipe; when it would be gradually retarded by the friction, and would flow out and down, in long spirals of gradually diminishing pitch, to again rise in the center. Now, as the average forward velocity of the water must in all cases be the same (provided the area of the pipe is unchanged), it follows that this spiral motion must give an increased actual velocity to the water proportional to the secant of the average angle which the different threads of the stream make with the axis of the pipe, and, as the energy of a moving body is proportional to the square of its velocity, the energy absorbed by the water will increase in even more rapid ratio. Mr. Brinckerhoff.

This is one source of increased resistance on curves. Another is to be found in the increased friction due to the greater velocity with which the external water, owing to its spiral motion, moves over the surface of the pipe, and this also increases as the square of the velocity.

Another source of resistance is probably to be found in the irregular disturbance of the water at the points where the spiral currents separate and where they come together again. As it is not conceivable that these currents will turn at sharp right angles where they meet and leave the walls of the pipe, there must exist at those points, viewed in cross-section, triangular areas, of greater or less extent, in which there are all sorts of eddies and counter currents, with a corresponding absorption of energy and increase of resistance.

From these considerations, it would seem to be sufficiently clear why curvature increases the resistance to the flow of water in a pipe. It now remains to see what relation the resistance bears to the radius of curvature, which is really the interesting part of the whole question. The centrifugal force in a unit section of the water is inversely as the radius, but the length of time that the centrifugal force acts upon the unit of section is directly as the radius. It is found that the energy in a falling body is proportional to the square of the time during which the force of gravity acts upon it, and, if the same reasoning may be applied to the case in hand, it will be found that the energy of the spiral motion developed in a unit section will be directly proportioned to the length of the curve, and hence to the radius of curvature. The resistance, of course, will be in a similar ratio. But this does not mean that the energy in the water when it leaves the curve has been increased by an amount exactly proportional to the length of the curve, for much of the energy will be absorbed as fast as generated by the increased friction on the internal surface of the pipe due to the increase of the superficial velocity by the spiral motion. In fact, as there is no way to determine the proportion which these three factors of resistance bear to each other, it

Mr. Brinckerhoff.

is impossible to say more than that the total resistance will increase in some proportion to the length of the curve, that is, with the radius of curvature. The diagram, Fig. 90, showing the relative excess loss of head in curves of 90° in 80 diameters length, would seem to indicate that for radii above 10 or 15 diameters the resistance increases almost uniformly with the length of the radius, and very nearly in direct proportion thereto, plus a constant of about 25 per cent. Below a radius of about 2½ diameters, the resistance begins to greatly increase in inverse proportion to the radius. This may, perhaps, be in part accounted for by a material change in the condition of the flow. With a very short radius and a high velocity the water may leave the concave side of the pipe altogether. This will, to that extent, reduce the effective section of the waterway and, by adding to the velocity, increase the resistance; and if the space should be filled with a turmoil of little eddies, as is likely to be the case with high pressures, the resistance will be still further increased.

Having now shown the probable cause, not only for the resistance on curves, but also for the increase of that resistance with the radius of curvature, it remains to be seen if any means can be devised by which that excess of resistance may be reduced. Two expedients will suggest themselves. First, to increase the area of the pipe on curves in such a ratio that the excess of energy, due to the increased velocity of the spiral motion, shall be balanced by the reduction of energy due to the slower forward motion of the water as a whole. This would mean, practically, that the average velocity of the water was not changed, but that it advanced in spiral lines instead of in lines parallel to the axis of the curve. As the spiral motion must be gradually produced, and probably even more gradually checked, the enlargement should be gradual from the commencement of the curve, and the reduction should also be gradual from the termination of the curve. This would avoid most of the resistance due to the production and absorption of energy. If the enlargement were such that the spiral velocity remained the same at the surface as in the straight pipe, then the friction would be increased only in proportion to the diameter of the pipe, obviously less than if the bore were not enlarged at the curve. The third cause of resistance, the little groups of eddies at the convex and concave sides of the pipe, could be gotten rid of by simply displacing them by V-shaped ribs which would gently part the spiral currents and as gently permit them to flow together. That the enlargement at the curves and the introduction of the ribs will cause some reduction of resistance, seems altogether probable. That they can entirely remove it is altogether improbable, but the experiment is worth trying, and experiment alone can determine the exact form and size of the section of least resistance for curved pipes.

Mr. Cole.

EDWARD S. COLE, Esq. (by letter).—This valuable paper has been studied by the writer with intense interest, as it describes a course of experimental research similar to one carried on by him independently during almost the identical period of time, viz.: October, 1896, to March, 1901, with, however, two years' interruption in his work, during 1898 and 1899.

In 1896 there was need of means for measuring the flow of water in the water-works pipe system of Terre Haute, Ind., for which John A. Cole, M. Am. Soc. C. E., was consulting engineer, and after considering the use of the Venturi, the Deacon, and the ordinary disc meters, it was decided that, for the purposes of a waste-water survey, it would be advisable to find some more economical method of measuring the flow of water in pipes.

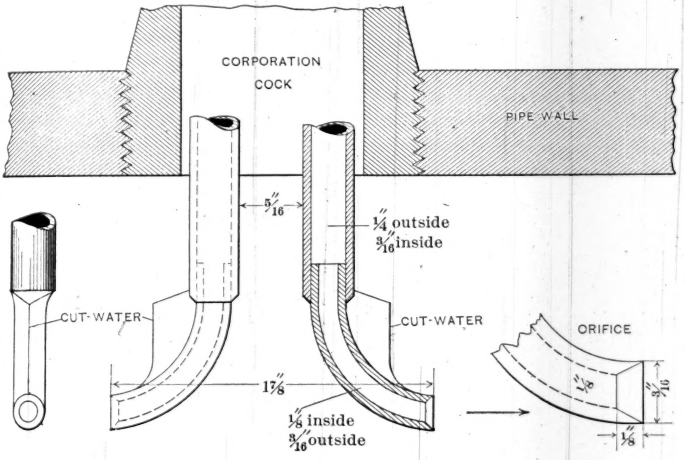


FIG. 104.

The writer, who is associated with Mr. John A. Cole, finally acted on a suggestion obtained from Professor R. C. Carpenter's work, "Experimental Engineering," regarding the Pitot tube, as applied to water mains.

A search was made among technical papers, etc., and correspondence was carried on with hydraulic engineers, for the purpose of ascertaining what had been done by others in this field, but little help could be found, aside from descriptions of the open-stream form of instrument, by Darcy, in the *Mémoires* of the Société des Ingénieurs Civils de France, and by several more recent investigators in open-stream work.

Mr. Cole.

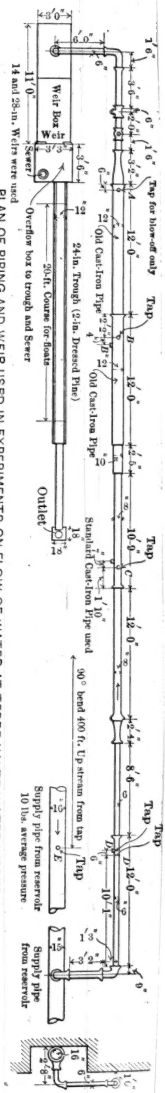
The writer, therefore, had no choice but to work out alone the many little details, which at first presented exceedingly great difficulties, especially when the object sought was accuracy at the lowest possible velocities.

Two orifice tubes were finally adopted which could be introduced through a 1-in. corporation cock by means of a special cap, each with a hook bent 90° and provided with a "cut-water," as shown in Fig. 104. Many series of calibrations were made with orifices and tubes of various diameters, but, finally, a choice was made of an orifice having an internal diameter of $\frac{1}{8}$-in., made of common $\frac{3}{16}$-in. brass tubing, fitted to $\frac{1}{4}$-in. tubes, as shown in Fig. 104.

It was found that an orifice and tube of this size gave more reliable indications at low velocities, and that there was less trouble with negative readings or failure to return to equilibrium at no flow.

A glass U-tube, 22 ins. long and having an internal diameter of $\frac{5}{16}$ in., was used from the first with excellent results. Connections were made with $\frac{1}{4}$-in. cloth-insertion rubber tubing, and a blow-off was located at the top of each connection to remove air. It was found that small brass pinch-cocks were best adapted to shutting off the water at these blow-offs, and, in fact, after repeated failures, it was decided that no form of ordinary valve was at all suitable, owing to the likelihood of the occurrence of minute seepage or leaks.

The lower half of the U-tube was filled with a mixture of carbon tetrachloride and gasoline, with a specific gravity of 1.25 at ordinary temperatures. Careful notes were made of the temperature at which observations were taken, and temperature corrections were applied where found necessary, although in some places the specific gravity changed to suit the temperature at which the tests were being made, so that no substantial correction was required. The object was to



PLAN OF PIPING AND WEIR USED IN EXPERIMENTS ON FLOW OF WATER AT TERRE HAUTE WATER-WORKS.

Fig. 105.

Mr. Cole.

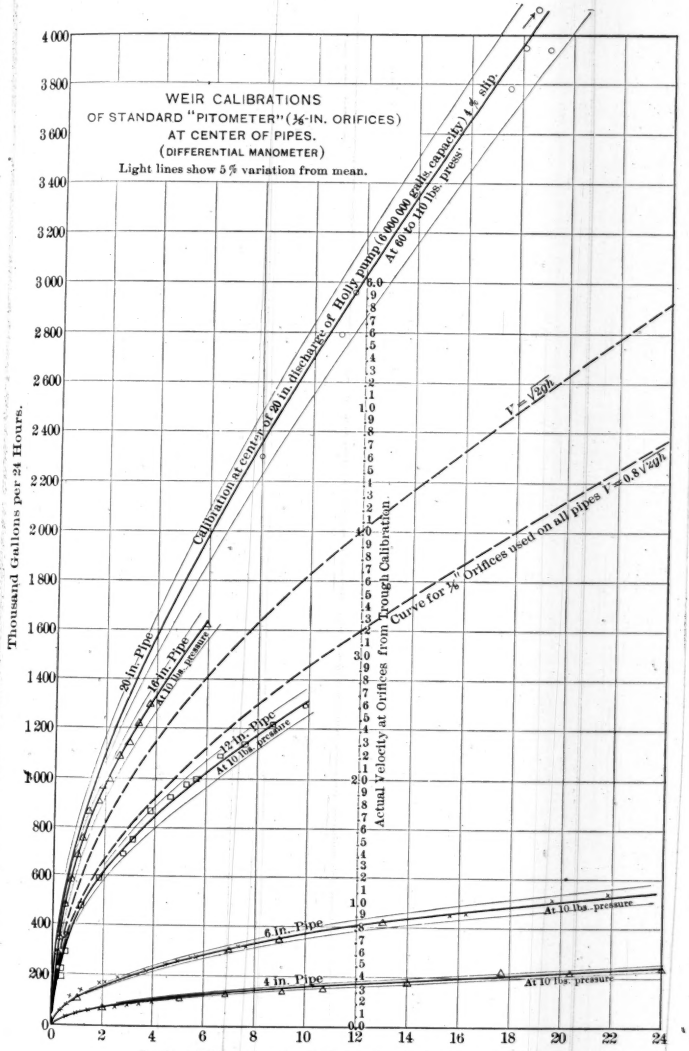


FIG. 106.

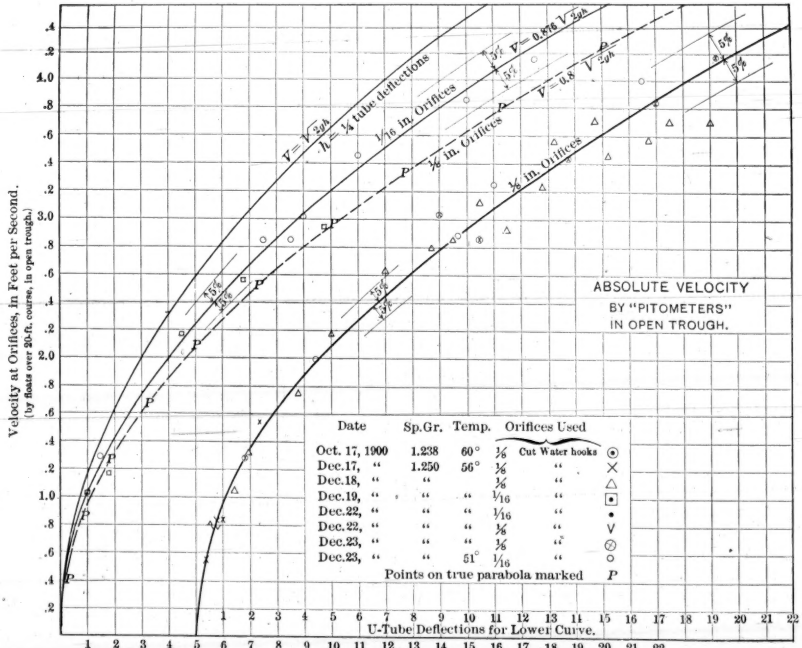


FIG. 107.

Mr. Cole.

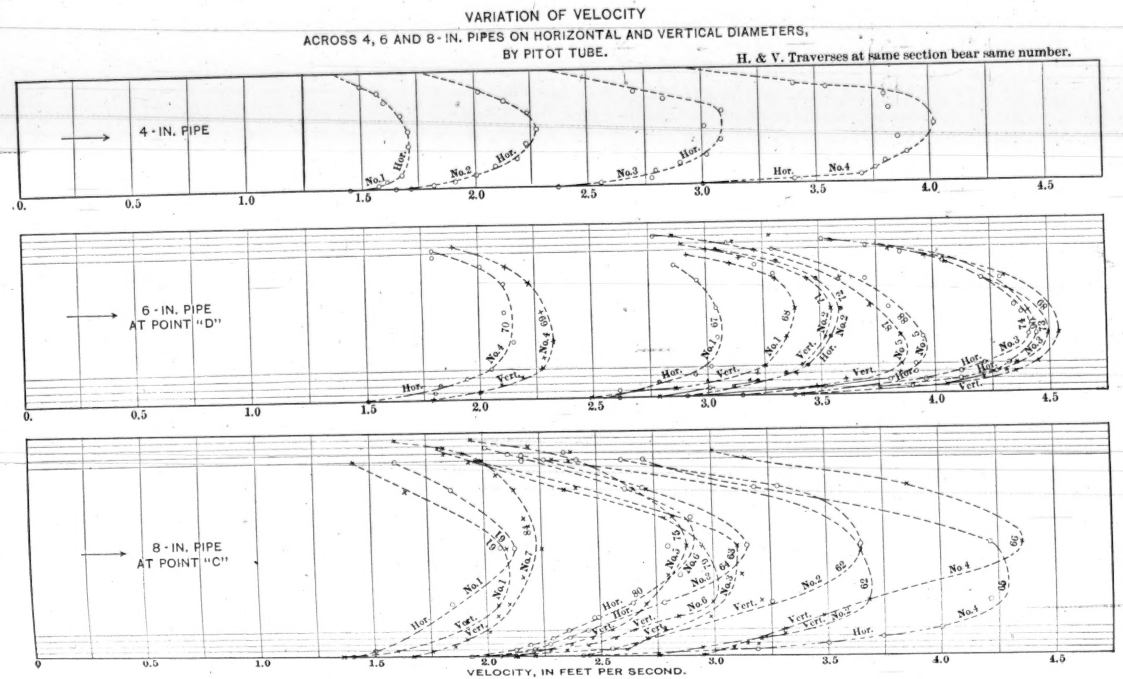

VARIATION OF VELOCITY
ACROSS 4, 6 AND 8-IN. PIPES ON HORIZONTAL AND VERTICAL DIAMETERS,
BY PITOT TUBE.
H. & V. Traverses at same section bear same number.
4-IN. PIPE
6-IN. PIPE AT POINT "D"
8-IN. PIPE AT POINT "C"
Hor.
Vert.
No.1
No.2
No.3
No.4
0.
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
VELOCITY, IN FEET PER SECOND.

FIG. 108.

Mr. Cole. maintain uniformly a differential effect within the U-tube, equivalent to four times the simple water head. This ratio of 4 to 1 in the manometer was used in all the calibrations, and was found most convenient for the range of velocities used.

In order to obtain a rating of the Pitot tube, a series of standard cast-iron water pipes was placed in line and provided with a supply of water from a reservoir giving a constant pressure of approximately 10 lbs. The discharge was measured over carefully calibrated rectangular weirs, and means were provided for regulating the flow through the line of pipes. In this way many series of experiments were made, comparing the deflection of the U-tube at the center of the pipe with the discharge, as shown by the weir for each of the various sizes of pipe tested.

The general plan of the piping, with the various connections, and the location of the several Pitot tubes, is shown in Fig. 105, the tubes being inserted at the mid-length of each size of pipe, a blow-off cock being placed at *A* to insure the removal of all air from the top of the 12-in. pipe, which was the highest point in the line.

Fig. 106 shows five of a long series of calibration curves which were obtained on this and other lines of pipe. It should be remarked that the deflections in the U-tube plotted on these curves are averages of the vibrations noted in the tube during each flow, which continued for about 5 minutes, the weir flow being observed simultaneously.

Fig. 107 shows the rating of the standard form of Pitot tube in a long, open trough, the tube being inserted upward through the bottom of the trough, with orifices located about $\frac{1}{2}$ in. below the surface of the running stream. The velocity of the water was taken carefully by means of floats, and plotted directly with the observed deflections of the U-tube.

This diagram shows the calibration curves of a tube with $\frac{1}{8}$-in. orifices and also with $\frac{1}{16}$-in. orifices. They have been plotted side by side to show the variation in the coefficient. The curve of the $\frac{1}{8}$-in. instrument has been adopted as the standard; this instrument being that with which the various pipe calibrations were made.

The open-stream calibration curve determines the absolute velocity at the orifice, wherever it may be located within a pipe, and use has been made of this curve to plot a series of traverse surveys, as exhibited in Figs. 108 and 109.

Special study has been made as to the ratio of mean to center velocity within the pipes calibrated, and the results of these investigations are somewhat at variance with the authors' Conclusion *C*. In the writer's work the ratio of mean to center velocity is not constant for pipes of all sizes, though it may be for various velocities in the same pipe. A few examples have been taken from the accompanying calibration curves, with the computed ratios, for the purpose of illustrat-

Mr. Cole.

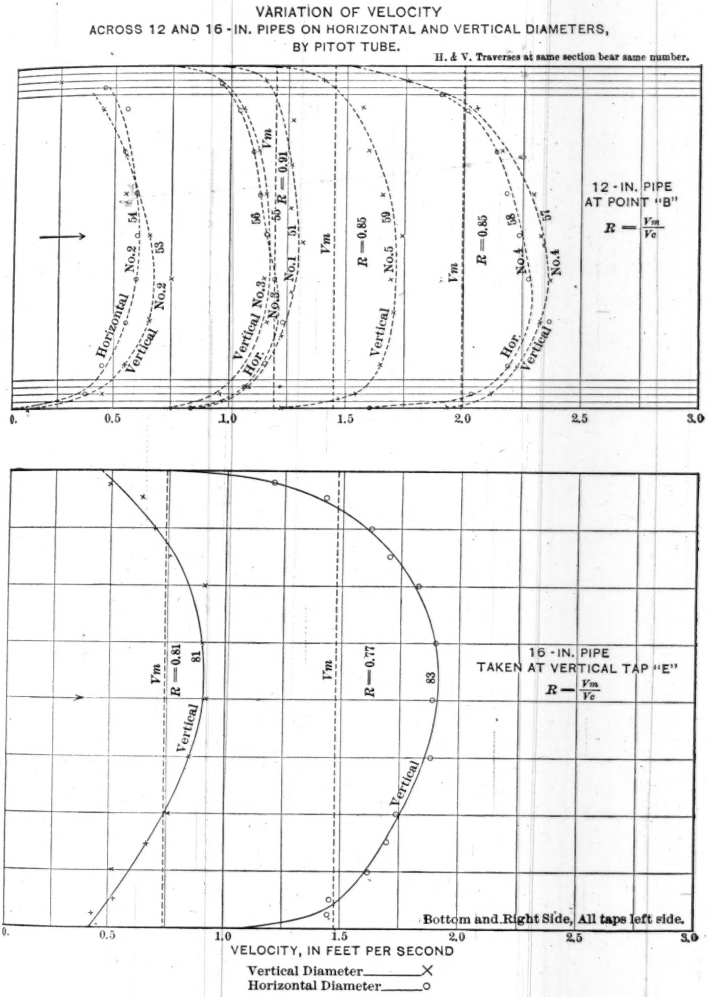


FIG. 109.

Mr. Cole. ing this point (see Fig. 110). The ratios of the mean to the center velocities are shown in Table No. 69.

Attention should be called to the fact that, in the pipes calibrated, the 4, 6 and 16-in. pipes were new, while the 12-in. had been in use for some years, and was slightly tuberculated.

The diameters of these pipes were taken before putting them in

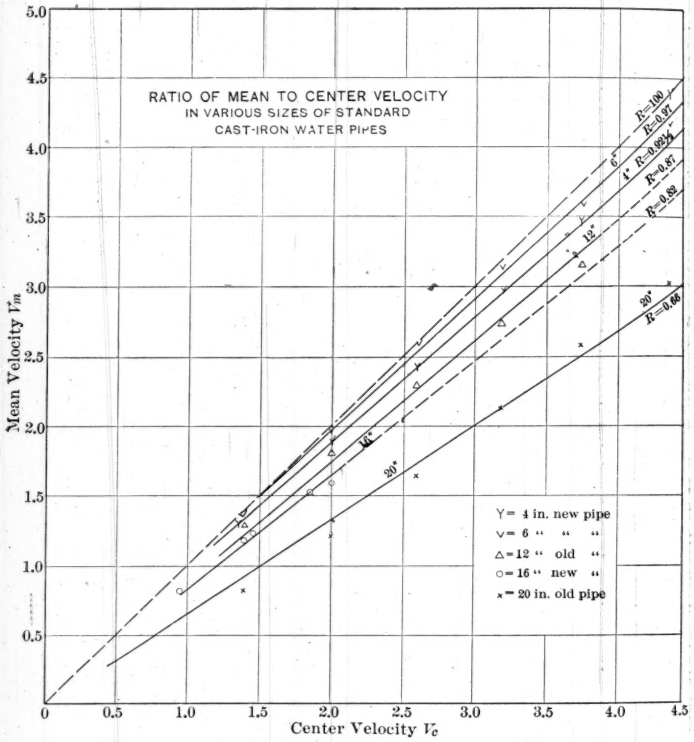
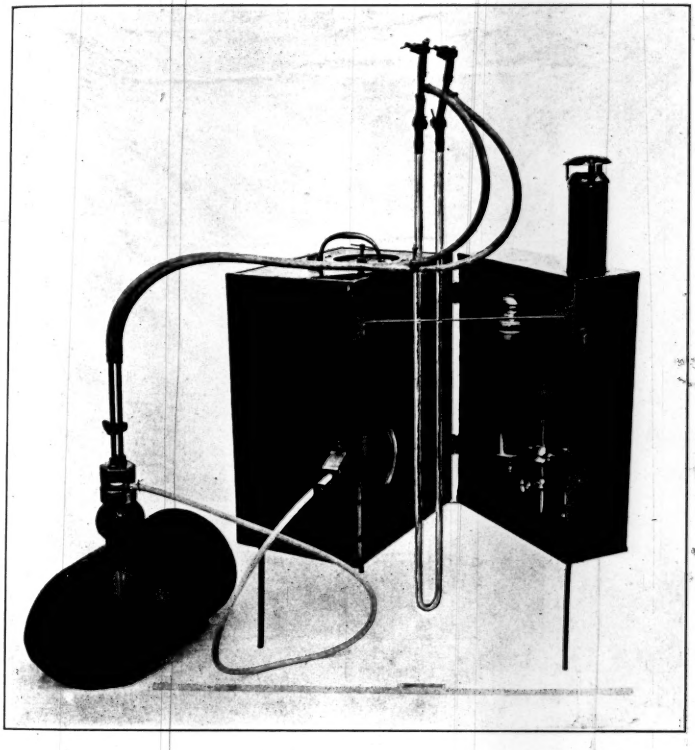


FIG. 110.

place, but no correction for the slight deviation from the standard diameter was made in this work.

It may be of interest to add here, that considerable time was spent in perfecting a recording device for this instrument. It was found that photography was the only practicable means of registering continuously the deflections in the U-tube. Plate VIII shows the recording



THE PHOTO-PITOMETER.

instrument, which, for lack of a better name, has been called the photo-pitometer. This instrument combines, upon one photographic diagram, a record of the static variations within the pipe, with a record of the U-tube deflections, the latter, of necessity, being half-deflections as shown on the U-tube. The instrument has shown itself capable of producing a good continuous record upon ordinary Velox paper, which is convenient for this use. Mr. Cole.

TABLE No. 69.—RATIOS OF MEAN TO CENTER VELOCITY.

Deflection.	Gallons per 24 hours.	Mean velocity.	Center velocity.	Approximate ratio of mean to center velocity.
		4-IN. PIPE.		
4 ins.	100 000	1.79	1.85	0.965
17 "	200 000	3.57	3.85	0.93
21 "	220 000	3.92	4.30	0.91
		6-IN. PIPE.		
4 ins.	237 000	1.87	1.85	1.01
12 "	404 000	3.17	3.20	0.99
22 "	540 000	4.26	4.43	0.96
		12-IN. PIPE.		
2 ins.	655 000	1.29	1.33	0.97
5 "	960 000	1.89	2.07	0.91
8 "	1 215 000	2.37	2.60	0.91
		16-IN. PIPE.		
1 in.	740 000	0.82	0.95	0.86
2½ ins.	1 110 000	1.23	1.46	0.84
4 "	1 375 000	1.53	1.85	0.83

Notches, in the slit through which the light passes on its way to the paper, are so located that they produce horizontal rulings upon the record, corresponding to the calibrated deflections taken from the curves. The combination of static-pressure variation with velocity is a useful feature of this instrument, as it aids in interpreting the cause of any given variation in the flow.

During the winter of 1900–1901, the writer was ably assisted by Messrs. C. B. Burdick and T. C. Phillips, to whom acknowledgments are due for their assistance in the experimental work, and to John A. Cole, M. Am. Soc. C. E., for providing the opportunity for these interesting but expensive studies.

E. E. HASKELL, M. Am. Soc. C. E. (by letter).—The authors are certainly deserving of great credit for the great number of new data contained in this valuable paper. A feature of it which has particularly appealed to the writer is the rating of the Pitot tubes, this being akin to the rating of current meters. E. C. Murphy, Assoc. M. Am. Soc. C. E., in his discussion has pointed out the wide range of values Mr. Haskell.

Mr. Haskell. of the coefficient, *M*, of any tube when rated in the tank. The criticism the writer would offer on this method of rating is, that the tube had to travel a curved path, and that the volume of water used, or cross-section of the tank, was too small.

Believing that this still-water method, which is generally that used in rating current meters, was worthy of further consideration, the writer obtained permission from Major W. L. Fisk, Corps of Engineers, U. S A., M. Am. Soc. C. E., in charge of the United States Lake Survey, to experiment with a tube in connection with the rating of some the Survey current meters. C. W. Hubbell, Assoc. M. Am. Soc. C. E., Engineer to the Board of Water Commissioners of Detroit, kindly loaned Tube *A*, with Gauge No. 4, used by the authors, and it was rated, with the results given in Table No. 70.

TABLE No. 70.—REDUCTION OF OBSERVATIONS OF PITOT TUBE *A* MADE AT DETROIT, MICH., AUGUST 14TH, 1901.

No. and direction of runs.	Duration of run, in seconds.	Velocity head, in inches.	U True velocity, in feet per second.	V Velocity by Pitot tube, in feet per second.	Ratio $\frac{U}{V}$, Coefficient of Pitot tube.	Ratio $\frac{U}{V}$, for pairs of runs.	v.	$v\,v$.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1 S.....	151	1.03	1.99	2.35	0.847	0.811	−0.020	0.000400
2 N.....	136	1.51	2.21	2.85	0.775			
3 S.....	112	1.98	2.68	3.26	0.822	0.824	−0.007	0.000049
4 N.....	93	2.85	3.23	3.91	0.826			
5 S.....	72	4.56	4.17	4.95	0.842	0.824	−0.007	0.000049
6 N.....	80	4.03	3.75	4.65	0.806			
7 S.....	84	3.51	3.57	4.34	0.823	0.820	−0.011	0.000121
8 N.....	74	4.82	4.05	5.09	0.796			
9 S.....	140	1.15	2.14	2.48	0.863	0.818	−0.013	0.000169
10 N.....	112	2.24	2.68	3.47	0.772			
11 S.....	216	0.49	1.39	1.62	0.858	0.844	+0.013	0.000169
12 N.....	185	0.71	1.62	1.95	0.831			
13 S.....	286	0.27	1.05	1.20	0.875	0.890	+0.059	0.003481
14 N.....	313	0.26	0.96	1.18	0.905			
15 S.....	148	1.10	2.03	2.43	0.835	0.826	−0.005	0.000025
16 N.....	137	1.34	2.19	2.68	0.817			
17 S.....	92	2.85	3.26	3.91	0.834	0.824	−0.007	0.000049
18 N.....	94	2.87	3.19	3.92	0.814			
19 S.....	152	1.07	1.97	2.40	0.821	0.811	−0.020	0.000400
20 N.....	133	1.48	2.26	2.82	0.801			
21 S.....	66	5.29	4.55	5.33	0.854	0.843	+0.012	0.000144
22 N.....	63	6.10	4.76	5.72	0.832			
23 S.....	117	1.90	2.56	3.19	0.803	0.805	−0.026	0.000676
24 N.....	107	2.24	2.80	3.47	0.807			
25 S.....	62	6.00	4.84	5.67	0.854	0.864	+0.033	0.001089
26 N.....	74	4.96	4.05	5.11	0.875			
.....	.....	.....	.....	.....	0.831	.....	.....	0.006821

$$\text{Probable error (pairs)} = 0.6745\sqrt{\frac{v\,v}{n\,(n-1)}} = 0.6745\sqrt{\frac{0.006821}{13\,(13-1)}} = \pm 0.004.$$

Mr. Haskell.

The observations for this rating were made in the canal at the Detroit Water-Works by drawing the tube attached to a boom in front of a rowboat over a 300-ft. course, at velocities varying from 0.96 to 4.84 ft. per second, as indicated in Column 4.

The canal is an arm of the Detroit River, running approximately at right angles to it, several hundred feet long, about 45 ft. wide and 14 ft. deep, and has vertical sides. The canal is open to the river, and had no current in it except that caused by a change of stage of water in the river, which was perceptible at times in the course of the observations. To eliminate the effect of this current, the runs were made alternately forward and back, as shown in Column 1. For a guide for directing the boat, a No. 12 telegraph wire, about 600 ft. long, was stretched taut, with block and tackle, over the center line of the canal, about $2\frac{1}{2}$ ft. above the surface of the water. A fair-sized rowboat, of good model, was fitted with a guide at both bow and stern for sliding along this wire, and also with a boom projecting far enough over the bow, so that the tube, with its point about 3 ft. below the surface of the water, could be carried in a vertical position about 6 ft. in front of the boat. The gauge was placed in a vertical position near the center of the boat, and was convenient for the observer to read. The base of 300 ft. was marked off by wires stretched across the canal, leaving approximately 150 ft. at either end for obtaining the proper speed for an observation and for stopping the boat after the run was completed. The two water columns of the gauge were read at intervals of about 5 seconds, the number of readings varying from 12 to 60, depending upon the duration of the run. In reducing the observations, the mean of the readings taken on any one run was taken as the mean velocity head for that observation.

Table No. 70 gives a tabulation of the runs, their respective true velocities U, Pitot tube velocities V, the tube coefficient, and the probable error, considering the observations combined in pairs.

This rating is also shown graphically in Fig. 111, the point C indicating the center of gravity of all the observations, the point A the center of gravity of the observations above C, and the point B, the center of gravity of the observations below C. The point C falls on the straight line, $B\,A$, the equation of which is, $U = 1.199\,V + 0.545$.

The observations for this rating were made by Mr. W. Edward Wilson, Recorder, United States Lake Survey, practically on a first trial. The result is very gratifying, and, to the writer, it seems to be strong evidence of the value of this method of rating.

George Y. Wisner, M. Am. Soc. C. E., in his discussion of this paper, makes some very pertinent remarks upon this subject of rating tubes and current meters, for, generally speaking, the writer's experience with current meters has been that engineers are not careful enough upon this point, nor do they take as good care of their instruments as they should. The writer cannot share Mr. Wisner's views in regard to

Mr. Haskell.

the rating of current meters in still water, however, and submits the following observations for consideration. In June, 1890, Direction-Current Meter No. 3, belonging to the Coast and Geodetic Survey, was rated by the writer, and the observations with their reduction are given in Table No. 71. This rating is also shown graphically in Fig. 111.

Shortly after this rating, a test for accuracy of velocities measured with this meter was made, under the direction of Dr. T. C. Mendenhall, then Superintendent of the Survey, by measuring the same current with it and with floats. This test was made in Vineyard Sound from the Coast Survey Steamer *Blake*. For an observation, the meter was run ten minutes, and the resulting mean velocity was compared with the mean velocity given from the number of floats that could be run in this interval of time.

TABLE No. 71—Reduction of Observations for Rating Direction Current Meter No. 3; Made at New York, N. Y., June 10th, 1890.

Number of revolutions.	Time, in seconds.	y = velocity, in feet per second.	x = revolutions per second.	xy	x^2	Computed y.	v.	vv.	Remarks.
118	396.25	0.505	0.298	0.150	0.088	0.495	+0.010	0.0001	Length of base = 200 ft.
126	340.50	0.587	0.370	0.217	0.137	0.571	+0.016	0.0003	
134	293.25	0.682	0.457	0.312	0.209	0.662	+0.020	0.0004	
151	222.00	0.901	0.680	0.613	0.462	0.894	+0.007	0.0000	
149	219.00	0.913	0.680	0.621	0.462	0.894	+0.019	0.0004	
158	214.75	0.931	0.736	0.685	0.542	0.953	−0.022	0.0005	Meter 5 ft. in front of bow of boat and 2 ft. under surface of water.
153	212.50	0.941	0.720	0.678	0.518	0.936	+0.005	0.0000	
155	203.00	0.985	0.764	0.752	0.584	0.982	+0.003	0.0000	
159	201.00	0.995	0.791	0.787	0.626	1.010	−0.015	0.0002	
163	165.00	1.212	0.988	1.197	0.976	1.216	−0.004	0.0000	
170	143.75	1.391	1.183	1.645	1.399	1.419	−0.028	0.0008	
175	97.50	2.051	1.837	3.768	3.375	2.102	−0.051	0.0026	
174	93.75	2.133	1.856	3.959	3.445	2.122	+0.011	0.0001	
178	88.00	2.273	2.023	4.597	4.093	2.296	−0.023	0.0005	
177	79.00	2.532	2.241	5.672	5.022	2.524	+0.008	0.0001	
179	66.25	3.019	2.702	8.157	7.301	3.005	+0.014	0.0002	
178	65.50	3.053	2.718	8.298	7.388	3.022	+0.031	0.0010	
180	58.00	3.448	3.104	10.702	9.635	3.425	+0.023	0.0006	
182	42.50	4.706	4.282	20.152	18.336	4.654	+0.052	0.0027	
183	41.50	4.819	4.410	21.252	19.448	4.788	+0.031	0.0010	
184	41.50	4.819	4.434	21.367	19.660	4.813	+0.006	0.0000	
185	39.00	5.128	4.744	24.326	22.506	5.137	−0.009	0.0001	
186	37.50	5.333	4.960	26.453	24.602	5.362	−0.029	0.0008	
187	33.00	6.061	5.667	34.343	32.115	6.100	−0.039	0.0015	
187	32.50	6.154	5.754	35.408	33.109	6.191	−0.037	0.0014	
186	27.50	7.273	6.764	49.190	45.752	7.246	+0.027	0.0007	
187	27.50	7.273	6.800	49.455	46.240	7.283	−0.010	0.0001	
.....	.....	80.118	71.963	334.756	308.080	.....	.....	0.0161	

n = number of observations = 27.

$$\Sigma (x^2)\, a + \Sigma (x)\, b = \Sigma (x\, y)$$
$$\Sigma (x)\, a + n\, b = \Sigma (y).$$

The resulting equations for determining a and b are:

$$30.080\, a + 71.963\, b = 334.756$$
$$71.963\, a + 27\, b = 80.118$$

and the values, with their probable errors, are:

$$a = 1.044 \pm 0.0115$$
$$b = 0.184 \pm 0.0053.$$

PITOT TUBE AND CURRENT-METER RATING IN STILL WATER.

○ North Runs of Tube ● South Runs of Tube ⊙ Center of Gravity of Tube Runs

Fig. 111.

Mr. Haskell. The results of the test were as follows, the velocities being expressed in feet per second.

Float.	Meter.
1.04	1.05
1.29	1.28
1.55	1.55
1.94	1.96
2.56	2.59
2.65	2.64
2.76	2.74
2.80	2.82
3.08	3.11

Other tests might be offered, but this one is probably sufficient for the present discussion.

The writer's experience with meters of the propeller-wheel type has been such as to give him perfect confidence in the results obtained with them, if the meters have been used by a careful observer, have been carefully rated in still water of considerable area and good depth, and have been well cared for. He firmly believes that the method of still-water rating on a long straight-away course will be found well suited to the Pitot tube.

Mr. Fisher. WAGER FISHER, Jun. Am. Soc. C. E. (by letter).—Mr. Wisner raises a question as to the effect of capillarity on the accuracy of differential gauge readings. He says:

"It is a well-known principle that capillarity will cause water in small tubes to stand higher than in the reservoir with which connected. This being the case, at what diameter of tube does this effect become sufficient to affect materially the results of the observations?"

The writer, in the spring of 1899, at Cornell University, with specially designed apparatus, and under the direction of Gardner S. Williams, M. Am. Soc. C. E., made a series of observations on tubes ranging in diameter from 0.07 to 0.84 in., over differences in head from zero to 2½ ft. The complete data are on file with the College of Civil Engineering, at Ithaca, N. Y.

The experiments showed that in measuring changes of level in and between reservoirs of glass 7¾ ins. and 5 ins. in diameter, respectively, to which gauges of large and small bore were connected for the purpose, the results were always identical; notwithstanding the fact that in the small tubes capillarity made the water column stand as much as ½ in. above its source, yet any difference of level between reservoirs was represented by precisely the same difference between the gauge readings on the water columns in the glass tubes. In fact, it was shown that dirt and individual peculiarities of tubes produce far greater errors than any supposed errors due to capillarity, should they be discoverable.

The same line of experiments was extended to differential gauges in

which oil was used to magnify the difference of head, and precisely similar results were secured. Mr. Fisher.

As to the accuracy of the observations made, it is proper to state that a cathetometer, with telescope, reading to $\frac{1}{1000}$ cm., was used to measure the heights in the reservoirs and tubes.

It may be said, therefore, that a difference of head of 1 ft. indicated by a tube having an interior diameter of $\frac{1}{16}$ in. indicates an actual difference of 1 ft. of head in the reservoir.

ROBERT E. HORTON, Esq. (by letter).—This paper constitutes an important contribution to the technique of hydraulic measurement. A small class of New England engineers was, perhaps, familiar with the uses of the Pitot tube as an instrument of precision. Most of the profession, it is believed, were hitherto inclined to look upon it as a scientific toy. Those who have read this paper may differ in opinion as to the range of legitimate application of the Pitot tube in practice. To the writer, its scope of usefulness seems to be a narrow one, in fact, is almost limited to the purpose for which the authors of the paper have applied it, viz., determining the flow in pipe lines and small closed conduits or penstocks, in cases where an open-channel measurement, at either end of the conduit, by weir, current meter, or other means is impracticable. In this way, it may be applied to determining the friction head in pipes and penstocks, or the loss of head due to bends, specials, or gate-valves. These are all points on which a great deal of additional information is desired. Mr. Horton.

The difficulties attendant upon the use of the Pitot tube are very great, as has been clearly set forth by the authors, and so many operations are required, in reducing the data, as to discourage its use in all cases where simpler, if less scientific or precise, means can be resorted to. The tube "point" must be of standard form, otherwise, known calibration coefficients will not apply. The pipe or conduit should be of precisely equal diameters at both points of measuring the head. If oil is used, the coefficients for reducing the tube readings are not directly proportional to the specific gravity of the two fluids used, but must be separately determined by elaborate experiments. The use of long hose connections, especially those of small diameter, is undesirable on account of the great difficulty of becoming perfectly sure that the hose and connections are entirely free from entrained air.

The authors deserve great credit for their endeavor to place before engineers the necessary data for reducing Pitot tube measurements when made under suitable or standard conditions. They have made the Pitot tube available for use in the class of cases above described where all other methods fail.

It is interesting to compare the Pitot tube with the current meter; the former instrument being applicable to closed and the latter to open channels.

Mr. Horton.

1. Both are used to determine the velocity of flow in a large number of small elements of the cross-section to be measured; the volume of flow being obtained by summation of the elementary discharges.

2. The current meter determines the existing velocity directly. The Pitot tube indicates the head produced by, or required to produce the existing velocity.

3. The current meter (of the "Price" variety) indicates the velocity of flow correctly only when its axis is parallel to the thread of the current.* The Pitot tube indicates the statical head, or statical and velocity heads combined, according as the axis of the orifice is normal to or parallel with the flow of the current.

4. The current meter has been shown to give different results when held rigidly in the current than when allowed freedom to move in one or more directions. It is believed that the rigidity with which the Pitot is held, especially in flowing water containing swirls or impulses, will affect the result in some degree.

5. The current meter indicates the velocity of the passing water regardless of its direction of flow. In deep or turbid water, where the meter is invisible, a direction indicator may be applied.† In cross-currents or eddies the Pitot tube fails to indicate correctly, either the direction or velocity of the water. This appears to the writer to constitute one of the chief difficulties in its use.

6. The authors have shown the rating coefficient of the Pitot tube to have different values when drawn through still water and when calibrated in flowing water. Recent experiments indicate that the same may be true of the current meter. Practically all flowing water, especially in small pipes and channels, appears to contain a certain amount of what might be termed, "internal motion," consisting of small swirls, eddies or vortices. The effect upon an instrument inserted to determine the velocity has been described as being what would result if the water moved in independent "chunks." When rated in still water, the current meter, or Pitot tube, does not encounter these swirls or eddies.

The authors conclude that the transverse curve of velocity in the pipes upon which they experimented has the form of an ellipse. In this connection, it may be well to consider further the nature and effect of the so-called internal motion of the water. In a closed pipe or conduit, the velocity of the particles around the circumference is retarded by friction. These in turn retard their neighbors next inside, and so on to the center of the pipe or channel. This is not the whole story, however, in the case of fluids as limpid as water, as may be demonstrated ocularly by placing aniline dye near the margin of a small stream

* Nineteenth Annual Report, U. S. Geol. Survey, Part 4, Hydrography, pp. 27 to 30, inclusive.

† Report of Chief of Engrs., U. S. Army, 1900, p. 5333.

of flowing water.* Little groups of particles near the border are differentiated from the mass; at the same time they are carried on with it. Their motion holds them together in what might be termed a dynamical system, namely, a "vortex." The laws of motion of vortices were investigated by Helmholtz, who showed that each bears a definite burden of kinetic energy, abstracted, in this case, from the energy of the mass, constituting a definite loss of head. The vortex may be carried on a considerable distance before it is broken up by the friction of the fluid particles themselves and its energy imparted to the individual particles and converted into heat. Mr. Horton.

In a highly viscous fluid, the generation of these vortices is prevented and the surface retardation is transmitted from lamina to

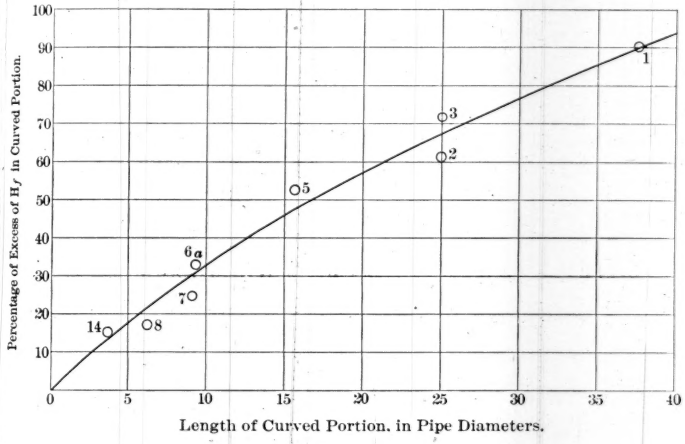


FIG. 112.

lamina, growing less and less as distance from the border increases. In such cases it is possible that the law of inward retardation may be such that the equation of the transverse velocity curve might be definitely determined. In the case of water, a large number of vertical velocity curves in open streams of varying depth, velocity and character of perimeter, obtained by the writer by means of the current meter, indicates that the velocity curve may vary from a flat arc to an equilateral hyperbola having as its asymptotes a diameter and an element of the channel perimeter. For certain sets of conditions, the curve will closely approach a parabola or a semi-ellipse, as stated by the

* "The Character of Fluid Motion," by Professor Hele-Shaw, *Proceedings*, Inst. Naval Architects, 1898.

Mr. Horton. authors. Either curve may be useful as a working hypothesis in particular cases.

The curve in Fig. 90 does not appear to represent the data fairly inasmuch as a large majority of the plotted points lie above the line as drawn. The line drawn is not a smooth curve, but has a sudden change of curvature near the point 6*a* and 2–3. It is a question whether the data would not be represented better by a line passing through the origin. Furthermore, the upward curve at the foot does not seem to be justified by the plotted points, unless it is desired to give some weight to the form of curve adopted by earlier experimenters. In order to get at more nearly the true significance of the data *per se*, Fig. 112 has been prepared, using, for the sake of uniformity, the data for 30-in. pipes only. A mean curve has been drawn through the plotted points by judgment, without any attempt at determining its position mathematically. Table No. 72 will give some idea of the relative weight of the plotted points, as stated by the authors.

TABLE No. 72.—DATA FOR CURVE EXCESS IN 200 FT. OF 30-IN. PIPE.

Number.	Description.	Percentage of excess.	Remarks.
1	60-ft. radius	90.2	
2	40-ft. radius	61.5	Castings very smooth.
3	Second large compound	71.8	
4	First large compound	119.5	Contains gate, etc.; rejected.
5	25-ft. radius	52.7	
6*a*	15-ft. radius	33.0	Corrected for specials.
7	Small compound	24.8	
8	10-ft. radius	17.3	
14	5-ft. radius	15.7	

This curve is not offered as a substitute for that given by the authors. It is merely designed to bring out more clearly the character of the data for the 30-in. pipes. The practical conclusion is that, taking into consideration the greatest relative length of quadrantal bends of large radius, and the difference in smoothness, if any, of the interior surface of bends, compounds, and special castings as compared with straight lengths, the resultant excess of friction head over a straight pipe of the given length increases as the radius and length of the bend increases. The data do not appear to be sufficiently numerous or concordant to enable working coefficients to be taken out with precision.

Mr. White. W. M. WHITE, Esq. (by letter).—This paper is of such wide scope and varied treatment that one can hardly be expected to discuss intelligently all its important features.

The authors express a doubt as to the correct theoretical formula for use in the reduction of the Pitot tube readings. There seems to be

quite a diversity of opinion as to whether the formula is $v = \sqrt{2gh}$, according to the velocity-head theory, or whether it is $v = \sqrt{gh}$, according to the impact theory. Mr. White.

In a paper,* by the writer, a series of experiments is given which seems to prove clearly that the theoretical and practical formula is $v = \sqrt{2gh}$ for Pitot tube points. The experiments show that velocity head is converted into static head exactly according to the law $v = \sqrt{2gh}$, whether the effect of the impinging jet be considered according to the impact theory or according to the velocity-head theory.

The introduction of a constant, as $v = \Phi\sqrt{2gh}$, is nearly always necessary on account of some suction action at the pressure, or side, openings of the tube.

The authors give ample evidence that the ratings of some Pitot tubes, obtained in open canals, do not apply when the tubes are used in closed conduits under pressure.

There can be only two causes affecting this result, when the cross-section of the conduit is not materially affected by the introduction of the tube: First, some changes in the law for the conversion of velocity head into static head at the point, due to the increased pressure; or, second, some change in the value of the suction action at the pressure opening due to the increased pressure. That the law for the conversion of velocity head into static head can be affected by pressure is hardly conceivable, because the pressure at the point is increased by an equal amount in all directions. The suction action is caused by irregularities of the tube near the pressure openings, as, for instance, in Tube No. 6, where the abrupt curves of the tube at the pressure openings cause a suction action equal to 40% of the velocity head of the impinging water. These irregularities distort the flow of the water, causing it to take a direction away from the openings. This flow, striking the surrounding water with constant impact, relieves the pressure at the openings. When the pressure of the surrounding water is small, the effect of this impact is large, comparatively. When the pressure of the surrounding wall of water is increased, the effect of this distorting force is small, comparatively, and hence changes the coefficient of the tube.

In June, 1900, the writer made some tests† with a Pitot tube enclosed in a 52-in. pipe. The pressure within the pipe was a little less than atmospheric pressure. The tube was worked under an average pressure of 2 ft. of water. The side of the pipe was tapped for a static opening, corresponding to a ring piezometer of the authors' description, and a pipe led to the Pitot tube gauge, connecting it with the

* *Journal of the Association of Engineering Societies*, August, 1901.

† *Journal of the Association of Engineering Societies*, October, 1900.

Mr. White. point and pressure pipes. The point and static readings were treated as those of a Pitot tube having a constant of unity, and the constant of the point and pressure was calculated to be 0.856. The Pitot tube was afterward calibrated in an open canal under a head of 2 ft., and its constant was determined as 0.849. That is to say, the point of the Pitot tube converted velocity head into static head according to the law $v = 0.993 \sqrt{2gh}$. This is well within the limits accorded to the errors of observation. An examination of the paper will show the same thing when those tubes having points which are surfaces of revolution are considered with ring openings for pressure readings. For four observations, the constant of the point does not vary from unity more than 3 per cent. In fact, a traverse in the 2-in. pipe agrees with the writer's deductions within one-half of 1 per cent. From this it appears to be clear that the difference of the ratings in open canals and closed conduits is caused by some action at the pressure openings due to increased pressure.

The authors have brought out several forms of tubes, and have tested them under the same conditions. The most instructive tests are those made in the 2-in. pipe with pressure and ring piezometer readings for the same point. A study of the tables shows that those points having surfaces of revolution, for instance, Tubes Nos. 3 and 6, convert velocity head into static head, according to $v = \sqrt{2gh}$. Tubes *C*, *D* and *E*, having points which are not surfaces of revolution, do not always give the full velocity head, as shown by Table No. 10.

The reason for this is not far to seek. In Fig. 113 are shown the stream lines of flow and the curves of pressure on a rectangular plate. By referring to these it is seen that the greatest pressure occurs where the velocity of flow is least. Along the line *A B* there will be no flow, and the pressure will be a maximum. The maximum pressure that can be obtained is equal to the velocity head of the flowing stream. Orifices placed along the line *A B* will give the maximum pressure; but orifices placed at *G* and *H* will not give the total velocity head, because the water does not come to rest at those points. It appears, from the results of the calibrations, that the orifices of Tubes *C*, *D* and *E*, were not placed so that they would give the maximum pressure obtainable, because the constants of the points are slightly greater than unity. In a circular flat plate, Fig. 114, there will be only one point, where there can be placed an orifice which will give the total velocity head, and that is at the center.

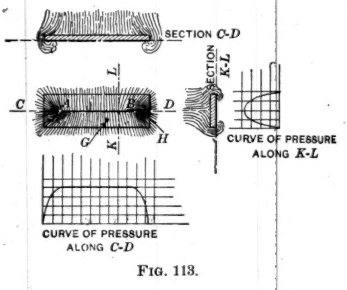


FIG. 113.

Mr. White.

There will be a point, or points, in any plane surface where an orifice can be placed so that it will give the full velocity head, but there will also be many other points in the same plane where this will not be the case, and, consequently, to the writer, it seems best to adopt only surfaces of revolution for Pitot tube points.

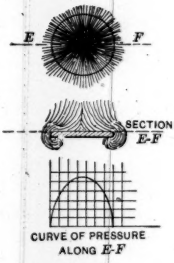


FIG 114.

The writer bases his deductions, of the streamline flow from a rectangular plate, on the following experiment on the variation of pressure on a flat circular plate:

For this experiment a flat circular plate, 8 ins. in diameter, having ten $\frac{1}{32}$-in. holes spaced along a diameter, was used. These holes were connected by nipples and rubber tubing to vertical glass tubes, as shown in Fig. 115.

The apparatus was placed over the side of a rowboat, and partially submerged. As the boat was driven forward, water rose in the glass tubes, due to the impact of the water on the plate, and outlined the pressure curve, as shown in Figs. 114 and 115.

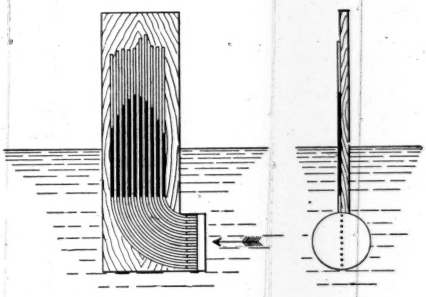


FIG. 115.

In the authors' summation at the end of the paper, Conclusion *F*, is given, as follows:

"The demonstration that Pitot tubes must have their coefficients determined, whether they consist of a point opening alone, or both point and pressure openings."

In the light of recent experiments, this claim must fail, in so far as it applies to points having surfaces of revolution; but, whatever new points may be brought out concerning the Pitot tube, the fact remains that the authors used the tubes at their disposal in such a manner as to obtain correct results.

The amount of painstaking care and labor devoted to the calibration and correction of the instruments used in these experiments is marvelous. In this line of work the authors have set a high standard of excellence which is well worthy of emulation.

Messrs. Saph and Schoder.

A. V. SAPH, Assoc. Am. Soc. C. E., and E. W. SCHODER, Jun. Am. Soc. C. E. (by letter).—The writers became associated in experimental hydraulics in the spring of 1901, while pursuing graduate work in the

Messrs. Saph and Schoder.

College of Civil Engineering, Cornell University. Their attention was first directed toward the line of experimentation herein described by Gardner S. Williams, M. Am. Soc. C. E., Engineer in Charge of the Hydraulic Laboratory. They gladly fell in with his suggestion to utilize certain parts of the laboratory apparatus which were well adapted for the performance of accurate experiments along the same lines as those dealt with in the paper.

The idea of the writers, at the beginning of the work, was to find out, by means of Pitot tube traverses, in what manner the normal conditions of the flow of water in pipes were altered by passage around a curve, and in what manner the altered conditions again resumed the normal in the succeeding tangent. Along with this, they planned to perform comprehensive experiments on the loss of head due to 180° curves of various radii.

General Description of Apparatus Used.

The experiments were made on a system of 2-in. seamless-drawn brass pipe, the plan of which is shown in Figs. 116, 117, 118 and 119, and which is described in part on pages 18 and 20 of the paper. Four different arrangements are shown, but the greater part of the work was done with the system arranged as illustrated in Figs. 118 and 119. In all cases the up-stream tangent remained the same for a distance of 58.146 ft. from the P. C. of the curve, while changes in the down-stream section were made as the work progressed, and a better knowledge of conditions was gained.

Description of Joints, Piezometers and Gauges.—All joints were made with the idea of obtaining, as nearly as possible, a continuous pipe. The pipe ends were turned off squarely, and threaded to receive the flanges. The threads were cut off near the ends to allow adjustment. When joining two pipes, one of two abutting ends was allowed to project beyond its flange, while, at the other end, the flange projected beyond the end of the pipe. The pipes being placed in position, the flanges were bolted and the pipes screwed to a tight bearing. The ridge on the inside at the junction of two pipes was, in all cases, very small; that in the extreme case between Curve No. 6 and the tangents being 0.007 in. In a number of piezometer specials, made of short pieces of pipe, the ridges could hardly be detected by the finger tips.

The piezometer specials used are illustrated on page 17, and are also shown in Plate IX, Fig. 1. The latter shows, also, an air-chamber connected to a ring piezometer, Pitot tube *G* in position in a stuffing-box, and the flanges for connection with the pipes. These flanges are the same as those mentioned in the preceding paragraph. The gauges (all of the differential type), and the hose connections between the gauges and the piezometers, need no further description

Messrs. Saph and Schoder.

ARRANGEMENTS OF 2-INCH BRASS PIPE.

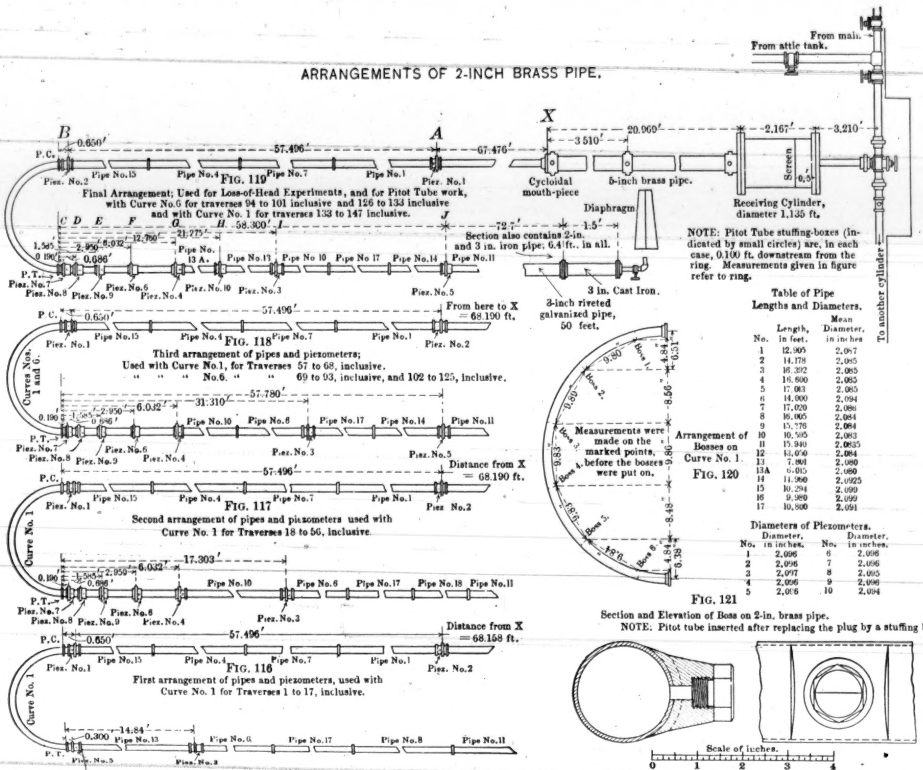


Table of Pipe Lengths and Diameters.

No.	Length, in feet.	Mean Diameter, in inches
1	12.905	2.087
2	14.178	2.085
3	16.392	2.085
4	16.800	2.085
5	17.001	2.085
6	14.000	2.094
7	17.020	2.086
8	16.005	2.084
9	15.176	2.084
10	16.595	2.083
11	15.940	2.0925
12	14.050	2.084
13	7.881	2.080
13A	6.005	2.089
14	14.960	2.0925
15	16.294	2.099
16	9.989	2.099
17	10.800	2.091

Diameters of Piezometers.

No.	Diameter, in inches.	No.	Diameter, in inches.
1	2.096	6	2.096
2	2.096	7	2.096
3	2.097	8	2.095
4	2.096	9	2.096
5	2.096	10	2.094

FIGS. 116 TO 121.

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than that given in the paper, except to state that the gauge scale unit in all cases was the double centimeter. The weighing apparatus has also been described by the authors, on page 20.

The Curves.—Six different curves were experimented upon. They were made of 2-in. seamless-drawn brass pipe, bent into an arc of 180°. The dimensions of these curves are given in Table No. 73. Seven different radii were measured on each curve, the values given in the table being the means of these. Two inside diameters at right angles to one another were measured at each end, the mean of the four measurements on each curve being taken as the true mean. To check this, outside caliperings on two diameters were taken at seven different sections on each curve. These measurements remained quite constant, and it was judged that the inside diameter was the same for all sections.

TABLE No. 73.

(1) Number of curve.	(2) Radius, in feet.	(3) Radius, in pipe diameters.	(4) Diameter, in inches.	(5) Length from Piezometer No. 2 to Piezometer No. 7, in feet.
1	1.6598	9.582	2.078	6.053
2	1.3177	7.631	2.077	4.960
3	0.9974	5.771	2.074	3.973
4	0.8229	4.759	2.075	3.425
5	0.7500	4.327	2.080	3.196
6	0.6588	3.838	2.060	2.910

Bosses on Curve No. 1.—In order to enter Curve No. 1 at different points with a Pitot tube, bosses were soldered to the curve at six places, as shown in Fig. 120. Bosses Nos. 1 and 6 are 15° from the P. C. and P. T., respectively, and the others are placed at intervals of 30 degrees. The construction of these bosses is shown in Fig. 121. When it was desired to insert the Pitot tube, the plug was removed and a stuffing-box, of the pattern shown in Plate IX, Fig. 1, was inserted. This construction allowed horizontal traverses to be taken at the different points on the curve. The ends of the plugs were carefully ground, so that when screwed into place they fitted the inside of the curve accurately.

Work with Pitot Tubes.

Description of Tubes Used.—The first portion of the work was done with Tube *E*, described on page 8, and illustrated in Fig. 4 and Plate I, and also in Plate IX, Fig. 2. Traverses 1 to 66, inclusive, were made with this tube.

PLATE IX.
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ON FLOW OF WATER IN PIPES.

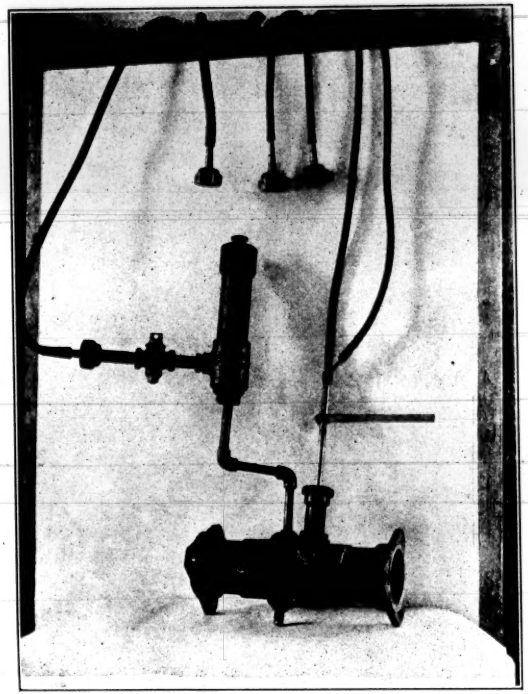

FIG. 1.—PITOT TUBE *G*, STUFFING-BOX AND HOSE CONNECTIONS.

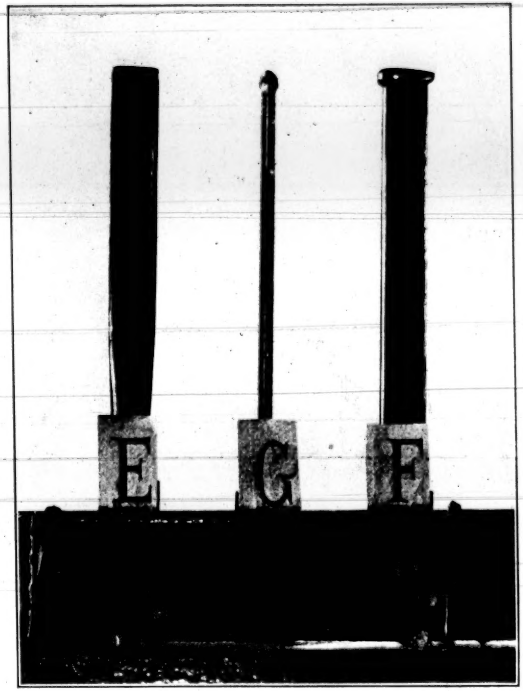

FIG. 2.—PITOT TUBES *E*, *G* AND *F*.

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Later, two tubes, *F* and *G*, of the point and pressure type, were built. They are as nearly alike as an expert mechanician could make them. In addition to the two side-pressure openings, seen in Plate IX, Fig. 2, there is a third opening at the bottom of the tube, all three openings communicating, through a small hole in the blade of the tube, to a single pressure connection at the top. Plate IX, Fig. 1 shows Tube *G*, the stuffing-box, and the hose connections. If only a point and pressure traverse was taken, the ring piezometer was not used, and the point and pressure connections were attached to a double instead of a triple gauge. Traverses 67 to 148, inclusive, were made with Tubes *F* and *G*.

Methods of Observation.—In making a traverse, the tube was set on the desired point and carefully aligned by means of the pointer on the stem. Several minutes were allowed for the gauge to settle, before commencing readings. In general, six observations, at intervals of 15 to 30 seconds, were taken on each point, an observation consisting of a reading of the meniscus in each column of the gauge.

Throughout the traverse the flow was kept constant, and, at intervals, the discharge was diverted to the weighing tank, the time being taken with an ordinary watch. The temperature of the water was taken in the weighing tank with an accurate thermometer, immediately after filling.

Before the beginning and after the end of each traverse the zero or level readings of the columns were recorded. It may be remarked in passing that no traverse was begun with an initial difference of more than 0.02 double centimeter, that being, in the writers' opinion, the limit of initial error allowable in laboratory work, a greater difference being probably due to the presence of air, or to leaks in the connections.

Methods of Reduction.—In reducing weights to velocities the weight per cubic foot of water was taken from a table, given by the late Hamilton Smith, Jr., M. Am. Soc. C. E., for distilled water at various temperatures. It had been determined previously that the specific gravity of the water used, at 68° Fahr., was identical, to the fourth decimal place, with that of distilled water at the same temperature.

In reducing observed heads to velocities, the latter were taken from a curve plotted according to the relation $V = \sqrt{2\,g\,h}$. The third place was gotten by estimation, and may be somewhat in error. In obtaining the mean velocity by tube at any cross-section, it was assumed that the head observed at any point acts over a ring, the width of which is the distance between two points, and which extends an equal distance on each side of the point. This method of approximation gives results about 0.8 of 1% too low (*i. e.*, for normal flow). The method does not apply in cases of distorted, unsymmetrical flow, as will be shown later.

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Results of Pitot-Tube Experiments.

The evidence of 148 Pitot-tube traverses shows that the disturbance caused by a smooth curve is not of the highly complex and indefinite nature ordinarily ascribed to it. The point of maximum velocity is merely shifted toward the convex side of the pipe, and with this shifting there is of necessity a variation in the shape of the velocity "contours." These, in normal flow in straight pipe, are concentric circles. The change in their shape after passage around a curve is admirably shown in Fig. 122. Here it is seen that the maximum velocity is shifted from the center to a point about five-sevenths of the radius toward the convex side. The "contours" are no longer circles, but are flattened out, while the ends are forced outward toward the top and bottom and back toward the concave side. There seems to be a peculiar combination of the efforts of the faster moving water to remain in the center, its normal position, and at the same time to get to the convex wall of the curve. As might have been expected, there is symmetry about a horizontal plane through the axis of the curve.

The experimental evidence, showing how this final distorted condition has been reached, and how it again reaches the normal condition, will next be examined. In Fig. 124 are plotted eight horizontal traverses; one just up stream from the P.C. of Curve No. 1, six in the curve itself, and one just down stream from the P. T. Table No. 74 gives the condensed data for these traverses. The traverses are plotted so that their maxima are separated by a distance proportional to the distances between the points where the traverses were taken. Then a curve is sketched through the points of maximum velocity, showing graphically how the distortion takes place in passing around the curve.

It will be seen that the greater part of the distortion has taken place between 15° and 45° from the P.T. Now, the axis and the concave side of the up-stream tangent, when prolonged, cut the outer wall of the curve at points about 18° and 26°, respectively, from the P.C. It thus appears that most of the distortion occurs in the region where the stream as a whole is first being deviated from its former direction. It is very evident that there is little change in the second quadrant of the curve. The flowing water has already accommodated itself to the curvilinear motion, and there is thereafter a similar constancy (under different conditions, of course) to that which exists in the flow in straight pipe.

The water, having passed around the curve, enters the down-stream tangent under the fully distorted conditions just described. But there is no evidence showing a rebound, or the existence of complicated eddies or spirals. On the other hand, the distortion in a most gradual manner changes back to normal conditions, and, having done so, the water flows on as though no curve had ever changed its course. On these points the experimental evidence is quite conclusive.

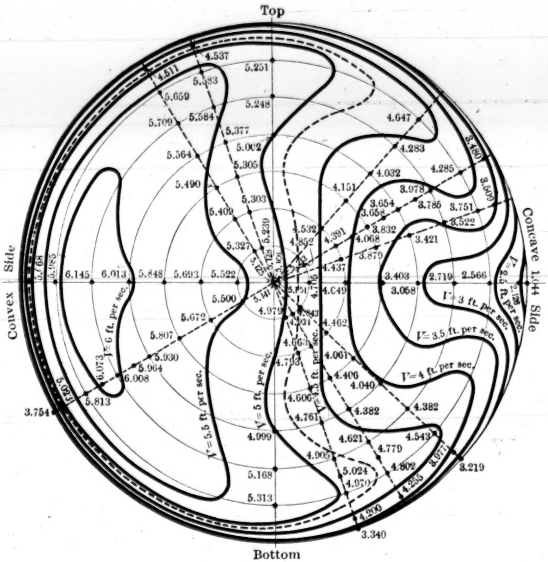

FIG. 122.

Diagram, showing velocity contours just beyond a 180° curve, in 2-in. brass pipe. The points plotted above are obtained from the formula $V=\sqrt{2gh}$, h being the observed difference of head (point and ring by Tube G) at the various points of the cross-section, at a distance 0.290 (= 1.66 diameters) from the P.T. of Curve No. 1 (Radius = 1.659 = 9.58 diameters.)

The extreme range of mean velocity (by weight) was 4.817 to 4.859 ft. per second. Average mean velocity = 4.840 ft. per second (indicated in sketch by broken line.)

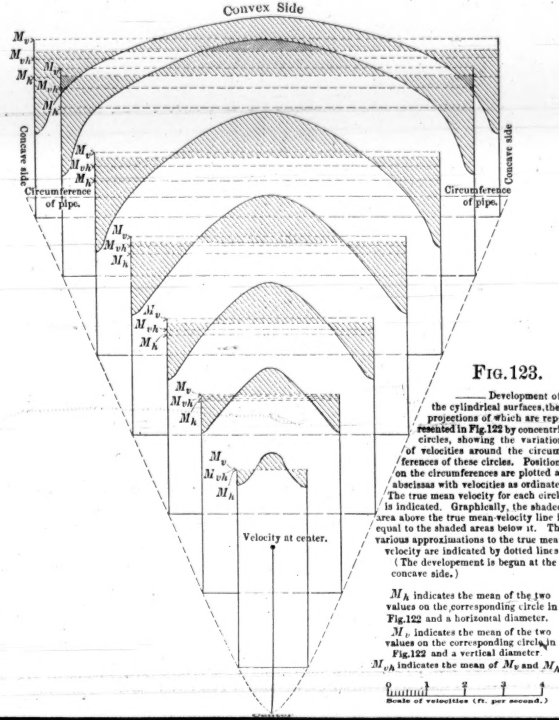

FIG. 123.

Development of the cylindrical surfaces, the projections of which are represented in Fig. 122 by concentric circles, showing the variation of velocities around the circumferences of these circles. Positions on the circumferences are plotted as abscissas with velocities as ordinates. The true mean velocity for each circle is indicated. Graphically, the shaded area above the true mean-velocity line is equal to the shaded areas below it. The various approximations to the true mean velocity are indicated by dotted lines. (The developement is begun at the concave side.)

M_h indicates the mean of the two values on the corresponding circle in Fig. 122 and a horizontal diameter.

M_v indicates the mean of the two values on the corresponding circle in Fig. 122 and a vertical diameter.

M_{vh} indicates the mean of M_v and M_h

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Fig. 125, in which the traverses are plotted, as in Fig. 124, shows graphically the gradual return to normal conditions in the down-stream tangent, for both horizontal and vertical traverses. In order to get traverses as nearly horizontal as possible it was necessary to use Traverses 18 and 45 by Tube *E* beyond Curve No. 1. The observed velocities in these traverses were reduced to what they would have been if Tube *F* had been used with point and pressure, instead of point and ring. This, however, in no way changes the approximate position of the point of maximum velocity. The occasional apparent disregard of the maximum shown by the plotted traverses, in sketching the locus of the maximum velocity, will be explained presently.

Further evidence of this gradual return is exhibited in Table No. 75, giving the essential data of all traverses.

These traverses, both horizontal and vertical, show the gradual change back to normal conditions. The horizontal traverses show the maximum velocity to be always on the convex side, until, at a distance of 31.400 ft. (181 diameters) from the P. T., if not before, the flow has become symmetrical and remains so thereafter. The traverses show the same general distribution of velocities for all mean velocities in the range of the experiments (from about 1 ft. per second to about 5 ft. per second), so that there is no evidence that a variation of velocity within these limits affects the distance through which distortion extends. A peculiar thing, however, is noticed concerning the point and pressure traverses made with Tubes *F* and *G*, in which a slightly distorted flow is indicated where normal conditions exist. The simultaneous point and ring traverses show symmetry, while point and pressure traverses invariably show the maximum to be on the side of the center away from the entry. This result is due to the effect upon the pressure openings of the disturbance caused by the presence in the pipe of the tube itself.

The effect just mentioned is of much importance. The design of the pressure openings, if possible, should be such that they will at all times communicate a pressure bearing a constant ratio to the mean pressure at any given point. As to what constitutes a well-designed tube, it seems certain that the pressure openings should be symmetrically placed about the axis of the point opening, and, moreover, they should be so placed that they may be affected as nearly equally as possible by the disturbances set up by the passage of the water past the end of the tube. These two considerations would indicate that Tubes *F* and *G* are not of the best design, which is further shown by the evidence just mentioned regarding their peculiar indications where normal flow is known to exist. The writers made a traverse, at at a point 58.30 ft. down stream from Curve No. 1, with Tube *G*, the two side-pressure openings having been closed with sealing wax, leaving only the bottom opening. The point and pressure traverse showed

TABLE No. 74.—Traverses with Pitot Tube *G*, in and near Curve No. 1, in 2-Inch Brass Pipe.

Length of Curve, 180° = 5.213 ft. = 30.133 Pipe Diameters on Axis of Pipe.
Radius " " 1.659 " = 9.58 " " at " " "
Mean Diameter of Cross-section of Pipe in Curve, 2.078 ins. = 0.1732 ft.

Traverse Number.	Location.	Entry.	By Pitot Tube.					By Weight.		Remarks.
			Maximum Velocity.							
			Position.		Amount, Feet per Second.	Ratio, $\frac{V_m}{V_{max.}}$	Mean velocity, V_m Feet per second.	Mean velocity, U Feet per second.	Ratio: $\frac{U}{V_m}$	
			$r \div R$	Side.						
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
					By Point and Pressure.					
120..	Piez. 2 = 3.139 Diams. above P. C.	Cv. 45° above Hor.	0.000	Center.	7.024	0.809	5.682	4.922	0.866	Normal.
126..	Boss 1 = 2.511 " below "	" Horizontal.	0.143	Convex.	7.010	0.813	5.700	4.900	0.860	Distorted.
134..	" 2 = 7.533 " " "	" "	0.572 0.715	"	6.783	0.746	5.054	4.928	0.975	"
140..	" 3 = 12.555 " " "	" "	0.715	"	6.780	0.778	5.275	4.924	0.933	"
135..	" 4 = 17.577 " " "	" "	0.715	"	6.898	0.767	5.289	4.918	0.929	"
139..	" 5 = 22.600 " " "	" "	0.715	"	6.977	0.753	5.250	4.920	0.937	"
137..	" 6 = 27.621 " " "	" "	0.715	"	6.939	0.740	5.138	4.899	0.953	"
138..	Piez. 7 = 1.618 " " P. T.	" "	0.715	"	7.116	0.718	5.110	4.817	0.943	"
148..	" 7 = 1.618 " " "	" 1° above Hor.	0.715	"	7.069	0.713	5.043	4.829	0.956	"
141..	" 4 = 1.676 " " "	Cx. 30° below "	0.643	"	6.880	0.822	5.655	4.849	0.857	"
145..	" 4 = 1.676 " " "	Cv. 49° above "	0.715	"	6.739	0.865	5.832	4.838	0.829	"
144..	" 4 = 1.676 " " "	" 71° below "	0.858	"	6.564	0.888	5.828	4.856	0.833	"
					By Point and Ring.					
120..	Piez. 2 = 3.139 Diams. above P. C.	Cv. 45° above Hor.	0.000	Center.	6.072	0.822	4.983	4.922	0.988	Normal.
148..	" 7 = 1.618 " below P. T.	" 1° below "	0.715	Convex.	6.157	0.707	4.352	4.829	1.110	Distorted.

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the maximum to be a little to the side of the center, away from the entry, while the point and ring traverse showed normal flow with the maximum at the center. This would seem to indicate that the presence of the bottom opening was the cause of the similar action of Tubes *F* and *G* with all three openings in use. However, a traverse made with Tube *C* (described and illustrated in the paper), showed the same effect, only much exaggerated, owing to the larger size of the tube relatively to the pipe. In this tube the openings are symmetrically placed, but they are situated near the rear of the cut-water blade, and close to the end of the tube. Now, if the tendency of the water near the end of the tube to flow around the end, rather than flow across perpendicularly to the blade, is different at different points in the cross-section of the flowing water, then it is evident that these pressure openings will not give uniform indications.

To illustrate, imagine the tube on a point beyond center. The tendency of the water to flow around the end of the tube is opposed by the circumstance that the water beyond the end is moving with a lower velocity than the water at the end. Now imagine the tube drawn back to the corresponding point on the entry side of center. Here, the water is flowing with the same velocity as on the opposite side; but the velocity beyond the end of the tube is now greater than the velocity at the end, and the tendency of the water to flow around is increased. In the first case, the velocity past the pressure openings is increased, and the pressure, therefore, is lowered, giving a greater observed difference of head between the point and pressure columns of the gauge In the second case, the velocity past the pressure openings is decreased, and the pressure increased, giving a smaller observed difference between point and pressure. These effects, of course, are smaller, for the same tube, in a large pipe than in a small one, because, in a given distance, there is less change of velocity. In addition to this effect, the velocity past the pressure openings is slightly higher on the side opposite the entry than on the corresponding point on the side next to the entry, because the area of the cross-section is less, owing to the greater area of the tube in the pipe. This last effect becomes quite pronounced for large tubes in a small pipe.

For the foregoing reasons, then, it is seen that the conditions actually existing in this pipe are not correctly represented by point and pressure traverses. The apparent maximum is shifted away from its true position toward the side opposite the entry. In Fig. 125 the effect of Tubes *F* and *G* has therefore been considered in sketching the locus of the position of maximum velocity.

Effects of Curves of Different Radius.—The traverses made beyond Curves Nos. 1 and 6 showed the same general distorted conditions. This, together with the evidence of the traverses in Curve No. 1, show-

FIG. 124

Traverses in Horizontal Plane of Curve No. 1, showing approximate displacement of Maximum Velocity from Center or Normal Position.
Length of Curve, 180° or 5.213 ft. = 30.133 diameters. Radius of Curvature, 1.660 ft. = 9.58 Diameters. Diameter of Pipe 2.078 ins. = 0.1732 ft.
Entry- Horizontal, Concave.

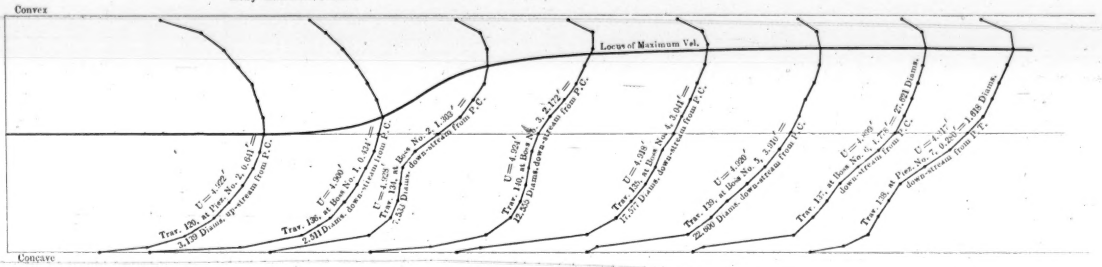
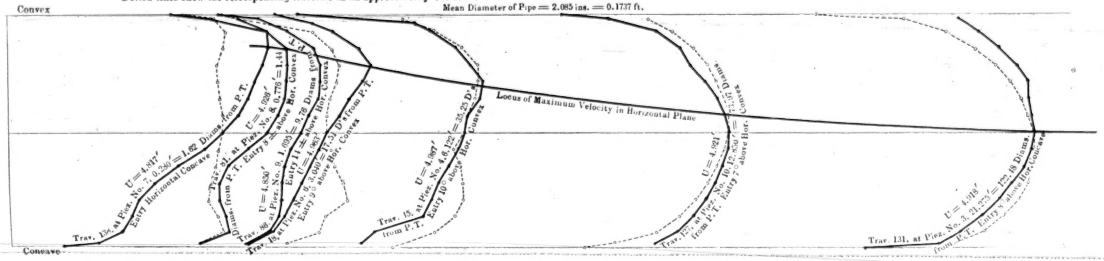


FIG. 125

Full lines show traverses in an approximately Horizontal Plane, in Tangent following 180° Curve, showing approximate return of Maximum Velocity to Center or Normal Position.
Dotted lines show the corresponding traverses in an approximately Vertical Plane.
Mean Diameter of Pipe = 2.085 ins. = 0.1737 ft.



26..	"	1.695	Cv. 84° +	0.715	Cx.	5.444	0.909	.....	4.935	5.035	1.020	.....
25..	"	1.695	.....	0.715	"	3.077	0.911	.....	2.802	2.854	0.947	.....
24..	"	1.695	.....	0.396	"	1.080	0.893	.....	0.962	0.958	0.990	.....
29..	"	1.695	Cx. 6° +	0.715	"	6.153	0.764	.....	4.701	5.031	1.069	.....
27..	"	1.695	.....	0.715	"	2.980	0.789	.....	2.354	2.368	1.004	.....
28..	"	1.695	.....	0.715	"	1.360	0.743	.....	1.010	1.033	1.022	.....
21..	"	3.040	Cv. 81° +	0.572	"	5.588	0.875	.....	4.891	4.957	1.014	.....
22..	"	3.040	.....	0.572	"	3.563	0.892	.....	3.179	3.085	0.971	.....
20..	"	3.040	.....	0.429	"	2.117	0.883	.....	1.870	1.851	0.990	.....
18..	"	3.040	Cv. 9° +	0.572	"	6.000	0.782	.....	4.699	4.963	1.058	.....
19..	"	3.040	.....	0.572	"	5.453	0.769	.....	4.192	4.423	1.055	.....
23..	"	3.040	.....	0.429	"	1.977	0.850	.....	1.681	1.614	0.960	.....
48..	"	6.122	Cv. 80° +	0.429	"	5.835	0.884	.....	5.160	5.196	1.007	.....
50..	"	6.122	.....	0.429	"	3.178	0.893	.....	2.838	2.663	0.929	.....
49..	"	6.122	.....	0.143	"	1.360	0.859	.....	1.167	1.118	0.958	.....
45..	"	6.122	Cx. 10° +	0.429	"	5.594	0.778	.....	4.361	4.987	1.144	.....
46..	"	6.122	.....	0.429	"	3.142	0.820	.....	2.578	2.577	0.999	.....
47..	"	6.122	.....	0.286 / 0.000	"	1.197	0.832	.....	0.996	0.999	1.035	.....
17..	"	14.930	Cv. 52° +	0.143	Cv.	5.426	0.822	.....	4.466	4.948	1.106	.....
15..	"	14.930	.....	0.143	Cx.	3.740	0.810	.....	3.022	3.326	1.101	.....
16..	"	14.930	.....	0.143	Cv.	1.917	0.870	.....	1.668	1.645	0.986	.....
14..	"	14.930	Cx. 38° +	0.000	C.	5.393	0.835	.....	4.498	4.977	1.106	.....
13..	"	14.930	.....	0.143	Cx.	3.450	0.795	.....	2.753	2.887	1.049	.....
12..	"	14.930	.....	0.143	Cx.	1.330	0.786	0.8197	1.066	1.054	0.989	.....
53..	"	17.393	Cv. 85° +	0.000	C.	5.743	0.834	.....	4.789	5.111	1.067	.....
51..	"	17.393	.....	0.143	Cv.	3.355	0.848	.....	2.845	2.686	0.944	.....
52..	"	17.393	.....	0.286 Cv. / 0.286 Cx.		1.280	0.857	.....	1.097	1.019	0.929	.....
54..	"	17.393	Cx. 5° +	0.000	C.	5.924	0.834	.....	4.942	5.115	1.035	.....
56..	"	17.393	.....	0.000	C.	3.269	0.848	.....	2.772	2.627	0.948	.....
55..	"	17.393	.....	0.000 / 0.143	Cx.	1.413	0.821	0.8403	1.160	1.146	0.988	.....
63..	"	31.400	Cv. 57° +	0.000	C.	4.042	0.819	.....	3.314	3.395	1.027	.....
61..	"	31.400	.....	0.000	"	2.996	0.833	.....	2.497	2.501	1.002	.....
60..	"	31.400	Cx. 33° +	0.000	"	5.900	0.845	.....	4.988	5.096	1.022	.....
62..	"	31.400	.....	0.000	"	1.355	0.861	0.8395	1.166	1.081	(0.927)*	1.0170
64..	"	57.970	Cv. 10° +	0.000	"	5.974	0.822	.....	4.889	5.053	1.033	.....
101..	*Fr.*	57.970	Cv. 9° +	0.141	Cv.	5.075	0.829	.....	4.208	4.220	1.004	.....
100..	"	57.970	Cv. 9° +	0.000	C.	4.126	0.815	.....	3.364	3.419	1.017	.....
66..	*E.*	57.970	Cv. 13° +	0.000	"	4.127	0.826	.....	3.410	3.416	1.004	.....
68..	*Fr.*	57.970	Cv. 10° +	0.000	"	3.275	0.815	.....	2.669	2.699	1.011	.....
65..	*E.*	57.970	Cv. 10° +	0.000	"	1.440	0.823	0.8217	1.185	1.172	0.989	1.0097

*Observation rejected in computing mean on account of low velocity.

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TABLE No. 75.—Traverses in 2-Inch Brass Pipe, Showing Curvature Effects.

Made at the Hydraulic Laboratory of the College of Civil Engineering of Cornell University.

Trav. No.	Tube.	Distance from P. T., in feet.	Entry.	By Pitot Tube. Maximum Velocity. Position. $r \div R$	Maximum Velocity. Amount, Feet per Second.	Ratio: $\frac{V_m}{V_{max.}}$ Obs.	Ratio: $\frac{V_m}{V_{max.}}$ Mean.	Mean Velocity, V_m Feet per Second.	By Weight. Mean Velocity, U, Feet per Second.	Ratios: $\frac{\text{Mean Vel. by Weight}}{\text{Mean Vel. by Tube}} = \frac{U}{V_m}$ Obs.	Mean.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
59..	*E.*	0.280	Cx. 82° +	0.858 Cx.	5.557	0.922	.....	5.113	5.108	0.999	.....
40..	"	0.280	.....	0.143 " 0.429 "	4.325	0.932	.....	4.032	4.951	(1.228)	.....
57..	"	0.280	.....	0.858 "	3.752	0.921	.....	3.455	3.376	0.977	.....
38..	"	0.280	.....	0.715 "	3.053	0.898	.....	2.743	3.366	(1.227)	.....
58..	"	0.280	.....	0.858 "	1.385	0.839	.....	1.162	1.313	1.131	.....
39..	"	0.280	.....	0.858 "	0.967	0.937	.....	0.916	1.048	(1.144)	.....
43..	"	0.280	Cx. 8° —	0.715 "	6.074	0.704	.....	4.275	4.938	1.155	.....
54..	"	0.280	.....	0.715 "	4.243	0.718	.....	3.047	3.507	1.151	.....
41..	"	0.280	.....	0.858 "	3.333	0.776	.....	2.588	2.610	1.009	.....
42..	"	0.280	.....	0.715 "	1.461	0.673	.....	0.983	1.189	1.209	.....
7..	"	0.390	Cv. 55° +	0.858 "	5.782	0.843	.....	4.877	4.978	1.021	.....
8..	"	0.390	.....	0.858 "	4.529	0.854	.....	3.868	3.904	1.009	.....
5..	"	0.390	.....	0.572 "	3.800	0.875	.....	3.326	3.402	1.023	.....
6..	"	0.390	.....	0.715 "	2.275	0.850	.....	1.934	1.911	0.988	.....
1..	"	0.390	Cx. 35° +	0.712 "	6.070	0.810	.....	4.921	4.919	1.000	.....
3..	"	0.390	.....	0.715 "	4.138	0.801	.....	3.314	3.298	0.995	.....
4..	"	0.390	.....	0.715 "	2.988	0.786	.....	2.349	2.391	1.018	.....
2..	"	0.390	.....	0.712 "	2.300	0.804	.....	1.849	1.845	0.998	.....
34..	"	0.776	Cx. 87° +	0.858 "	5.388	0.941	.....	5.069	5.064	0.999	.....
37..	"	0.776	.....	0.858 "	4.180	0.945	.....	3.952	3.961	1.005	.....
36..	"	0.776	.....	0.858 "	3.020	0.929	.....	2.807	2.623	0.934	.....
35..	"	0.776	.....	0.715 "	1.720	0.919	.....	1.580	1.561	0.988	.....
33..	"	0.776	Cx. 3° —	0.715 "	5.848	0.752	.....	4.400	4.679	1.056	.....
30..	"	0.776	.....	0.715 "	3.200	0.813	.....	2.601	2.473	0.951	.....
32..	"	0.776	.....	0.858 "	2.075	0.730	.....	1.515	1.613	1.064	.....
31..	"	0.776	.....	0.715 " 0.572 "	0.935	0.811	.....	0.758	0.655	0.864	.....

82..	"	1.695	Cv. 76° +	0.572	"	6.690	0.872	.....	5.835	4.921	0.843	.....
83..	"	1.695	.....	0.572	"	3.660	0.855	.....	3.129	2.728	0.874	.....
84..	"	1.695	.....	0.429	"	1.424	0.860	.....	1.224	1.038	0.848	.....
86..	"	1.695	Cx. 14° +	0.429 / 0.572	"	6.665	0.858	.....	5.720	4.850	0.848	.....
87..	"	1.695	.....	0.572	"	3.644	0.834	.....	3.039	2.663	0.875	.....
85..	"	1.695	.....	0.572 / 0.429	"	1.453	0.844	.....	1.227	1.080	0.880	.....
92..	"	3.040	Cv. 68° +	0.429	"	6.805	0.863	.....	5.870	4.921	0.838	.....
91..	"	3.040	.....	0.286	"	3.556	0.863	.....	3.063	2.618	6.855	.....
93..	"	3.040	.....	0.286	"	1.619	0.836	.....	1.352	1.167	0.863	.....
88..	"	3 040	Cx. 22° +	0.286	"	6.776	0.847	.....	5.743	4.922	0.857	.....
90..	"	3.040	.....	0.429	"	3.667	0.838	.....	3.076	2.697	0.877	.....
89..	"	3.040	.....	0.286	"	1.453	0.834	.....	1.211	1.094	0.903	.....
114..	*G.*	6.122	Cx. 57° +	0.000	C.	6.732	0.826	.....	5.555	4.921	0.886	.....
115..	"	6.122	.....	0.000	"	3.742	0.824	.....	3.082	2.728	0.885	.....
116..	"	6.122	.....	0.000 / 0.143	" Cv.	1.453	0.826	.....	1.201	1.038	0.864	.....
118..	"	6.122	Cv.33 ° +	0.143	Cx.	6.773	0.852	.....	5.769	4.850	0.841	.....
119..	"	6.122	.....	0.143 / 0.286	"	3.873	0.845	.....	3.272	2.663	0.814	.....
117..	"	6.122	.....	0.143 / 0.286	"	1.580	0.852	.....	1.344	1.080	0.804	.....
99..	*F.*	12.850	Cv. 83° +	0.143	"	6.999	0.825	.....	5.769	4.918	0.852	.....
97..	"	12.850	.....	0.143	"	3.559	0.826	.....	2.939	2.501	0.852	.....
98..	"	12.850	.....	0.143 / 0.000	Cx. / C.	1.892	0.812	.....	1.537	1.292	0.840	.....
127..	*G.*	12.850	Cx. 7° +	0.000	"	7.038	0.822	.....	5.782	4.921	0.851	.....
128..	"	12.850	.....	0.000	"	3.932	0.820	.....	3.224	2.674	0.829	.....
126..	"	12.850	.....	0.143	Cv.	1.670	0.823	0.8213	1.375	1.140	0.829	.....
95..	*F.*	21.275	Cx. 82° +	0.143	"	7.056	0.815	.....	5.748	4.921	0.857	.....
96..	"	21.275	.....	0.143	"	3.748	0.810	.....	2.037	2.674	0.881	.....
94..	"	21.275	.....	0.000	C.	1.619	0.807	.....	1.305	1.140	0.874	0.8704
131.	*G.*	21.275	Cv. 8° +	0.000	"	6.982	0.826	.....	5.771	4.918	0.852	.....
129..	"	21.275	.....	0.143	Cx.	3.605	0.816	.....	2.939	2.501	0.851	.....
130..	"	21.275	.....	0.143 / 0.143	Cx. / Cv.	1.804	0.824	0.8163	1.485	1.292	0.870	0.8577

* Observation rejected in computing mean on account of low velocity.
† Up stream from P. C., showing normal conditions of flow.

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TABLE No. 75—(*Continued*).

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
133..	*Gr.*	57.487+	Cv. 45° +	0.000 C.	5.200	0.823	.....	4.278	4.220	0.986	.....
132..	"	57.487+	.....	0.000 "	4.136	0.833	.....	3.442	3.419	0.993	.....
120..	"	0.641+	.....	0.000 "	6.072	0.822	.....	4.993	4.922	0.988	.....
9..	*E.*	0.641+	Cv. 38° +	0.000 "	5.692	0.821	.....	4.675	4.906	1.049	.....
122..	*Gr.*	0.641+	Cv. 45° +	0.000 "	3.380	0.821	.....	2.773	2.697	0.973	.....
10..	*E.*	0.641+	Cv. 38° +	0.000 " / 0.143 Cv.	2.339	0.825	.....	1.928	1.968	1.025	.....
121..	*Gr.*	0.641+	Cv. 45° +	0.000 C.	1.453	0.828	.....	1.207	1.094	(0.909)*	.....
124..	"	0.641+	Cx. 45° +	0.000 "	6.082	0.819	.....	4.983	4.921	0.988	.....
11..	*E.*	0.641+	Cx. 38° +	0.000 "	5.483	0.836	.....	4.583	4.786	1.044	.....
123..	*Gr.*	0.641+	Cx. 45° +	0.000 "	3.259	0.823	.....	2.684	2.618	0.975	.....
125..	"	0.641+	.....	0.000 " / 0.143 Cx.	1.424	0.795	0.8224	1.131	1.167	1.032	1.0053
67..	*F.*	0.641+	Cx. 80° +	0.000 C.	7.231	0.816	.....	5.902	5.061	0.857	.....
68..	"	57.970+	Cv. 10° +	0.143 Cx.	3.899	0.806	.....	3.142	2.699	0.859	.....
133..	*G.*	57.487+	Cv. 45° +	0.000 C.	5.990	0.823	.....	4.933	4.220	0.855	.....
132..	"	57.487+	.....	0.143 Cx.	4.782	0.831	.....	3.976	3.419	0.860	.....
69..	*F.*	0.641+	Cv. 0° +	0.000 C.	7.107	0.827	.....	5.876	5.022	0.855	.....
120..	*G.*	0.641+	Cv. 45° +	0.000 "	7.024	0.809	.....	5.682	4.922	0.866	.....
122..	"	0.641+	.....	0.143 Cx.	3.866	0.820	.....	3.168	2.697	0.851	.....
121..	"	0.641+	.....	0.000 C.	1.619	0.814	.....	1.316	1.094	0.831	.....
124..	"	0.641+	Cx. 45° +	0.000 "	6.902	0.821	.....	5.729	4.921	0.859	.....
123..	"	0.641+	.....	0.143 Cv.	3.748	0.828	.....	3.100	2.618	0.845	.....
125..	"	0.641+	.....	0.000 C.	1.670	0.788	0.8166	1.315	1.167	0.887	0.8568
75..	*F.*	0.280	Cv. 73° +	0.429 Cx.	6.631	0.910	.....	6.027	5.160	0.856	.....
74..	"	0.280	.....	0.143 "	3.326	0.924	.....	3.074	2.696	0.877	.....
73..	"	0.280	.....	0.286 "	1.706	0.885	.....	1.510	1.358	0.901	.....
70..	"	0.280	Cx. 17° +	0.429 "	7.167	0.809	.....	5.796	5.108	0.881	.....
72..	"	0.280	.....	0.572 "	3.590	0.772	.....	2.772	2.563	0.925	.....
71..	"	0.280	.....	0.572 " / 0.429 "	1.424	0.737	.....	1.050	1.038	0.988	.....
76..	"	0.776	Cv. 82° +	0.429 "	6.386	0.894	.....	5.711	4.919	0.861	.....
77..	"	0.776	.....	0.429 "	3.113	0.871	.....	2.713	2.392	0.882	.....
78..	"	0.776	.....	0.715 to 0.286 Cx.	1.249	0.878	.....	1.096	0.943	0.860	.....
81..	"	0.776	Cx. 8° +	0.572 Cx.	6.898	0.828	.....	5.704	4.926	0.865	.....
80..	"	0.776	.....	0.572 "	3.665	0.833	.....	3.036	2.365	0.868	.....
79..	"	0.776	.....	0.572 " / 0.429 "	1.540	0.790	.....	1.217	1.808	0.890	.....

TABLE No. 75—(*Concluded*).

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
112..	*G.*	31.400	Cv. 57° +	0.000 C.	6.964	0.828	.....	5.766	4.930	0.855	.....
111..	"	31.400	.....	0.000 "	3.861	0.828	.....	3.198	2.686	0.840	.....
113..	"	31.400	.....	0.143 Cx.	1.804	0.841	.....	1.516	1.231	0.812	.....
108..	"	31.400	Cx. 33° +	0.143 "	6.960	0.817	.....	5.683	4.955	0.872	.....
109..	"	31.400	.....	0.143 "	3.815	0.817	.....	3.123	2.636	0.844	.....
110..	"	31.400	.....	0.143 "	1.607	0.808	0.8172	1.298	1.086	0.837	0.8433
102..	"	57.870	Cx. 80° +	0.000 C.	6.978	0.822	.....	5.736	4.919	0.857	.....
103..	"	57.870	.....	0.000 " / 0.143 Cv.	3.360	0.823	.....	2.765	2.392	0.865	.....
104..	"	57.870	.....	0.000 C.	1.395	0.838	.....	1.170	0.943	(0.807)	.....
107..	"	57.870	Cv. 10° +	0.000 "	6.987	0.827	.....	5.770	4.926	0.854	.....
106..	"	57.870	.....	0.000 "	3.749	0.809	.....	3.033	2.635	0.869	.....
105..	"	57.870	.....	0.000 "	1.567	0.809	0.8213	1.267	1.083	0.855	0.8600
101..	*F.*	58.390	Cv. 9° +	0.143 Cv.	6.020	0.824	.....	4.960	4.220	0.851	.....
100..	"	58.390	.....	0.143 "	4.900	0.824	0.8240	4.038	3.419	0.847	0.8490

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ing that the full distortion is reached in the first quadrant, rendered Pitot-tube experiments on the curves of intermediate radius unnecessary. The loss-of-head experiments, which will be discussed later, furnish additional evidence regarding the similarity of effect of the curves of different radii. Messrs. Saph and Schoder.

The Pitot Tube as a Meter.—Thus far, all that has been said concerns only the relative indications of the Pitot tube. However, the numerous traverses, mostly in cases of distorted flow, are also of value in indicating the action of the tube as a meter in such cases.

In order to use a traverse on a single diameter, to correctly measure the flow in the whole cross-section, it is necessary to know the manner of the distribution of velocities throughout the whole cross-section. In normal flow the conditions are the same for every diameter, and the matter of gauging the flow becomes quite simple. It is only necessary to apply the proper factors to each of the velocities deduced from the observed heads along any diameter. In cases of distorted flow the same factors cannot be used, and the determination of the proper ones involves, as just mentioned, a knowledge of the conditions throughout the cross-section.

Fig. 123 shows the variation of velocities around the circumferences of concentric circles under the distorted conditions shown in Fig. 122. The true mean velocity for each circle was gotten by applying a planimeter to the area bounded by the base line (or rectified circumference), the end ordinates and the velocity curve. The area thus obtained was divided by the length of the base, to get the height of the mean-velocity ordinate. The graphical interpretation of the mean-velocity line is that the area above it, and bounded by it and the curve, is equal to the areas below it, and bounded by it, the end ordinates prolonged and the curve. These areas are shaded in Fig. 123.

The error introduced by assuming that the mean of two velocities at points equi-distant from and on opposite sides of the center is the mean velocity for the circle passing through these points is clearly shown in the diagram. Invariably, the points on the vertical diameter give too high a mean velocity, while the points on the horizontal diameter, or the mean of four points on both diameters, give too low a mean velocity. It is a curious fact that the observations on a diameter 30° above or below the horizontal may be treated as though the flow were symmetrical, and will give the discharge as accurately as though such were the case. Whether this is of general application or not, the writers will not venture to say.

It is evident, then, that the ratios $\frac{U}{V_m}$ given in Tables Nos. 74 and 75 are incorrect for the whole cross-section, except in cases of symmetrical flow. These ratios have been worked out in order to examine the range of resulting error, and the wide variation of the ratios

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among themselves would of itself have been sufficient cause to question the validity of using such a method of reduction to obtain the true discharge.

Considering the traverses where nearly symmetrical flow existed, the ratios of the mean velocity by tube to the mean by weight, and the ratio of mean by tube to maximum by tube, seem to indicate that the latter part of the authors' Conclusion *B* and their Conclusion *C* are conservative statements regarding the accuracy of the Pitot tube as a meter in cases of symmetrical flow.

Comparison of Conditions of Flow in Small and Large Pipes.—All the traverses made by the writers under normal-flow conditions show an approximately elliptical velocity distribution. The average ratio of the mean velocity to the maximum obtained by the writers from their normal flow traverses is 0.827, somewhat lower than the ratio given in Conclusion *C* of the paper. This ratio agrees well with that obtained by the authors for the same size and kind of pipe. Their value is 0.8139, given in Table No. 4. This latter value is obtained from a combination of a few traverses by each of four tubes, and, in the writers' opinion, is not as reliable as the value 0.827 which was obtained from nearly four times as many experiments more consistent among themselves.

In distorted flow caused by curves, Figs. 11 and 12 show the distribution of velocities at points about 4 and 10 diameters, respectively, down stream from 40-ft. and 15-ft. radius 90° curves. There is a remarkable similarity between these and the horizontal traverses by the writers at corresponding points down stream from Curves Nos. 1 and 6, shown in Fig. 125. It must be remembered that where the entry is not exactly horizontal the traverse will show peculiarities due to the lack of symmetry of the velocity distribution about a vertical. A glance at Fig. 122 will render this more apparent. Thus it seems to be entirely in accord with experimental evidence to say that the general nature of the distortion due to curves is the same, whether we are dealing with a 2-in. or a 30-in. pipe. The matter of the loss of head, however, seems to depend on other factors than the relative distortion. It will be considered next.

Experiments on Loss of Head.

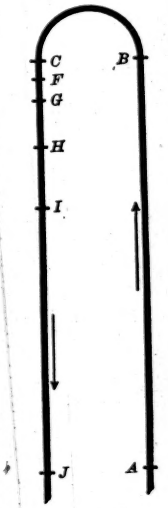
For the loss-of-head experiments the system of pipes and piezometers was arranged as shown in Fig. 119. The only change necessary was the substitution of the different curves, descriptions and measurements of which have already been given. The losses of head were measured in nine different sections: *AB*, *BC*, *CI*, *CF*, *FG*, *BJ*, *IJ*, *GH* and *HI*. Observations were taken on four sections at a time, and the sets of simultaneous observations for a particular velocity consti-

No. of curve.	No. of experiment.	Temperature, Fahr.	Section C–F. Velocity, in feet per second.	Section C–F. Observed head, in double centimeters.	Section C–F. Head at 56° Fahr., in double centimeters.	Section F–G. Velocity, in feet per second.	Section F–G. Observed head, in double centimeters.	Section F–G. Head at 56° Fahr., in double centimeters.	Section G–H. Velocity, in feet per second.	Section G–H. Observed head, in double centimeters.	Section G–H. Head at 56° Fahr., in double centimeters.	Section H–I. Velocity, in feet per second.	Section H–I. Observed head, in double centimeters.	Section H–I. Head at 56° Fahr., in double centimeters.
			$l = 2.760$ $d = 0.1746$			$l = 3.082$ $d = 0.1747$			$l = 6.728$ $d = 0.1733$			$l = 8.515$ $d = 0.1733$		
6	10	56.7	4.924	1.88	1.88	4.919	2.02	2.08	4.995	4.60	4.61	4.995	5.58	5.59
	11	56.2	4.487	1.58	1.58	4.482	1.70	1.70	4.551	3.93	3.98	4.551	4.73	4.73
	12	56.2	4.009	1.29	1.29	4.006	1.37	1.37	4.068	3.22	3.22	4.068	3.88	3.88
	13	56.4	3.099	0.82	0.82	3.096	0.88	0.88	3.144	2.08	2.08	3.144	2.48	2.48
	14	56.2	2.267	0.49	0.42	2.265	0.51	0.51	2.299	1.18	1.18	2.299	1.42	1.42
	15	56.5	1.520	0.23	0.23	1.519	0.25	0.25	1.542	0.57	0.57	1.542	0.72	0.72
	16	57.0	0.988	0.11	0.11	0.981	0.10	0.10	0.946	0.94	0.94	0.946	0.80	0.80
			$l = 2.760$ $d = 0.1746$			$l = 3.082$ $d = 0.1747$			$l = 6.728$ $d = 0.1733$			$l = 8.515$ $d = 0.1733$		
5	17	57.0	4.924	1.87	1.88	4.919	1.99	2.00	4.995	4.58	4.60	4.995	5.61	5.63
	18	56.5	4.359	1.49	1.49	4.355	1.58	1.58	4.392	3.64	3.65	4.392	4.46	4.47
	19	56.5	3.855	1.20	1.20	3.852	1.31	1.31	3.911	2.98	2.99	3.911	3.54	3.55
	20	57.7	3.169	0.82	0.83	3.166	0.95	0.95	3.215	2.05	2.06	3.215	2.57	2.59
	21	57.7	2.478	0.52	0.52	2.475	0.61	0.61	2.514	1.31	1.32	2.514	1.64	1.65
	22	57.8	1.598	0.23	0.23	1.596	0.28	0.28	1.621	0.60	0.60	1.621	0.74	0.74
	23	58.0	0.958	0.09	0.09	0.957	0.11	0.11	0.972	0.25	0.25	0.972	0.28	0.28
			$l = 2.760$ $d = 0.1746$			$l = 3.082$ $d = 0.1747$			$l = 6.728$ $d = 0.1733$			$l = 8.515$ $d = 0.1733$		
4	39	58.2	4.924	1.87	1.88	4.919	1.98	2.00	4.995	4.49	4.53	4.995	5.58	5.57
	40	58.1	4.558	1.62	1.63	4.548	1.71	1.73	4.618	3.95	3.98	4.618	4.82	4.85
	41	58.1	4.017	1.28	1.29	4.013	1.36	1.37	4.075	3.15	3.17	4.075	3.84	3.87
	42	58.3	3.360	0.92	0.93	3.357	0.97	0.98	3.409	2.28	2.30	3.409	2.82	2.84
	43	58.1	2.632	0.55	0.55	2.629	0.50	0.50	2.670	1.60	1.61	2.670	1.73	1.74
	44	58.3	1.617	0.26	0.26	1.616	0.24	0.24	1.641	0.64	0.65	1.641	0.76	0.77
	45	58.3	1.069	0.10	0.10	1.060	0.10	0.10	1.077	0.31	0.31	1.077	0.37	0.37
			$l = 2.760$ $d = 0.1746$			$l = 3.082$ $d = 0.1747$			$l = 6.728$ $d = 0.1733$			$l = 8.515$ $d = 0.1733$		
3	72	52.8	4.892	1.95	1.98	4.888	2.03	2.01	4.963	4.56	4.51	4.963	5.67	5.60
	73	52.0	4.059	1.39	1.37	4.055	1.44	1.42	4.117	3.27	3.23	4.117	4.04	3.98
	74	52.0	3.302	0.92	0.91	3.299	0.95	0.94	3.350	2.30	2.27	3.350	2.74	2.70
	75	52.0	2.221	0.45	0.44	2.219	0.48	0.47	2.253	1.10	1.08	2.253	1.35	1.33
	76	52.5	1.498	0.24	0.24	1.491	0.24	0.24	1.515	0.52	0.51	1.515	0.69	0.68
	77	52.5	0.814	0.09	0.09	0.818	0.04	0.04	0.826	0.16	0.16	0.826	0.23	0.23
			$l = 2.760$ $d = 0.1746$			$l = 3.082$ $d = 0.1747$			$l = 6.728$ $d = 0.1733$			$l = 8.515$ $d = 0.1733$		
2	66	52.9	4.894	1.93	1.91	4.889	2.04	2.02	4.965	4.57	4.57	4.965	5.64	5.58
	67	51.9	4.256	1.49	1.47	4.252	1.57	1.55	4.318	3.58	3.58	4.318	4.41	4.35
	68	51.9	3.655	1.15	1.13	3.652	1.17	1.15	3.708	2.73	2.69	3.708	3.37	3.33
	69	51.8	2.880	0.74	0.73	2.878	0.76	0.75	2.922	1.79	1.77	2.922	2.22	2.19
	70	52.0	1.972	0.39	0.38	1.970	0.39	0.39	2.001	0.90	0.89	2.001	1.13	1.11
	71	52.9	0.863	0.09	0.09	0.883	0.09	0.09	0.896	0.21	0.21	0.896	0.28	0.28
			$l = 2.760$ $d = 0.1746$			$l = 3.082$ $d = 0.1747$			$l = 6.728$ $d = 0.1733$			$l = 8.515$ $d = 0.1733$		
1	46	57.5	4.900	1.89	1.90	4.896	2.00	2.01	4.971	4.50	4.52	4.971	5.56	5.59
	47	57.2	4.396	1.46	1.47	4.392	1.61	1.63	4.358	3.56	3.58	4.358	4.38	4.40
	48	57.2	3.660	1.10	1.10	3.656	1.22	1.23	3.713	2.70	2.71	3.713	3.29	3.30
	49	57.0	2.849	0.68	0.68	2.847	0.78	0.78	2.891	1.71	1.72	2.891	2.11	2.13
	50	57.5	1.647	0.25	0.25	1.646	0.31	0.31	1.671	0.64	0.64	1.671	0.83	0.84
	51	57.8	0.818	0.07	0.07	0.817	0.09	0.09	0.830	0.19	0.19	0.830	0.21	0.21

TABLE No. 76.—Loss of Head Experiments.

Section H–I.			No. of experiment.	Temperature, Fahr.	Section B–C.			Section C–I.			Section B–J.			Section I–J.			Section A–B.		
Velocity, in feet per second.	Observed head, in double centimeters.	Head at 56° Fahr., in double centimeters.			Velocity, in feet per second.	Observed head, in double centimeters.	Head at 56° Fahr., in double centimeters.	Velocity, in feet per second.	Observed head, in double centimeters.	Head at 56° Fahr., in double centimeters.	Velocity, in feet per second.	Observed head, in double centimeters.	Head at 56° Fahr., in double centimeters.	Velocity, in feet per second.	Observed head, in double centimeters.	Head at 56° Fahr., in double centimeters.	Velocity, in feet per second.	Observed head, in double centimeters.	Head at 56° Fahr., in double centimeters.
$l = 8.515$ $d = 0.1733$					$l = 2.910$ $d = 0.1717$			$l = 21.085$ $d = 0.1737$			$l = 61.020$ $d = 0.1737$						$l = 57.496$ $d = 0.1743$		
4.995	5.58	5.59	1	56.4	5.090	3.03	3.08	4.974	14.11	14.13	4.974	41.76	41.82				4.945	38.21	38.27
4.551	4.73	4.73	2	55.9	4.742	2.75	2.75	4.633	12.36	12.36	4.633	36.94	36.98				4.607	33.79	33.78
4.068	3.88	3.88	3	56.0	4.334	2.37	2.37	4.235	10.59	10.59	4.235	31.48	31.48				4.211	28.81	28.81
3.144	2.48	2.48	4	55.9	3.760	1.78	1.78	3.674	8.23	8.23	3.674	24.50	24.49				3.653	22.41	22.40
2.299	1.42	1.42	5	56.0	3.183	1.33	1.33	3.110	6.09	6.09	3.110	18.17	18.17				3.098	16.68	16.68
1.542	0.72	0.72	6	56.0	2.102	0.64	0.64	2.054	2.90	2.90	2.054	8.68	8.68				2.042	8.00	8.00
0.946	0.30	0.30	7	56.2	1.536	0.39	0.39	1.491	1.63	1.63	1.491	4.95	4.95				1.482	4.57	4.57
			8	56.7	0.986	0.23	0.23	0.963	0.75	0.75	0.963	2.29	2.29				0.958	2.13	2.13
			9	57.0	0.645	0.14	0.14	0.631	0.35	0.35	0.631	1.08	1.08				0.627	1.01	1.01
$l = 8.515$ $d = 0.1733$					$l = 3.196$ $d = 0.1733$			$l = 21.085$ $d = 0.1737$			$l = 61.306$ $d = 0.1737$			$l = 37.025$ $d = 0.1739$					
4.995	5.61	5.63	24	58.4	5.004	3.14	3.17	4.985	13.95	14.07	4.985	41.68	42.05	4.975	24.56	24.78			
4.392	4.46	4.47	25	58.6	4.503	2.64	2.67	4.486	11.56	11.67	4.486	34.70	35.03	4.477	20.50	20.69			
3.911	3.54	3.55	26	58.4	3.988	2.13	2.15	3.973	9.36	9.44	3.973	28.13	28.38	3.965	16.57	16.72			
3.215	2.57	2.59	27	58.4	3.158	1.42	1.43	3.146	6.17	6.22	3.146	18.62	18.78	3.140	10.97	11.07			
2.514	1.64	1.65	28	58.6	2.345	0.83	0.84	2.336	3.59	3.62	2.336	10.88	10.98	2.332	6.41	6.47			
1.621	0.74	0.74	29	58.6	1.476	0.36	0.36	1.470	1.55	1.56	1.470	4.79	4.84	1.468	2.85	2.88			
0.972	0.28	0.28	30	59.0	0.936	0.18	0.18	0.933	0.68	0.69	0.933	2.16	2.18	0.931	1.29	1.30			
			31	59.0	0.554	0.07	0.07	0.551	0.25	0.25	0.551	0.87	0.88	0.550	0.51	0.52			
$l = 8.515$ $d = 0.1733$					$l = 3.425$ $d = 0.1729$			$l = 21.085$ $d = 0.1737$			$l = 61.535$ $d = 0.1737$			$l = 37.025$ $d = 0.1739$					
4.995	5.53	5.57	32	59.1	5.013	3.25	3.29	4.970	13.95	14.11	4.970	41.61	42.08	4.960	24.32	24.60			
4.618	4.82	4.85	33	59.0	4.407	2.60	2.63	4.389	11.10	11.22	4.389	33.15	33.51	4.381	19.43	19.64			
4.075	3.84	3.87	34	59.0	3.950	2.18	2.21	3.916	9.12	9.22	3.916	27.32	27.62	3.909	16.08	16.21			
3.409	2.82	2.84	35	59.0	2.913	1.27	1.28	2.888	5.28	5.34	2.888	15.96	16.14	2.883	9.38	9.48			
2.670	1.73	1.74	36	59.0	2.021	0.63	0.64	2.008	2.77	2.80	2.008	8.32	8.41	2.000	4.90	4.95			
1.641	0.76	0.77	37	59.0	1.263	0.29	0.29	1.252	1.15	1.16	1.252	3.68	3.72	1.250	2.17	2.19			
1.077	0.37	0.37	38	59.0	0.796	0.11	0.11	0.790	0.40	0.40	0.790	1.96	1.97	0.718	0.81	0.82			
$l = 8.515$ $d = 0.1733$					$l = 3.973$ $d = 0.1728$			$l = 21.085$ $d = 0.1737$			$l = 62.083$ $d = 0.1737$			$l = 37.025$ $d = 0.1739$					
4.963	5.67	5.60	78	52.8	4.992	3.68	3.64	4.944	14.17	14.00	4.944	42.72	42.22	4.935	24.74	24.45			
4.117	4.04	3.98	79	52.0	4.348	2.90	2.86	4.306	11.13	10.99	4.306	33.56	33.07	4.298	19.48	19.19			
3.350	2.74	2.70	80	53.0	3.891	1.88	1.86	3.859	7.18	7.10	3.859	21.59	21.35	3.852	12.55	12.41			
2.258	1.35	1.33	81	53.0	2.758	1.31	1.30	2.732	4.97	4.92	2.732	14.91	14.78	2.726	8.69	8.59			
1.515	0.69	0.68	82	52.8	1.867	0.62	0.61	1.849	2.50	2.47	1.849	7.46	7.37	1.845	4.39	4.34			
0.896	0.23	0.23	83	53.1	1.110	0.23	0.23	1.099	1.00	0.99	1.099	3.00	2.97	1.097	1.76	1.74			
			84	53.2	0.311	0.08	0.08	0.308	0.09	0.09	0.308	0.35	0.35	0.307	0.21	0.21			
$l = 8.515$ d $= 0.1733$					$l = 4.980$ $d = 0.1727$			$l = 21.085$ $d = 0.1737$			$l = 63.090$ $d = 0.1737$			$l = 37.025$ $d = 0.1739$					
4.965	5.64	5.58	59	54.7	5.024	4.43	4.41	4.967	14.11	14.04	4.967	43.19	42.98	4.957	24.56	24.44			
4.318	4.41	4.35	60	52.4	4.598	3.86	3.81	4.545	12.19	11.99	4.545	37.33	36.84	4.538	21.26	20.98			
3.708	3.37	3.33	61	51.8	3.983	3.02	2.98	3.937	9.47	9.34	3.937	29.02	28.70	3.930	16.52	16.29			
2.922	2.22	2.19	62	52.0	3.175	2.07	2.04	3.139	6.33	6.24	3.139	19.53	19.24	3.133	11.10	10.94			
2.001	1.13	1.11	63	52.4	2.043	0.94	0.93	2.019	2.83	2.79	2.019	8.92	8.80	2.016	5.11	5.04			
0.896	0.28	0.28	64	57.9	1.314	0.38	0.38	1.299	1.30	1.31	1.299	4.01	4.04	1.297	2.31	2.32			
			65	54.9	0.814	0.16	0.16	0.805	0.51	0.51	0.805	1.73	1.72	0.808	1.01	1.01			
$l = 8.515$ $d = 0.1733$					$l = 6.053$ $d = 0.1732$			$l = 21.085$ $d = 0.1737$			$l = 64.163$ $d = 0.1737$			$l = 37.025$ $d = 0.1739$					
4.971	5.56	5.59	52	56.5	5.001	5.18	5.19	4.970	14.04	14.07	4.970	43.75	43.88	4.961	24.47	24.51			
4.358	4.38	4.40	53	55.5	4.392	4.18	4.17	4.365	11.19	11.17	4.365	35.09	35.08	4.356	19.68	19.59			
3.713	3.29	3.30	54	55.1	3.605	2.98	2.97	3.583	7.97	7.94	3.583	24.97	24.89	3.577	14.01	13.96			
2.891	2.11	2.12	55	55.1	2.812	1.90	1.89	2.795	5.10	5.08	2.795	16.03	15.98	2.789	9.01	8.98			
1.671	0.83	0.84	56	55.1	1.742	0.75	0.75	1.731	2.17	2.16	1.731	6.80	6.78	1.729	3.87	3.86			
0.830	0.21	0.21	57	55.2	1.256	0.42	0.42	1.248	1.19	1.19	1.248	3.85	3.84	1.246	2.23	2.22			
			58	55.2	0.603	0.11	0.11	0.600	0.30	0.30	0.600	1.07	1.07	0.599	0.66	0.66			

Section A-B was taken instead of Section I-J, when Curve No. 6 was in position.



tute what is called an experiment. Thus, in Table No. 76 (Plate X) the same number will be found under four sections for the same curve. All heads were measured in water. Messrs. Saph and Schoder.

Methods of Observation.—The same precautions were taken to have the gauges in perfect working order as in the Pitot-tube work. Each velocity was maintained for a period of from 15 to 20 minutes, and, during this time, the gauges were read from twenty to sixty times.

Two different methods of reading were used. One of the writers preferred to watch closely the vibrations of the menisci for a short time, and to set the riders on what he judged to be the mean position. The other preferred to take readings on the successive maxima and minima, and considered that the average of a large number of readings so taken would give the true mean. These two methods, however, give substantially the same results, as may be seen by summing the observed heads on Sections *BC*, *CI* and *IJ*, which should give the same value as the observed head on Section *BJ*. The differences are so small as to be negligible. It may be stated, in addition, that two of the gauges had glass tubes of large diameter (1 in.), and two had tubes of small diameter ($\frac{3}{8}$ in.). Hence the vibrations were not of the same magnitude. The fact that the results checked so closely would indicate that the size of the glass tubes is not a vital matter. Nor does the length of the connecting hose seem to make any difference, the lengths used varying from 10 to 60 ft.

In general, two weights were taken for each experiment, one by each of the writers, and these were averaged unless there was a greater difference than about $\frac{1}{2}$ of 1 per cent. In the latter event the gauge readings were consulted, to find whether these showed a corresponding change. If no change was found a check weight was taken before altering the flow. The average variation of the weights was less than $\frac{1}{4}$ of 1 per cent.

Method of Reduction.—The method of reducing the weights to velocities was essentially the same as for the Pitot-tube work. The mean diameters used in the reduction are given in Table No. 76 (Plate X). These were obtained from the measured diameters of pipes given in Figs. 116, 117, 118 and 119, each diameter being allowed to influence the result in proportion to the corresponding length of pipe.

Temperature Effect, and the Corrections for it.—Other experiments performed by the writers on 2-in. brass pipe have shown that the effect of temperature on loss of head is a very important factor. This effect has been pointed out by foreign experimenters, especially Hagen, but generally it has been overlooked or disregarded.

In the experiments to be described the range of temperature was only 7° Fahr., but this range is enough to introduce effects which are far from negligible. In Fig. 126 the observed losses in Section *IJ* were plotted as ordinates, with the corresponding velocities, in feet per

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second, raised to the power 1.75, as abscissas. This power of the velocity was used because it had been found previously that the law for the loss of lead in this pipe could be expressed approximately by the formula $H_f = m\ V^{1.75}$. The effect of temperature, the degrees Fahrenheit being marked on each observation plotted, is quite clearly shown.

For any particular temperature, the points lie approximately on a straight line. The points for 52° and also those for 59° were connected by short lines, and a number of the differences in ordinates were scaled off. These differences were divided by the corresponding heads and by 7, the temperature difference. These values were then averaged, giving a correction of 0.00366 of a gauge-scale unit for each unit of head and degree of temperature. The correction applied in each case was that necessary to bring all the observations to a common basis of 56° Fahr. There is, of course, a slight error introduced in assuming that for a particular head the correction per degree for different temperatures is the same. For the small range of temperatures in these experiments this error is inappreciable.

The writers are at the present time conducting a series of experiments in which the effect of temperature on loss of head is being more fully investigated.

Plotting of Results.—All the essential data in connection with these experiments are contained in Table No. 76 (Plate X). From a plotting of these data on logarithmic paper it was found that the points for Section AB lie on a straight line, which indicates that the law for the loss of head in straight pipe under normal conditions can be represented by the formula $H_f = m\ V^n$, where, as shown in the paper, n is the slope of the line, and log. m is the intercept on the y-axis. The origin is the point for which $H_f = 1$ and $V = 1$.

The line for Section AB had the slope $n = 1.759$, as nearly as could be scaled from the logarithmic plotting, and $m = 2.29$; however, when the heads were plotted as ordinates on ordinary cross-section paper, with the values of $V^{1.759}$ as abscissas, the more accurate value, 2.296, was obtained for m. This value has been adopted. Hence the formula for the loss of head in a straight section of 2.090-in. brass pipe, under normal conditions of flow, for a temperature of 56° Fahr., may be represented by the expression, $H_f = 2.296\ V^{1.759}$, where H_f is in double centimeters and V is in feet per second, and where the length of the pipe is 57.496 ft., the length of Section AB; or H'_f, in feet per 1 000 ft. of length, $= 2.62\ V^{1.759}$.

The same value for n was found to hold for Section $I\,J$, and the fact that all the points, even for the different curves, fell nearly in straight lines indicates that there is very little, if any, effect, due to the curves, on the law of flow in that section. This question will be further considered presently. Section $B\,J$, which includes the curves,

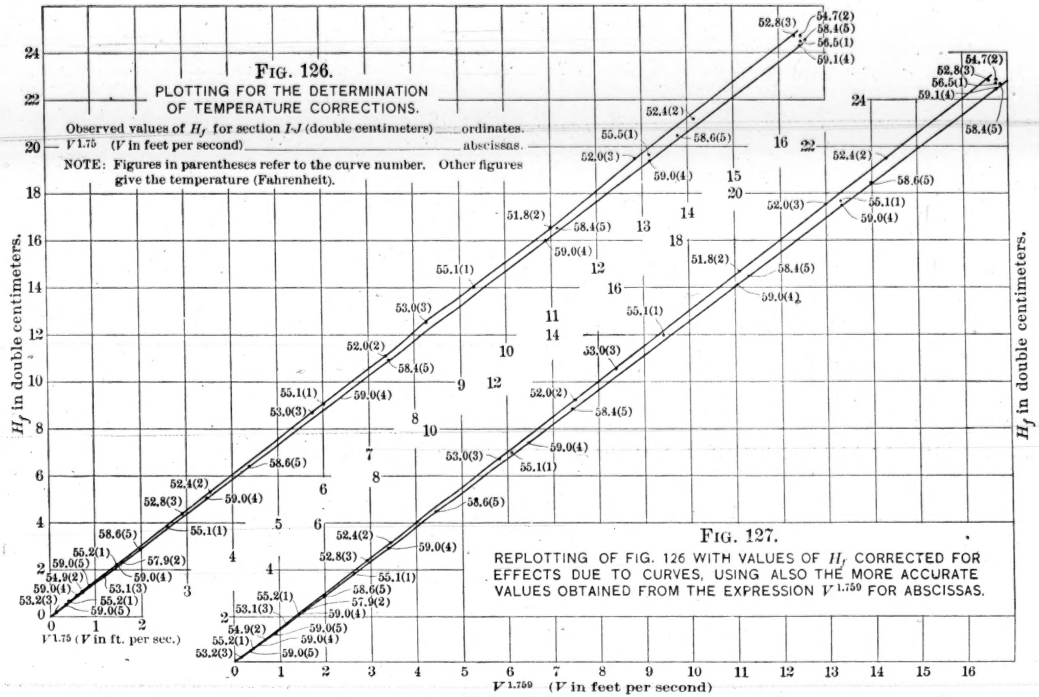

FIG. 126.
PLOTTING FOR THE DETERMINATION OF TEMPERATURE CORRECTIONS.

Observed values of H_f for section I-J (double centimeters) ..... ordinates.
$V^{1.75}$ (V in feet per second) ..... abscissas.

NOTE: Figures in parentheses refer to the curve number. Other figures give the temperature (Fahrenheit).

FIG. 127.
REPLOTTING OF FIG. 126 WITH VALUES OF H_f CORRECTED FOR EFFECTS DUE TO CURVES, USING ALSO THE MORE ACCURATE VALUES OBTAINED FROM THE EXPRESSION $V^{1.759}$ FOR ABSCISSAS.

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also gave straight lines approximately parallel to the line for Section *A B*. The plotted points for the other sections seemed, in a general way, to lie on lines with approximately the same slope as Section *A B*, but they showed considerable irregularity, which was quite pronounced for the lower velocities. These irregularities may be partly due to errors in reading small heads, where an error quite small in itself becomes relatively large in plotting. It will be seen also that the points which show such irregularities were plotted on a part of the logarithmic paper where a small error is greatly magnified owing to the distorted scale.

Excesses of Loss of Head due to Curves.—The losses which would occur in a straight pipe, equal in length to the length of Section *B J*, under normal conditions, were calculated from the relation,

$$H_f = 2.296\ V^{1.759} \times \frac{\text{length of } B\ J}{\text{length of } A\ B'}$$

the different lengths of *B J* being taken from Table No. 76. The resulting values are given in Table No. 77, which also contains the observed losses on Section *B J* after being corrected for temperature, and the excess losses over *A B* for equal length, obtained by subtraction. This table gives also the excesses for Section *I J* over *A B* for equal length.

The reason for taking Section *B J*, in calculating the total excess loss of head due to the curves, is that the conditions were known to be normal at both ends of this section. All the piezometers between the piezometer at *J* and the P. T. of the curves give incorrect indications, because the flow has not yet become normal at the places where they are located. This matter will be considered more fully, later.

The excesses for Section *I J*, given in Table No. 76, are found to be negative in most cases. This indicates that the distortion due to the curves has not entirely disappeared at *I*, a distance of 21.275 ft., or 102 diameters, down stream from the P. T. These negative values, moreover, show that the effect due to the curves is of such a nature as to cause a ring piezometer to communicate a lower pressure than it would communicate under normal conditions of flow.

The fact that the conditions of flow were not entirely normal in Section *I J*, the observed losses in which were used to determine the correction for temperature, gave rise to the suspicion that the differences shown in Fig. 126 might be due partly to the effect of the curves, especially as the temperature was fairly constant for the experiments on this section for any particular curve. Accordingly, the observed heads for Section *I J* were corrected by these excesses. Fig. 127 shows the plotting of these finally corrected heads, using also the more accurate values obtained from the expression $V^{1.759}$. It will be seen that this final correction of observed heads, as obtained by this method, which is really one of successive approximations, reduces Section *I J*

to the basis of entirely normal conditions, retaining, however, the observed temperatures. The temperature correction determined from Fig. 127 is 0.00355 of a double centimeter, instead of 0.00366, as previously determined from Fig. 126. However, no revision of observations was necessary, the corrections which would be introduced being quite inappreciable. Messrs. Saph and Schoder.

Action of Piezometers under Distorted Conditions.—In order to show clearly how piezometers act when the conditions are abnormal, Table No. 78 has been prepared. The losses of head, which would be indicated by each of the different piezometers down stream from the P. T. of the curves, when taken in connection with the piezometer at *B*, were obtained by using some of the observations directly and making such changes in others as to bring them to a "simultaneous observation" basis. Sections *B J* and *B C* were taken as observed, and Section *B I* was obtained by subtracting *I J* from *B J*. The observed losses in *C F*, *F G* and *G H* were reduced in proportion to the 1.75th power of the ratio between the observed velocities and the adopted velocities for comparison. The resulting values have only to be combined by direct addition to get the values as given in Table No. 78.

The losses for the corresponding lengths of straight pipe, under normal conditions, were calculated as before. By subtraction, the excess loss, as indicated by any particular piezometer in connection with a common piezometer at *B*, is found.

All the excesses thus obtained are seen to be greater than the true excesses given in Table No. 77. This shows that every piezometer within the region of distorted flow gives too low an indication. And if, as seems reasonable, the true excess loss due to the curves has been accumulating throughout the whole region of distorted flow, then it is evident that the piezometer nearest the P. T. gives the most incorrect indications, for there is a remarkable constancy in the excesses indicated by piezometers situated anywhere in the region of distortion. Until the flow has become entirely normal, however, piezometric indications give misleading results. These, in some cases in the writers' experiments, amount to an indicated excess 50% above the true excess.

The Detroit Experiments.—It seems reasonable to expect that curve effects in large pipe extend as far as in small pipe, if not farther. Now, the writers' experiments show, beyond any question, that conditions are not yet normal at a distance of 100 diameters down stream from a curve.* While Pitot-tube traverses at a distance of 181 diameters down stream from Curve No. 1 appeared to be symmetrical, yet the coefficient of Tube *G*, obtained from these traverses, is lower than in cases of normal flow at *B* and *J*. From this it is judged that a distance

* Experiments by Messrs. A. Adams and W. E. Wilson in The College of Civil Engineering, Cornell University (see next discussion, p. 335), have shown that the effect of a reducer contraction dies out between 17 and 35 diameters down stream. M. Bazin's experiments on 32-in. pipe show that the effects of an entrance contraction practically disappear between 25 and 50 diameters down stream.

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					With Curve No. 8.								
					Total l = 62.068								
					Curve l = 3.133								
78.	.....	.....	.....	4.944	16.629	42.22	41.234	0.986	4.695	16.574	24.45	24.510	− 0.060
79.	.....	.....	.....	4.306	13.042	33.07	32.339	0.731	4.298	12.999	19.19	19.223	− 0.033
80.	.....	.....	.....	3.359	8.424	21.35	20.888	0.462	3.352	8.396	12.41	12.416	− 0.006
81.	.....	.....	.....	2.732	5.857	14.78	14.522	0.258	2.726	5.837	8.59	8.632	− 0.042
82.	.....	.....	.....	1.849'	2.947	7.37	7.308	0.062	1.845'	2.937	4.34	4.344	− 0.004
83.	.....	.....	.....	1.099	1.181	2.97	2.929	0.041	1.097	1.177	1.74	1.741	− 0.001
84.	.....	.....	.....	0.308	0.126	0.35	0.312	0.038	0.307	0.126	0.21	0.186	− 0.024
						With Curve No. 4.							
						Total l = 61.595							
						Curve l = 2.585							
32.	.....	.....	.....	4.970	16.782	42.08	41.946	0.834	4.960	16.725	24.60	24.735	− 0.135
33.	.....	.....	.....	4.369	13.379	33.51	32.883	0.627	4.361	13.325	19.64	19.719	− 0.079
34.	.....	.....	.....	3.916	11.038	27.02	27.128	0.492	3.909	11.000	16.21	16.311	− 0.100
35.	.....	.....	.....	2.888	6.460	16.14	15.879	0.261	2.883	6.439	9.48	9.522	− 0.042
36.	.....	.....	.....	2.003	3.395	8.41	8.344	0.066	2.000	3.384	4.95	5.004	− 0.054
37.	.....	.....	.....	1.232	1.485	3.72	3.650	0.070	1.230	1.480	2.17	2.189	− 0.019
38.	.....	.....	.....	0.720	0.561	1.37	1.378	0.008	0.718	0.559	0.82	0.825	− 0.005
						With Curve No. 5.							
						Total l = 61.306							
						Curve l = 2.356							
24.	.....	.....	.....	4.985	16.870	42.05	41.318	0.732	4.975	16.815	24.78	24.866	− 0.086
25.	.....	.....	.....	4.499	14.013	35.08	34.312	0.718	4.477	13.966	20.69	20.654	+ 0.036
26.	.....	.....	.....	3.573	11.310	28.38'	27.716	0.668	3.565	11.281	16.72	16.682	+ 0.038
27.	.....	.....	.....	3.146	7.509	18.78	18.385	0.395	3.140	7.411	11.07	10.959	+ 0.111
28.	.....	.....	.....	2.336	4.449	10.98	10.894	0.086	2.332	4.394	6.47	6.557	− 0.087
29.	.....	.....	.....	1.470	1.970	4.84	4.824	0.016	1.468	1.964	2.88	2.904	− 0.024
30.	.....	.....	.....	0.933	0.894	2.18	2.165	0.015	0.931	0.891	1.30	1.303	− 0.003
31.	.....	.....	.....	0.551	0.351	0.88	0.859	0.021	0.550	0.350	0.52	0.517	+ 0.003
						With Curve No. 6.							
						Total l = 61.020							
						Curve l = 2.070							
1.	.....	.....	.....	4.945	16.809	41.83	40.958	0.869	.....	.....	.....	.....	.....
2.	.....	.....	.....	4.607	14.894	36.93	36.153	0.777	.....	.....	.....	.....	.....
3.	.....	.....	.....	4.211	12.695	31.48	30.966	0.514	.....	.....	.....	.....	.....
4.	.....	.....	.....	3.653	9.767	24.40	24.049	0.351	.....	.....	.....	.....	.....
5.	.....	.....	.....	3.003	7.286	18.17	17.957	0.213	.....	.....	.....	.....	.....
6.	.....	.....	.....	2.049	3.510	8.96	8.643	0.317	.....	.....	.....	.....	.....
7.	.....	.....	.....	1.482	1.990	4.95	4.921	0.029	.....	.....	.....	.....	.....
8.	.....	.....	.....	0.954	0.927	2.33	2.320	0.008	.....	.....	.....	.....	.....
9.	.....	.....	.....	0.627	0.440	1.01	1.003	0.003	.....	.....	.....	.....	.....

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TABLE No. 77.

(1) Number of experiment.	Section *A B*. Up-stream Tangent. Average Diam.... 0.1742 ft. Total Length....57.496 ft. (2) Mean velocity for this section. Feet per second. *V*.	(3) $V^{1.759}$.	(4) Observed loss of head, in duo-centimeters. Corrected for temperature by Table No. 2 = H_f.	Section *B J*. Curve and Tangent. Average Diameter .... 0.1737 ft. Length of Tangent .... 58.110 ft. (5) Mean velocity for this section. Feet per second. *V*.	(6) $V^{1.759}$.	(7) Observed loss of head, in duo-centimeters. Corrected for temperature by Table No. 2 = H_f.	(8) H_f for equal length of straight pipe $= 2.296 \frac{B\,J}{A\,B} V^{1.759}$.	(9) Excess H_f over equal length of straight pipe = Col. 7 — Col. 8.	Section *I J*. Down-stream Tangent. Average Diameter.......... 0.1738 ft. Total Length.......... 37.025 ft. (10) Mean velocity for this section. Feet per second. *V*.	(11) $V^{1.759}$.	(12) Observed loss of head, in duo-centimeters. Corrected for temperature by Table No. 2 = H_f.	(13) H_f for equal length of Sec. *A B* $= 2.296 \frac{I J}{A B} V^{1.759}$.	(14) Excess H_f over equal length of Sec. *A B* = Col. 12 — Col. 13.
				With Curve No. 1. Total l = 64.163 Curve l = 5.213									
52	....	....	....	4.970	16.785	43.83	43.013	0.817	4.961	16.741	24.51	24.756	− 0.246
53	....	....	....	4.365	13.263	35.03	34.224	0.806	4.358	13.319	19.59	19.696	− 0.106
54	....	....	....	3.583	9.438	24.89	24.187	0.703	3.577	9.413	13.96	13.931	+ 0.029
55	....	....	....	2.795	6.096	15.98	15.623	0.357	2.789	6.079	8.98	8.989	− 0.009
56	....	....	....	1.731	2.626	6.78	6.730	0.050	1.729	2.619	3.86	3.873	− 0.013
57	....	....	....	1.248	1.477	3.84	3.786	0.054	1.246	1.473	2.22	2.179	+ 0.041
58	....	....	....	0.600	0.407	1.07	1.042	0.028	0.599	0.406	0.66	0.600	+ 0.060
				With Curve No. 2. Total l = 63.090 Curve l = 4.140									
59	....	....	....	4.967	16.766	42.98	42.246	0.734	4.957	16.720	24.44	24.725	− 0.285
60	....	....	....	4.545	14.341	36.84	36.138	0.702	4.538	14.301	20.98	21.149	− 0.169
61	....	....	....	3.937	11.141	28.70	28.074	0.626	3.930	11.108	16.29	16.421	− 0.131
62	....	....	....	3.139	7.479	19.24	18.846	0.394	3.133	7.454	10.94	11.023	− 0.083
63	....	....	....	2.019	3.443	8.80	8.675	0.125	2.016	3.481	5.04	5.074	− 0.034
64	....	....	....	1.299	1.585	4.04	3.994	0.046	1.297	1.580	2.82	2.836	− 0.016
65	....	....	....	0.805	0.683	1.72	1.720	0.000	0.808	0.680	1.01	1.006	− 0.004

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of at least 200 diameters of down-stream tangent is required before piezometric indications become reliable. It thus appears that most of the authors' values for excess loss of head due to curves are too high.

TABLE No. 78.

Curve.	Mean velocity, in feet per second.	Length of section, in feet.	Mean diameter, in inches.	Loss in double centimeters which piezometers would indicate.	Loss in the same length of tangent (up stream).	Excess which the piezometers would indicate.	Mean velocity, in feet per second.	Length of section, in feet.	Mean diameter, in inches.	Loss in double centimeters which piezometers would indicate.	Loss in same length of up-stream tangent.	Excess which the piezometers would indicate.
	Section *B–C.*						Section *B–F.*					
1	5.001	6.053	2.078	5.19	4.06	1.13	4.992	8.813	2.080	7.10	5.95	1.15
2	5.024	4.980	2.072	4.41	3.33	1.08	5.005	7.740	2.076	6.32	5.25	1.07
3	4.992	3.973	2.074	3.64	2.64	1.00	4.972	6.733	2.078	5.57	4.55	1.02
4	5.013	3.425	2.075	3.29	2.30	0.99	4.994	6.185	2.079	5.17	4.18	0.99
5	5.004	3.196	2.080	3.17	2.15	1.02	4.994	5.956	2.082	5.06	4.03	1.03
6	5.090	2.910	2.060	3.03	1.95	1.08	5.032	5.670	2.072	4.92	3.89	1.03
	Section *B–G.*						Section *B–H.*					
1	4.970	11.895	2.084	9.12	7.89	1.23	4.975	18.623	2.083	13.65	12.51	1.14
2	4.977	10.822	2.082	8.36	7.27	1.09	4.977	17.550	2.082	12.90	11.79	1.11
3	4.944	9.815	2.084	7.58	6.52	1.06	4.953	16.543	2.082	12.08	11.03	1.05
4	4.965	9.217	2.085	7.17	6.20	0.97	4.974	15.995	2.083	11.68	10.74	0.94
5	4.970	9.038	2.087	7.07	6.06	1.01	4.985	15.766	2.084	11.67	10.62	1.05
6	4.995	8.752	2.080	6.95	5.92	1.03	4.996	15.480	2.080	11.54	10.34	1.20
	Section *B–I.*						Section *B–J.*					
1	4.980	27.138	2.082	19.32	18.26	1.06	4.970	64.163	.....	43.83	43.01	0.82
2	4.981	26.066	2.081	18.54	17.54	1.00	4.967	63.090	.....	42.98	42.25	0.73
3	4.958	25.058	2.081	17.77	16.73	1.04	4.944	62.083	.....	42.22	41.23	0.99
4	4.979	24.510	2.082	17.48	16.48	1.00	4.970	61.535	.....	42.08	41.25	0.83
5	4.989	24.281	2.088	17.27	16.39	0.88	4.985	61.306	.....	42.05	41.32	0.73
6	4.996	23.995	2.080	17.16	16.02	1.14	4.974	61.020	.....	41.82	40.96	0.86

The points which have the most influence in determining the form of the curve given by the authors in Fig. 90, are those obtained from 30-in. pipe. Concerning these 30-in. curves, a reference to Fig. 23 will show that not only are the down-stream piezometers too close to the curves, but that the up-stream piezometer is in most cases affected by the preceding curve. The effect is to make both piezometers read low, but by different amounts.

For these curves, the basis of calculation seems to be wrong. The excesses were compared to the loss in the preceding tangent, except in two cases where the mean of the following tangent and the second preceding tangent was taken, all of which are probably too low. This means that the percentage excesses obtained are too high.

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The high losses which were obtained in curve sections affected by preceding curves may be due, also, in large part, to the fact that there is in every case a reversal of curvature. Both the authors' and the writers' experiments have shown that the effect of a curve is to shift the maximum velocity toward the convex side of the curve, and that this distorted flow extends a considerable distance down stream. Now, when the water approaches a curve with the maximum velocity on the opposite side, one would naturally expect a higher loss of head than if normal conditions existed in the up-stream tangent. This higher loss cannot justly be attributed to the single curve.

There are only two experiments on 16-in. curves. Both of these would be expected to give high excesses; the Porter Street and Cavalry Avenue curve chiefly on account of the down-stream piezometer, and the Cavalry Avenue and Army Avenue curve chiefly on account of the reversal of curvature effect. In addition, both were compared with the Army Avenue tangent, which is, undoubtedly, affected by the preceding curve.

Considering the 12-in. experiments, the down-stream piezometers are in all cases too close to the curves. Curve No. 16 is the most interesting of all, because it is the curve of the shortest radius in the whole series, and because it is the one which determines the upward turn of the plotted curve in Fig. 90. While this curve may be affected by a gate a short distance up stream, any corrections made on that account would have to be made on Curve No. 9, as well. The corrections for these gates would still further reduce the excesses obtained by the authors. Another point regarding these curves is that the up-stream piezometers were single corporation cocks tapped into the top of the pipe, while the down-stream piezometers were special castings of the ring type. In the other curves ⅛-in. cocks were used for both up-stream and down-stream piezometers. It is quite evident, from experiments by the writers, that a single pressure connection on the top of a pipe just beyond a curve communicates a lower pressure than a ring piezometer at the same point. A reference to Fig. 122, or to any of the vertical traverses made by the writers, within the region of distorted flow, Fig. 125, will show that the wall velocity at the top and bottom is greater than the mean of any other two opposite points. In other words, the pressure at the top is less than the mean wall pressure. Hence, a comparison of the indications for Curves Nos. 9 and 16 with Curves Nos. 11, 12 and 15 is not justifiable.

Excess Losses in the Writers' Experiments.—The total excess losses of head due to curves, as found by the experiments of the writers, and given in Table No. 77, show no tendency to follow any simple law. When plotted as percentages of the loss in 80 diameters of straight pipe under normal conditions, the plotted points give no suggestion of the authors' curve given in Fig. 90. Nor do the percentages so

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obtained amount, in any case, to more than a very small fraction of those obtained by the authors.

It seems improbable that a revision of the authors' data, if possible, would lower the calculated excesses to the neighborhood of the writers' values.

The Losses Occurring in the Curve Itself.—When the Pitot-tube traverses in Curve No. 1, referred to in connection with Fig. 124, were made, a triple gauge was used, the third column being attached to the ring piezometer at *B* (0.650 ft. up stream from the P. C.). By this means the variation in pressure around the curve, as shown by the ring-pressure openings of the tube during the traverses, was obtained. These pressure traverses, as they may be termed, showed that wherever the velocity was highest the pressure was lowest, and *vice versa*. However, the difference between the velocity heads at any two points in the same cross-section was not equal to the corresponding difference of pressure heads, as shown by the pressure openings of the tube. The pressure traverses were taken on all the bosses on Curve No. 1, but as the exact nature of the distortion at each was not known, except by a comparison of the horizontal traverses with normal conditions and with Fig. 122, it was impossible to make a close approximation to the summation of pressures throughout the cross-sections. By applying the mean of two corresponding points on opposite sides of the center to the area of the ring passing through them, and summing these products for each position around the curve, and finally summing the differences between any section and the one next to it, a remarkably close check was obtained on the observed loss of head between the ring piezometers at *B* and *C*. The partial differences were as follows:

	Piez. No. 2 at *B*.		Boss No. 1.		Boss No. 2.		Boss No. 3.		Boss No. 4.		Boss No. 5.		Boss No. 6.		Piez. No. 7 at *C*.
Differences, in double centimeters.....		1.081		0.579		0.871		0.827		0.911		0.544		0.546	

The sum of the differences gives 5.31, as against 5.42, the observed head for *B C*.

Now, it has been shown that the indications of a piezometer at *B* are lower than they would be under normal conditions, and the relation just pointed out indicates that the same is true of the indications of horizontal pressure traverses. Comparing only the differences between the bosses which are equally spaced around the curve, it would appear that the greatest loss occurs between Bosses Nos. 2 and 5, or

between 45° and 135°. But such a comparison is hardly just, on the basis of the present knowledge of conditions and of the peculiar action of the Pitot tubes used. Messrs. Saph and Schoder.

It may be remarked that there is in the foregoing a suggestion of a method for finding by Pitot-tube traverses the true loss of head between two points, one or both of which may be situated in places of abnormal flow.

In conclusion, the writers desire to express their high appreciation of the ever ready and valuable assistance and the enthusiastic interest of Professor Gardner S. Williams throughout the whole series of experiments, even when the indications seemed to be directly opposed to the findings of the Detroit experiments. They also desire to acknowledge the broad liberality at all times shown by the authorities of the College of Civil Engineering of Cornell University, in their attitude toward the experiments.*

ARTHUR ADAMS, Jun. Am. Soc. C. E., and W. E. WILSON, Esq. (by letter).—At the suggestion of G. S. Williams, M. Am. Soc. C. E., the writers, during the past year, have conducted a series of experiments along the lines described in this paper; in general, relative to the effect of obstructions in pipes upon the flow of water in them. The experiments, however, have been confined entirely to the effect of a contraction upon the distribution of velocities within the pipe-section, and have been carried on chiefly in a pipe of 2 ins. diameter. Messrs. Adams and Wilson.

The apparatus used was a part of that in the Hydraulic Laboratory of Cornell University. The water supply, system of piping, form of contractions, and means for the accurate measurement of discharge by weighing, are all described on pages 18 and 20. The plan of the piping also appears in Fig. 119. In the present case, however, the piezometers shown at the down-stream end of the curve were located next below the contraction at the inlet of the 2-in. brass pipe. The distances of the several piezometers from the contraction, both in feet and in diameters of pipe, are shown in Table No. 79.

In addition to the description of the piezometer given on page 20, and in Fig. 4, it is to be observed that each piezometer has two openings in the same diametric plane, and at right angles to each other, designated, respectively, as the "west" and "east" sides of the piezometer. Usually, the openings have been located so as to admit of nearly vertical and horizontal positions of the tube. The angles by which they differ from such vertical or horizontal positions have been measured, and are recorded in Column 4 of Table No. 79. The Pitot tube used was the Cornell Tube No. 2, or Tube *E*, of which the cali-

* The data of these observations, in their original form, are on file in the Hydraulic Laboratory of the College of Civil Engineering of Cornell University, together with much reduced material not included in this discussion, where they are open to examination at any time.

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bration and rating curve under normal conditions are given in Plate I. The gauge was a differential gauge, designated by the authors as "*C* double."

The method of procedure, including the insertion of the tube into the 2-in. pipe, the connection to the gauges, and "traversing," are described on page 12; and likewise, on page 21, the method of reducing the rating experiments (except that in obtaining the mean velocity of the section, V_m, by the tube, the center reading has been given one weight by the writers, instead of two as by the authors).

The writers had as their objects in these experiments: (1) To aid in extending the scope of the experiments carried on at Detroit; (2) to study the workings of the Pitot tube under other than normal conditions; and (3) to discover if the velocities in pipes, near causes of disturbance, are distributed according to a natural law, and whether that law depends upon the distance of the point of observation from the disturbance. After presenting the experimental data obtained, together with the numerical and graphical reductions, it will be attempted to show that such a law as mentioned above does exist.

In Table No. 79 are given the final results of the traverses in the various piezometers; the several reductions leading thereto are omitted, these being, as previously stated, quite similar to the corresponding reductions made by the authors. These results are shown graphically in Fig. 128 with an accompanying explanation. Column 7 of Table No. 79 contains the mean velocity of every traverse, and Column 8 the mean velocity for the corresponding period, as obtained by weighing the discharge. The center velocity by Pitot tube, as given in Column 11, is the mean of the first reading at the center, the middle reading at the center taken twice, and the final center reading. The remaining columns are self-explanatory. The preceding three velocities are denoted by V_m, U, and V_c, respectively.

It has been shown in this paper that the Pitot tube and appurtenances, under normal conditions of flow, make a consistent recorder of velocities existing in a pipe. By normal conditions are meant the conditions under which the water flows through a pipe line undisturbed by any obstruction. Such will give, for any section where they prevail, a constant ratio of the mean velocity of the section, V_m, to the velocity at the center, V_c, the latter being always a maximum for the section, as will be seen upon examining the traverses in Fig. 128. The authors, in rating the Cornell Tube No. 2, or Tube *E*, in Piezometer No. 4 in the 2-in. brass pipe, obtained for the ratio, $\frac{V_m}{V_c}$, 0.8139. This value, as will be shown later, was the result of experiments made under normal conditions. The writers, however, have obtained as the value of the ratio, $\frac{V_m}{V_c}$, 0.798, this being the mean of

TABLE No. 79.—PITOT-TUBE TRAVERSES BELOW CONTRACTION.

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Pipe, 2.096 ins. in diameter.

Reference number of traverse. (1)	Number of piezometer. (2)	Side of piezometer. (3)	Angle of inclination from vertical. (4)	Distance of piezometer from contraction, in feet. (5)	Distance of piezometer from contraction, in diameters. (6)	V_m, mean velocity by Pitot tube. (7)	U, mean velocity by weight. (8)	Coefficient, ratio of $\frac{U}{V_m}$ (9)	Mean, $\frac{U}{V_m}$ (10)	V_c, center velocity by Pitot tube. (11)	Ratio of $\frac{V_m}{V_c}$ (12)	Mean ratio of $\frac{V_m}{V_c}$ (13)
1..	7	West	58°	0.420	2.404	3.496	3.509	1.004		3.660	0.955	
2..	7	"	53	0.420	2.404	2.692	2.713	1.008		2.910	0.925	
3..	7	"	53	0.420	2.404	1.854	1.884	1.016		2.000	0.927	
4..	7	"	33	0.420	2.404	0.856	0.887	1.036	1.014	0.940	0.911	0.938
5..	7	East	37	0.420	2.404	5.823	5.867	1.008		6.060	0.961	
6..	7	"	37	0.420	2.404	4.081	4.070	1.007		4.220	0.955	
7..	7	"	37	0.420	2.404	1.547	1.577	1.019		1.661	0.931	
8..	8	West	55	0.913	5.227	3.406	3.604	1.058		3.760	0.906	
9..	8	"	55	0.913	5.227	1.539	1.573	1.022	1.037	1.700	0.905	0.910
10..	8	East	35	0.913	5.227	3.442	3.633	1.055		3.820	0.901	
11..	8	"	35	0.913	5.227	1.565	1.587	1.014		1.690	0.926	
12..	9	West	65	1.803	10.321	4.402	4.458	1.013		5.000	0.880	
13..	9	"	65	1.803	10.321	3.582	3.648	1.018		4.130	0.867	
14..	9	"	65	1.803	10.321	1.374	1.384	1.007	1.013	1.590	0.864	0.871
15..	9	East	25	1.803	10.321	3.523	3.604	1.023		4.080	0.863	
16..	9	"	25	1.803	10.321	1.334	1.337	1.002		1.510	0.883	
17..	6	West	69	3.158	18.078	4.123	4.095	0.993		4.920	0.838	
18..	6	"	69	3.158	18.078	2.888	2.799	0.986		3.380	0.840	
19..	6	"	69	3.158	18.078	1.212	1.183	0.975	1.002	1.463	0.828	0.831
20..	6	East	21	3.158	18.078	4.052	4.120	1.017		4.960	0.817	
21..	6	"	21	3.158	18.078	2.713	2.737	1.009		3.250	0.835	
22..	6	"	21	3.158	18.078	1.102	1.136	1.031		1.333	0.827	
23..	4	West	69	6.238	35.710	3.488	3.613	1.036		4.410	0.791	
24..	4	"	69	6.238	35.710	1.468	1.503	1.024	1.043	1.790	0.820	0.798
25..	4	East	21	6.238	35.710	3.444	3.615	1.050		4.380	0.786	
26..	4	"	21	6.238	35.710	1.517	1.609	1.061		1.910	0.794	
27..	10	West	40	21.118	120.89	3.366	3.464	1.029		4.230	0.796	
28..	10	"	40	21.118	120.89	3.452	3.474	1.006		4.160	0.830	
29..	10	"	40	21.118	120.89	4.925	4.956	1.006	1.027	5.920	0.832	0.803
30..	10	East	50	21.118	120.89	3.002	3.153	1.050		3.848	0.780	
31..	10	"	50	21.118	120.89	1.136	1.187	1.045		1.460	0.778	
32..	2	West	29	68.285	390.90	3.408	3.611	1.060		4.390	0.776	
33..	2	"	29	68.285	390.90	1.887	1.980	1.049	1.030	2.320	0.813	0.796
34..	2	East	61	68.285	590.90	4.516	4.577	1.014		5.680	0.795	
35..	2	"	61	68.285	390.90	1.274	1.272	0.998		1.590	0.801	
36..	1	"	.....	125.781	720.05	1.190	1.284	1.079	1.056	1.492	0.798	0.796
37..	1	"	.....	125.781	720.05	3.059	3.163	1.034		3.848	0.795	
								Mean 1.039 (Normal conditions.) Mean 1.017 (Abnormal conditions.)			Mean 0.798 (Normal conditions).	

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the observations made in all the piezometers included in normal flow. The apparent discrepancy between the two is explained by the fact that the authors, in computing the mean velocity of the section, V_m, have used the mean center velocity, V_c, twice, whereas the writers have used it but once, which, of course, decreases the ratio, $\frac{V_m}{V_c}$, since the maximum velocity of the section is at the center, one of the conditions of normal flow. Considering the traverses of the latter in Piezometer No. 4, and using the center velocity twice, the resulting ratio, $\frac{V_m}{V_c}$, is 0.804, and in Piezometer No. 6, it is 0.808, or a mean of the two = 0.806, which checks closely with the authors' value. They obtain for Tube *E*, in Piezometer No. 4 (see Table No. 4), a value of 0.808. It is to be remarked that the writers, in bringing forward this question of the weight of the center velocity, do not contemplate a contradiction of the authors' method of finding the mean velocity of section, V_m; for one of them, Mr. G. S. Williams, has given the writers reasons sufficiently good to warrant using V_c twice; but as the error introduced in either case is practically the same, one method giving an excess and the other a deficiency, the latter have made their computations on a one-weight basis; but the above point is advanced in order to explain the difference in the two values of the ratio, $\frac{V_m}{V_c}$. The value of the writers' rating coefficient is practically the same as that of the authors, namely, 1.039, as compared with 1.025, the authors' value. The former is larger because the ratio, $\frac{U}{V_m}$, increases as V_m (depending as it does upon the weights given the individual observations) decreases. With the center velocity used twice, the rating coefficient obtained by the writers in Piezometer No. 4, which was the piezometer in which the authors' rating experiments for Tube *E* were carried on, is 1.028.

Under conditions rendered abnormal by a contraction, the Pitot tube is as consistent a measurer of velocities in a pipe as under normal conditions. This consistency of measurement depends upon the constancy of its rating coefficient. In order to investigate this constancy of coefficient under abnormal conditions, the writers have plotted the observations in Fig. 129, the abscissas of which are the mean velocities of the section by tube, V_m, and the ordinates the mean velocities by weight, U, of the corresponding traverses. These are plotted separately for each piezometer, whether it be located at points where normal or abnormal conditions prevail. The numbers opposite the points refer to the number of the traverse as given in Table No. 79. A line connecting the points for any one piezometer, it is seen at once, would practically coincide with the straight averaging line which is drawn in the figure. Since such a graphic treatment has been assumed through-

DISTRIBUTION OF VELOCITIES IN 2-INCH BRASS PIPE SHOWING EFFECT OF CONTRACTION AT INLET.

Diameter of Pipe = 2.096 ins. = 0.1747 ft. Open Circles represent East and Filled Circles West Traverses.
Numbers of Traverses correspond to Table No. 79 Scale of Velocities = ⅓ Scale of Diameters.

FIG. 128.

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out the paper and discussions, to determine definitely the constancy of the Pitot-tube rating coefficient the evidence is conclusive that the Pitot tube and appurtenances are a consistent recorder of velocities in a pipe under normal and abnormal conditions alike, as long as the distortion is symmetrical with respect to the axis of the pipe.

It is to be noted that, under abnormal conditions produced by a contraction, the rating coefficient is 1.017, being the mean of the values for Piezometers Nos. 7, 8, 9, and 6 (see Table No. 79); whereas, under normal conditions the coefficient 1.039 is but little larger, the difference being 2.1 per cent.

Upon examining the traverses in the 2-in. pipe, as shown in Fig. 128, it is seen that an averaging curve drawn through the points of observation in any one traverse taken in Piezometer No. 7, which is the one nearest the contraction of the 5-in. into the 2-in. pipe, is very much flatter than a similar curve drawn through the points of observation of a traverse with velocities approximately equal to those in question in Piezometer No. 4, which is 5.82 ft., or 33 diameters, farther away from the contraction. Furthermore, it is seen that the velocity distribution curve elongates gradually in the direction of the pipe axis, as the point of observation moves away from the contraction. In order to show that this change in form is a regular one, the writers have plotted a curve, Fig. 130, with the distances of the piezometers, or points of observation, from the contraction, in diameters, as abscissas (a diameter measurement being the ratio of the distance of the point of observation to the mean diameter of the pipe); and the ratios, $\frac{V_m}{V_c}$, as ordinates. Examining this curve and Column 13 of Table No. 79, it is seen that the flow becomes constant, and therefore, as may reasonably be concluded, normal, somewhere between Piezometer No. 6 at 18 diameters, and Piezometer No. 4 at 35 diameters, the curve as drawn indicating the point of normal flow to be quite near the latter piezometer. Incidentally, it is of interest to observe that the curve produced back to intersect the y-axis of co-ordinates gives a ratio of $\frac{V_m}{V_c} = 1.000$ at 0.0 diameters; that is, the surface representing velocity distribution in the pipe, as the flow passes the contraction, is flat.

It will be noticed in Fig. 119 that a cylindrical regulator, 13.62 ins. in diameter, was located at the head of the 5-in. pipe leading to the 2-in. pipe, in which the foregoing experiments were carried on, thus forming a contraction and an obstruction to the water flowing into the former pipe, and producing abnormal conditions in it. To prove that this abnormal flow in the 5-in. pipe is eradicated, and that normal conditions again prevail before the water filaments arrive at the contraction into the 2-in. pipe, which fact, otherwise, might affect the observations in the latter, the writers made eleven traverses in the former, four in

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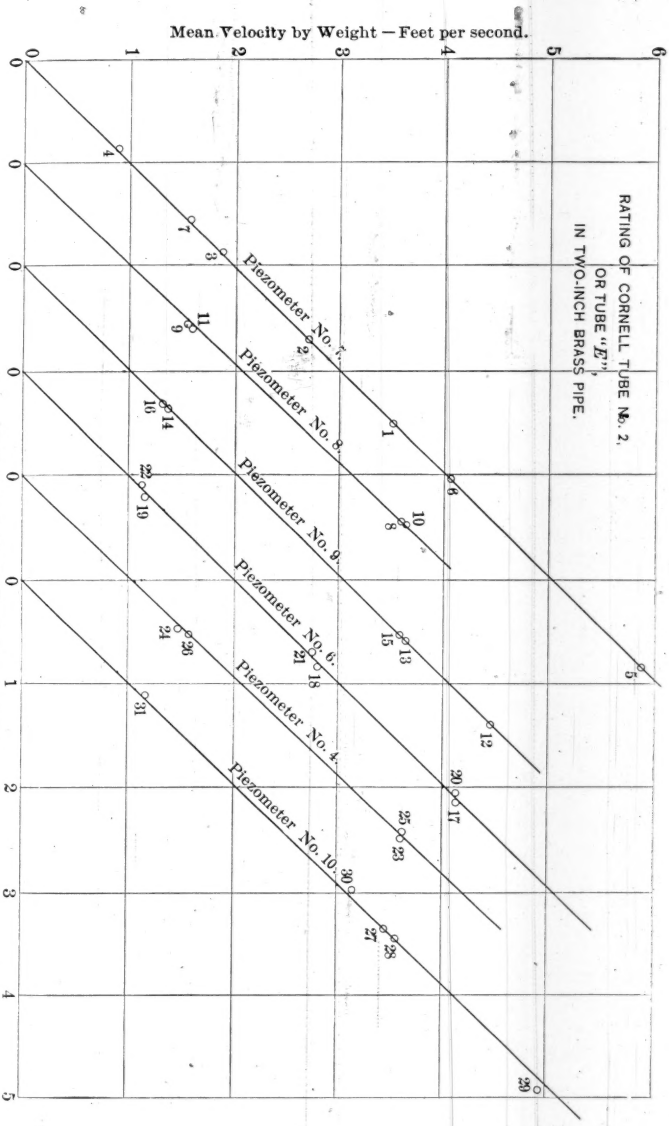

FIG. 129.

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each of two of the piezometers, and three in the third. The observations at the inlet piezometer, 0.4 diameter from the regulator contraction, give the ratio $\frac{V_m}{V_c}$ a value of 0.87; and at the piezometer 40 diameters, and also the piezometer 48 diameters, from the inlet, a value of 0.80. That is, the flow becomes constant, and normal conditions prevalent, in the 5-in. pipe before it reaches a point 40 diameters from the regulator contraction, and accordingly, the water filaments enter the contraction into the 2-in. pipe in an undisturbed state of equilibrium. This, also, corroborates the results cited in regard to normal flow in the 2-in. pipe.

From the data at hand there is no evidence that the higher the velocity the greater is the tendency to eradicate any distortion from the normal distribution as induced by a contraction, when any particular point of observation is considered, *i. e.*, at the same distance from the contraction. In other words, at first thought, it might be expected that as the velocity increased, the ratio $\frac{V_m}{V_c}$ would decrease. Examining the traverses in those piezometers past which the water flows with an abnormal distribution of velocities (see Table No. 79), it is seen that the ratio, $\frac{V_m}{V_c}$, for the highest velocity in Piezometer No.7, 0.961 for 6.060 ft. per second, is greater than the mean, 0.938, for that piezometer; and that in each of the other three piezometers the ratio for the respective maximum velocity is less than the mean for the piezometer. Moreover, it is to be noticed that the ratios have no uniformity whatever in the change of their values as the velocity decreases from the maximum. Hence, it is quite improbable that a greater range in velocities would affect the foregoing statements concerning the ratios $\frac{V_m}{V_c}$ and $\frac{U}{V_c}$, the variations in the table being probably due to experimental errors.*

Unless different forms of contraction, pipes of different material or condition, and different temperatures of water affect the flow enough to produce results, other things being equal, appreciably different from the above, which is doubtful, the statement can safely be made that in the smaller pipes, up to at least 5 ins. diameter, the water filaments disturbed from equilibrium by a contraction in the pipe line, attain equilibrium again after passing a point 35 diameters from the disturbance; and in such pipes the mean velocity of the flow, at any section where normal conditions prevail, is about 80% of the maximum velocity at the center of that section.

* A table giving these observations in detail is on file in the Library of the Society and the original notes are in the custody of the Hydraulic Laboratory of the College of Civil Engineering of Cornell University.

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M. Bazin, in his discussion on this paper, presents a number of traverses made in a pipe 31.496 ins. in diameter; at the head of it is a contraction from a large chamber into the 31½-in. pipe.

In order that a proper determination of the discharge may be obtained from these experiments, it becomes necessary to obtain a value for the velocity at the wall of the pipe, which was not observed, the nearest observation thereto, as shown by Table No. 68, being about 1 in. away.

In Table No. 60, however, an assigned value is given to V_p for a mean of all the traverses upon various diameters at any one shaft and during the same mean velocity of flow; these values are not the results of experiments, but are obtained by M. Bazin from his velocity distribution curves. In Table No. 80 are given the ratios of the velocity at the perimeter, V_p, to the velocity at the center, V_c, the values for both being taken from Table No. 60. This gives at Shaft A a mean ratio $\frac{V_p}{V_c} = 0.668$; and at Shafts B and C, together, 0.634. Shaft A is considered singly, because the difference in the value of this ratio shows that the same conditions do not prevail there as at Shafts B and C.

TABLE No. 80.—Relation of Velocity at Wall to Velocity at Center, in M. Bazin's Experiments.

Number of traverse.	Location.		Velocity at Center = V_c.	Velocity at Perimeter (from Table No. 60) = V_p.	Ratio: $\frac{V_p}{V_c}$	Mean: $\frac{V_p}{V_c}$	
	Number of shaft.	Number of discharge.				A.	B and C.
1 and 4.....	A.	1	1.249	0.820	0.657		
2 and 5.....	A.	2	1.665	1.130	0.678		
3 and 6.....	B.	1	1.246	0.805	0.646	Mean = 0.668	Mean = 0.635
7 and 10	B.	2	1.706	1.080	0.633		
8 and 11.....	C.	1	1.271	0.795	0.626		
9 and 12.....	C.	2	1.721	1.095	0.636		

To further prove that the flow is abnormal at Shaft A and approximately normal at Shafts B and C, the writers have taken the mean velocity, V_m, as computed by M. Bazin from his traverses, and computed the ratios $\frac{V_m}{V_c}$ (which are presented in Table No. 81). These results likewise indicate by the high value of the ratios at Shaft A

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and the lower and nearly equal values at Shafts B and C, that after passing Shaft A, and before reaching Shaft B, the flow changes from an abnormal to a nearly normal state, this change occurring between 25 and 50 diameters from the contraction.

TABLE No. 81.—EFFECT OF DISTORTION ON RATIO $\frac{V_m}{V_c}$, IN M. BAZIN'S EXPERIMENTS.

Number of traverse.	Location of Traverse.			Distance of shaft from contraction, in diameters.	Velocity at center, V_c, in feet per second.	Velocity at perimeter of pipe, in feet per second.	Mean velocity, V_m, in feet per second.	Ratio: $\frac{V_m}{V_c}$
	Name of shaft.	Number of discharge.	Diameter observed.					
1.....	*A*	1st	Vertical	25.6	4.085	2.042	3.635	0.890
2.....	*B*	1st	Vertical	50.6	4.104	1.847	3.478	0.847
3.....	*C*	1st	Vertical	75.6	4.167	1.875	3.566	0.856
4.....	*A*	1st	Horizontal	25.6	4.085	2.042	3.635	0.890
5.....	*B*	1st	Horizontal	50.6	4.104	1.847	3.478	0.847
6.....	*C*	1st	Horizontal	75.6	4.167	1.875	3.566	0.856
7.....	*A*	2d	Vertical	25.6	5.449	2.725	4.876	0.894
8.....	*B*	2d	Vertical	50.6	5.600	2.520	4.820	0.861
9.....	*C*	2d	Vertical	75.6	5.626	2.532	4.849	0.862
10.....	*A*	2d	Horizontal	25.6	5.449	2.725	4.876	0.894
11.....	*B*	2d	Horizontal	50.6	5.600	2.520	4.820	0.861
12.....	*C*	2d	Horizontal	75.6	5.626	2.532	4.849	0.862

Mean Ratios, $\frac{V_m}{V_c}$: at A, 0.892; at B, 0.854; at C, 0.859.

Mr. Edward S. Cole has also furnished some interesting data in regard to the distribution of velocities in pipes. Examining the plan of his system of experimental piping, Fig. 105, and remembering that, according to Messrs. Saph and Schoder (page 316), the influence of curvature extends considerably more than 100 diameters down stream, not to mention the effect of expansions (which, likely, is akin to that of contractions), it is to be noted that in only one instance is it probable that normal conditions would prevail at the various points of observation. In the 4-in. pipe, it has been learned that the traverse was made about 18 ft., or 54 diameters, from a 90° curve, which precludes the flow being normal at that point, for the reason already stated, together with the authors' own statement in regard to the influence of curvature. Fig. 108 confirms the presence of this abnormal distribution; hence it is not fair to compare Mr. Cole's ratio, $\frac{U}{V_c}$ (where V_c in this case, as well as the others included in Mr. Cole's work, has been corrected by introducing the coefficient of the tube) of 0.936 with the ratios under normal conditions given above. In the 6-in. pipe the

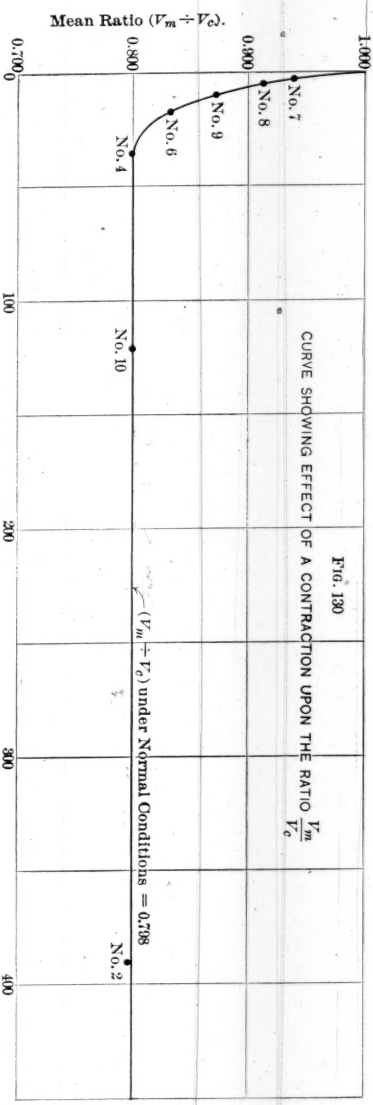

FIG. 130

CURVE SHOWING EFFECT OF A CONTRACTION UPON THE RATIO $\frac{V_m}{V_c}$

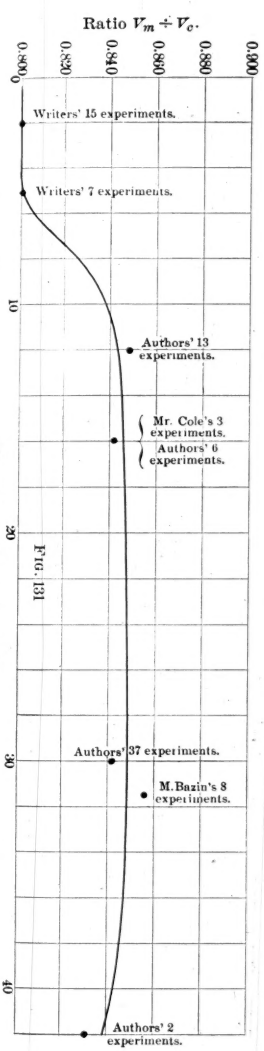

FIG. 131

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traverses were made at a point 10 ft., or 20 diameters, from a 90° curve, which Fig. 108 shows to have produced a distortion in the velocity distribution. The remarks made herein, in connection with the 4-in. pipe, can be applied to the 6-in. pipe. The data and ratios for the 8-in. pipe are omitted, but Fig. 108 shows quite clearly an abnormal distribution of velocities. For the 12-in. pipe, the mean ratio is 0.93, which indicates that the center velocity is either not a maximum or is less than normal. The traverses were made about 10 ft., or 10 diameters, from an expansion and about 60 diameters from a 90° curve, and may be considered to have been influenced by both the expansion and the curve. In the 16-in. pipe, the nearest disturbance was a curve 400 ft., or 300 diameters, up stream. From Table No. 69, the ratio, $\frac{U}{V_c}$, is found to be 0.843, and seems to represent normal conditions. It appears, therefore, that the most of Mr. Cole's experiments are not of general application, other than to illustrate the danger of assuming standard conditions in a pipe too near disturbances having the character mentioned above, and the errors in measurement likely to result from such an assumption.

TABLE No. 82.

Number.	Diameter of pipe, in inches.	Name of Experimenter.	$\frac{V_m}{V_c}$
1	2	Writers.	0.798
2	5	Writers.	0.800
3	12	Authors.	0.843
4	16	Cole.	0.843
5	16	Authors.	0.841
6	30	Authors.	0.842
7	31.5	Bazin.	0.856
8	42	Authors.	0.881

In Table No. 8 is given a series of values of the ratio, $\frac{V_m}{V_c}$, in pipes of various diameters, where normal conditions prevail. In attempting to discover if this ratio, $\frac{V_m}{V_c}$, is different for different diameters of pipe, and if its change is a uniform one, the writers have prepared Table No. 82, in which are collected the values of the ratio $\frac{V_m}{V_c}$ for all the different sized pipes mentioned in this discussion, omitting the cases of low velocities, for reasons explained by the authors, and a mean found for each set of experiments. From the latter table Fig. 131 is plotted, with the diameter of the pipe in inches as abscissas, and $\frac{V_m}{V_c}$

as ordinates. This curve is presented in order to show that it is hardly probable that the ratio, $\frac{V_m}{V_c}$, in pipes of different diameters, is the same. Messrs. Adams and Wilson.

The writers feel that from the experiments, data, and computation herewith presented, they can, in a few words, state:

1.—That the rating coefficient of the Pitot tube of the form used is constant under abnormal conditions produced by a contraction as well as under normal conditions; and, further, that it does not differ materially under either set of conditions.

2.—That a contraction does not displace the maximum velocity from the center of the section.

3.—That a contraction in a pipe line produces an abnormal distribution of velocities therein, and that in the process of eradication of this abnormal distribution, the change in the value of the ratio, $\frac{V_m}{V_c}$, is regular; and that normal flow is again attained after passing a point about 35 diameters from the contraction.

4.—That the ratio, $\frac{V_m}{V_c}$, for small pipes, is not 0.84, which is stated by the authors to apply to all pipes.

5.—That probably the ratio, $\frac{V_m}{V_c}$, increases directly as some function of the diameter of the pipe.

GARDNER S. WILLIAMS, M. Am. Soc. C. E., CLARENCE W. HUBBELL, Assoc. M. Am. Soc. C. E., and GEORGE H. FENKELL, Jun. Am. Soc. C. E. (by letter).—At the conclusion of the paper, the writers summarized what seemed to them to be the most interesting features of the investigation, under eleven heads. Messrs. Williams, Hubbell and Fenkell.

RESULTS *A*, *B* AND *D*.

The Oil Differential Gauge.—Regarding the results designated as *A*, *B* and *D*, which relate particularly to the apparatus used, although at the time of producing their instruments, and for a considerable period thereafter, the writers were entirely ignorant of the fact, it appears that the oil differential gauge was invented and patented by the late Henry Flad, Past-President, Am. Soc. C. E., of St. Louis, in 1885, as stated by Mr. Sherman, and again invented in 1900 by Mr. William M. White, of New Orleans, and a modification of it, as shown in the discussion, was used by Mr. Edward S. Cole, at Terre Haute, in 1896. It was also used by Professor Frederick W. Sperr,* in Ohio, some years previous to these experiments, for the measurement of the flow of gas in pipes. It appears that these several inventions of the

* Of the Michigan College of Mines, Houghton, Mich.

Messrs. Williams, Hubbell and Fenkell.

device, as well as that of the writers, were entirely independent of each other, unless that of Mr. Cole was connected with that of Mr. Flad. To the writers it appears in no wise surprising that so useful and simple an instrument should have been used before, and, undoubtedly, there are other instances which have not yet come to their attention; but, during the progress of the experiments described, the results and the apparatus were exhibited to a large number of engineers, some of them among the most eminent in this particular branch of the profession, and it does seem remarkable that none of them should have recalled the invention of the apparatus by a Past-President of this Society, and that no suggestion of its previous use should have come to the writers until after the publication of this paper.

From the evidence to be gathered from such reports and descriptions, of the use of the oil differential gauge by others than the writers, as are available, it appears that the latter were the first and only ones to appreciate the fact that the coefficient of this instrument cannot be obtained from the specific gravities of the liquids used. As shown in Table No. 14, with kerosene and water, the multiplication of the scale reading is about 4% greater than is accounted for by the difference in specific gravities, and, in the case of sperm oil, where the specific gravities were determined, by means of a delicate chemical balance and a suspended weight, much more accurately than would be possible with an hydrometer, the increase in the factor of multiplication was as much as 10 to 12 per cent.

This increase of multiplication seems to be due to the adhesion of the oil to the glass, and, very probably, may vary with the size of the tube; but, as in the writers' experiments, the oils were calibrated in the respective gauges by comparison with a water column, it was not considered necessary to investigate the matter further.

The Pitot Tube.—The foregoing remarks apply also to the Pitot tube. No previous use of this instrument in closed pipes was known to the writers at the time of beginning these experiments. Their sole knowledge of the application of this apparatus was obtained from M. Bazin's memoir * on the "Liquid Vein," and the "Experiments Relating to Hydraulics of Fire Streams,"† by John R. Freeman, M. Am. Soc. C. E., although, before the investigation was completed, they learned of the work of Darcy, Darcy and Bazin, and Hiram F. Mills, and also of that of Professor S. W. Robinson‡ on the Sangamon River, in Illinois, as well as other occasional experiments more or less satisfactory; most or all of which last were made under such abnormal conditions of flow as to render them useless for general application.

* "Experiments upon the Contraction of the Liquid Vein Issuing from an Orifice, and upon the Distribution of the Velocities within it," by H. Bazin, translated by John C. Trautwine, Jr., Assoc. Am. Soc. C. E.

† *Transactions*, Am. Soc. C. E., vol. xxi, p. 303.

‡ *Van Nostrand's Engineering Magazine*, 1878.

Messrs. Williams, Hubbel and Fenkell.

The claims for accuracy and applicability of the Pitot tube have been opposed by Mr. Henry, Mr. Horton and Mr. Kuichling.

Mr. Henry objects to the rubber connecting hose used by the writers, and to the absence in their work of the cocks at the bottom of the old Darcy gauge. He also criticises the velocity curves, and claims, apparently, that better results could have been obtained by the use of the current meter. If the last be the case, the experience of the writers with that instrument for some years has been misleading, and the plottings of Mr. Henry's observations in the Chicago Tunnel, the data of which have been kindly furnished by him, fail to show more consistent or symmetrical results than do the tube observations of the writers at considerably lower velocities. But, even granting Mr. Henry's claim for his instrument, the problem of its insertion in the pipe line remains to be solved.

Regarding the writers' traverses at low velocities, all of which show marked irregularities, it should be stated that for reasons contingent upon the conditions at the pumping station, the high velocities were run in the early morning and the low ones in the evening and late morning, so that the latter were always affected by a much greater consumption of water in territory beyond the experimental line than were the former; and, at the low velocities, the effect of even the same consumption would have been much greater.

The use of rubber hose by M. Bazin seems to relieve the writers of Mr. Henry's criticism for not having used the more cumbersome lead pipes which Darcy adopted before the days of three-ply cotton-insertion tubing; and the writers believe the use of shut-off cocks on vibrating columns to be likely to introduce considerable error. It is interesting to note, here, that the vibrations in the gauges were not in opposite phases in the two tubes, but, ordinarily, the upward vibrations took place simultaneously in both columns, and were probably due to the effect of the pumps. In Table No. 75, Messrs. Saph and Schoder present a series of comparisons between the discharge obtained by the Pitot tube and that obtained by weighing the water delivered under various conditions. This table affords a criterion of the accuracy of the instrument as a meter, which the writers stated to be as great as that of any other device, except the graduated tank and the weighing scale. In this table, for conditions of normal flow at velocities above 1.17 ft. per second, which seems to be the limit of accuracy of the water-column gauge used by the observers, the minimum differences of the columns being then about $\frac{1}{10}$ in., there are presented thirty-six traverses. The data of these are abstracted in Table No. 83, with those from twenty-one experiments with a 2-in. Venturi meter, made by the same observers, at the Hydraulic Laboratory of the College of Civil Engineering, Cornell University.

Messrs. Williams, Hubbell and Fenkell.

TABLE No. 83.—Comparison of Errors of Pitot Tubes and Venturi Meter.

Instrument.	Number of experiments.	Range of: True mean velocity, U, feet per second.	Range of: Coefficient: $U \div V_m$.	Mean coefficient: $U \div V_m$.	Percentage of variation from mean $\frac{U}{V_m}$.
(1)	(2)	(3)	(4)	(5)	(6)
Tube E, point and ring.....	9	1.172 to 5.053	0.989 to 1.033	1.015	—2.6 to +1.8
" Fr " " "	3	2.699 " 4.220	1.004 " 1.007	1.011	—0.6 " +0.6
" Gr " " "	6	2.618 " 4.922	0.973 " 0.993	0.984	—1.0 " +0.9
" F " " " pressure.....	5	2.699 " 5.031	0.847 " 0.859	0.854	—0.8 " +0.6
" G " " "	13	2.618 " 4.922	0.845 " 0.869	0.858	—1.5 " +1.8
All tube experiments.....	36	1.172 " 5.053	.....	.....	—2.6 " +1.8
2-in. Venturi meter, near curve.	14	0.400 " 4.871	0.942 " 0.985	0.967	—3.6 " +1.8
2-in. " normal flow.	7	0.400 " 4.721	0.932 " 0.974	0.960	—2.9 " +1.5
All meter experiments.....	21	0.400 " 4.871	0.932 " 0.985	0.965	—3.4 " +2.1

In these experiments with the Venturi meter the readings were taken with a mercury differential gauge, the smallest reading of scale differences being about 0.06 in., which, owing to the clearer meniscus with mercury, would be read with about the same accuracy as 0.10 in. in water, riders and markers being used in both cases, so that the foregoing comparison seems to be a fair one. The Venturi was experimented upon in two positions, one with the inlet 0.756 ft. from the outlet of Curve No. 1, Fig. 119, and the other 36.725 ft. down stream from the P. T. of the same curve.

It will be seen that changing the position of the instrument changed its coefficient about 1% all through its range, due apparently to the different distributions of velocities in the preceding pipe.

The weir, the current meter, the water meter, and the nozzle are the only other devices to be compared with the Pitot tube, and the results obtained with the first three are rarely if ever better than those presented herein, and two of them are not directly applicable to the flow of water in pipes. The nozzle may perhaps give somewhat better results, but is limited to comparatively small streams. It seems, therefore, that the writers' conclusion, that the Pitot tube, properly rated, and in the hands of a skilled observer, is as accurate as any other device for measuring water in a closed conduit, except the scale and tank, is fairly sustained.

The simplicity, portability, low cost of construction, and ease of application should make this apparatus a highly valuable instrument for occasional observations, either in open or closed channels, and it may also be found useful in establishing the proper coefficient to use with Venturi meters which have been set so as to receive streams

having velocities which are distributed abnormally, and others to which the ordinary coefficients may not apply. Messrs. Williams, Hubbell and Fenkell.

The insignificant effect of small errors in pointing, as will be shown, makes extreme care in setting the instrument in large-pipe work unnecessary; and, in the Detroit experiments, it was found possible, when traversing, to occupy one position a minute, so that, counting three observations on center, a 30-in. traverse was made in 23 minutes. Comparisons of such traverses with those in which 3-minute observations were taken on each point show the former to be nearly as reliable; as, for instance, Traverse No. 7, in Table No. 8, made by the latter method, gave a mean velocity of 2.808 ft., and Traverse No. 21, made by the former method, under the same conditions, one week later, gave a mean velocity of 2.806 ft. per second.

Result *E*.

Rating Methods.—Under the headings *C*, *E*, *F*, *G* and *H* there were set forth certain conclusions quite intimately related, and referring in general to those portions of the paper dealing with the measurement of velocity. Of these, Conclusions *E* and *F* are very closely connected, and will be considered first. Against the former, that ratings of the Pitot tube by dragging it through still water do not coincide with those obtained when the instrument is stationary and the water flows past it, criticisms have been raised as follows:

By Mr. Murphy: That the wide variation from the mean of the coefficients of individual observations in the open-trough ratings leads to the supposition that either the velocity was not uniform, or that a motion was imparted to the water in the trough, and that, by reason of more consistent results obtained in ratings of other instruments in open channels over a straight course than are shown in the pipe ratings of the tubes, the open-channel method of rating seems preferable.

By Mr. Henry: That the angle of obliquity of impact caused low readings in the open-trough ratings.

By Mr. Frizell: Who suggests that possibly the effect of centrifugal force was overlooked in the trough rating.

The extension of this Conclusion *E* to current meters has been suggested by Mr. Wisner, partially endorsed by Mr. Horton, and contested by Mr. Haskell.

Mr. White has presented valuable data, from which he concludes that neither this nor the following Conclusion *F*, that all tubes must be rated, applies except to certain forms which he considers unsuited for general use.

From the data contained in these several discussions and the original paper, Table No. 84 has been compiled.

Messrs. Williams, Hubbell and Fenkell.

TABLE No. 84.—Comparison of Still and Running-Water Ratings.

Instrument. (1)	Authority. (2)	Method of rating. (3)	Coefficients: Still. A. (4)	Running. B. (5)	Percentage of excess: $\frac{A-B}{B}$ (6)
Tube No. 5...	Writers.	Still water, open circumferential trough	0.859	.....	
" "...	"	Flowing water, in 16 and 30-in. pipe	.....	0.75	+14.5
" 6...	"	Still water, open circumferential trough	0.950	.....	
" "...	"	Flowing water, in 30-in. pipe	.....	0.84	+13.1
" 3...	"	Still water, open circumferential trough	0.926	.....	
" "...	"	Flowing water, in 2, 5 and 12-in. pipe	.....	0.89	+ 4.0
" A.....	"	Flowing water, in 5 and 30-in. pipe	.....	0.80	
" ".....	Haskell.	Still water; straight course, with boat	0.831	.....	+ 3.9
Darcy tube.	Darcy and Bazin.	Still water; straight course, with boat	1.034	.....	
"	" "	Open channel, flowing water, by floats	.....	1.008	
"	" "	Open channel, flowing water, by weir	.....	0.996	+ 3.4
Haskell meter....	Haskell.	Still water; straight course, with boat	.....	.....	
Haskell meter....	"	Open channel; flowing water, by float	.....	.....	+0.35
Price meter..	Murphy.	Still water; straight course, with car	.....	.....	
" ..	"	Open channel; flowing water; by weir	.....	.....	+1.47

From this it appears that in every case, both for tubes and current meters, the rating in still water gave the higher coefficient, however the respective ratings were made. This being the case, it seems to the writers that such differences are not accidental, and, in most cases, they are too great to be overlooked.

Effect of Obliquity.—Considering now the effect of obliquity of the instrument, which it appears was the explanation suggested by MM. Darcy and Bazin for the phenomenal results obtained in their ratings, the writers have made, since the publication of the original paper, a series of experiments by revolving the point of a very small tube about the stem, at the center of a 2-in. pipe in which a uniform flow was maintained when readings of the point, pressure, and ring columns of the gauge, were taken with the point set at different angles. From this it appears that a point having an exterior which is a surface of revolution, and having inner walls which are cylindrical, or an orifice at the center of a flat plate, transmits the component of velocity

normal to the plane of the opening as velocity head, or that the velocity thus indicated varies as the cosine of the angle of obliquity. Messrs. Williams, Hubbell and Fenkell.

Applying this result to the obliquity of impact in the case of the writers' open-trough ratings, it appears that, allowing for both point and pressure effect, the coefficients would be reduced, in the worst case, that of Tube No. 6, less than $\frac{1}{1000}$ of 1%, and the probability of this cause accounting for the difference of coefficient observed in the Darcy and Bazin experiments seems to the writers quite doubtful.

Tank Ratings.—In the tank ratings of the writers, as both point and pressure openings were at the same distance from the axis of revolution and were connected to the gauge by tubing of the same size and length, it appears that no correction for centrifugal force at the tube is required, as both sides of the gauge would be affected alike. A small correction, amounting to about $\frac{3}{10}$ of 1%, however, should be made for the different centrifugal effects upon the two columns of the gauge, owing to its position, the correction decreasing the still-water coefficients.

With reference to the variations of coefficient on the open-trough ratings, criticised by Mr. Murphy, it may be added that these experiments of the writers were intended to be only preliminary, as an apparatus to provide a more uniform motion by the aid of a falling weight was planned, but, these preliminary observations proving the methods of rating to be unsuited to the purpose of the work, the experiments were then discontinued. Such errors as do exist are believed to be mainly in the time, and, as this was read continuously, an error in one observation would affect the next also, but in the opposite way, and thus the averages would probably be reliable though the single observations were not.

Mr. Horton has suggested that the coefficients of meters rated on a rigid support and on a rope do not coincide, and this, as also the fact that ratings on either support, with uniform motion and with variable motion, give different results, is proven by experiments by one of the writers, wherein both a Price and a Haskell meter were rated in all four ways, and all records, time, distance in 10-ft. lengths, and revolutions, were taken by chronograph, thus eliminating, as far as possible, chances of error.

The writers, therefore, believe that Conclusion *E* is sustained, and that an instrument should be rated under conditions, as nearly as may be, similar to those under which it is to be used, and that dragging an instrument through still water does not conform with this condition when the apparatus is to be used to measure running water, either in closed or open channels.*

* In the light of present knowledge, if instruments can only be rated by dragging, the writers would prefer the coefficients established by dragging in opposite directions in running water to those obtained by dragging in still water.

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RESULT *F*.

Necessity of Rating.—In view of the data presented by Mr. White, and of the experiments of Messrs. Saph and Schoder, and Adams and Wilson, wherein instruments of certain forms were used with ring piezometers, as well as the experiments by the writers regarding obliquity, just referred to, it seems to be demonstrated satisfactorily that Conclusion *F* was too sweeping, and the writers agree that, as pointed out by Mr. Ferris, a proper form of point opening will have a coefficient of unity by the formula $h = \frac{V^2}{2g}$ when combined with a ring piezometer under normal or nearly normal flow. This form, apparently, may be any one which presents an orifice in the center of a small plane surface, which surface (and orifice) is normal to the direction of flow, and this, perhaps, may be extended to orifices in large planes. That this applies also to converging nozzles receiving the impact at their larger ends, the writers are not so well satisfied, for it seems to them that there may reasonably be a difference between the conditions of such a point, set in a jet where the water is free to be thrown off at all sides, or in an open channel where the free surface allows certain motions to take place, and those conditions existing when the apparatus is in a closed pipe under pressure.

Were it not that in this paper the writers have endeavored to confine themselves to the relation of fact, rather than the promulgation of theory, they might be tempted to offer some explanations on this point, but as such explanations would be only theories, not facts, it is considered unwise to present them here. It may be said, however, that their experiments with flat plates in closed pipes confirm, under such conditions, the experiments and conclusions of Mr. White, presented in Figs. 113, 114 and 115, so far as the character of the results is concerned.

RESULT *C*.

Ratio of Mean to Center Velocity.—Conclusions *C*, *G* and *H* are intimately related to the laws governing the internal motions of flowing water, laws which, unfortunately, are far from being thoroughly understood by the present generation, and an analysis of the experimental data bearing thereon leads to some seeming anomalies.

In Conclusion *C* it is stated that where observations were taken at the center of a circular pipe with a properly rated tube, if the flow were proceeding under normal conditions at ordinary velocities, the mean velocity would be obtained within 3% of correctness by multiplying the central velocity, deduced by the formula $V = C\sqrt{2gh}$, by 0.84.

Mr. Mills has shown (page 204) from his own experiments, which the writers believe to rank among the most reliable ever conducted,

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that the ratio of mean to maximum velocity varies somewhat with the velocity, the range given being from 0.8293 to 0.8589 for velocities from 1 to 12 ft. per second in 12-in. pipe. The mean of this range is 0.8441, and it is seen that the maximum variation from the value 0.84 is 1.3% below and 2.3% above, thus falling almost within half the limit given by the writers.

Mr. Mills also gives the mean ratios for conduits ranging in diameter from 5 to 8.75 ft., and of various degrees of roughness, for velocities falling between 2.25 and 3.65 ft. per second. These values range from 0.809 to 0.861, the mean being 0.835. The variation from 0.84 is 3.7% below in the case of a wrought-iron conduit with plates laid in and out, rivet heads projecting, "and much of the surface covered with tubercles," and a maximum of 2.5% above in the remainder of the series. Omitting the tuberculated conduit, the variation is from 0.8 of 1% to 2.5% above the value, 0.84.

Mr. Cole presents (page 283) data upon this ratio from his experiments, which he interprets as being at variance with the writers' conclusion, but, as is shown by Messrs. Adams and Wilson (page 332), and by the published traverses themselves, in all cases but one the distribution of velocities was abnormal, and hence the deduced ratio does not fall within the limits set by the writers, and in the remaining case, that of the 16-in. pipe, the mean value is 0.844, the range being from 0.83 to 0.86, in which the variations are 1.2% below and 2.4% above 0.84.

The discussion of M. Bazin presents further experimental data bearing on this point (pages 256 and 257). As is shown by Messrs. Adams and Wilson, the value of the ratio, $\frac{V_m}{V_c}$, for the 31½-in. conduit of cement, at the two sections B and C, where the distribution is practically normal, is 0 856; the range being from 0.847 to 0.862, or from 0.8 of 1% to 2.6% above the value of the writers.

In the Darcy experiments, presented by M Bazin (page 262), the values of this ratio are 0.817, 0.814, 0.908 and 0.826, all but the third being within 3.1% of the writers' value, and the third is questioned by M. Bazin.

In the experiment by John R. Freeman, M. Am. Soc. C. E., quoted by M. Bazin, page 264, the ratio is 0.835, or 0.6 of 1% below the writers' value.

Finally, in the experiments of MM. Darcy and Bazin,* are given the results of a series of observations, by Pitot tube, of the central velocity in two rectangular closed conduits under low pressure. The conduits were 31.49 ins. wide and 19.68 ins. deep, and 18.90 ins. wide and 11.81 ins. deep, and the mean values of the ratios, for velocities ranging from about 1 ft. to 6.5 ft. per second, are, respectively, 0.848 and 0.808, a range from 1% above to 3.8% below the writers' value.

* "Recherches Hydrauliques," pp. 168 and 170.

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So far as these data go, Conclusion *C* seems to be amply sustained and to be fairly applicable to the extreme cases of the tuberculated pipes and rectangular conduits, but Messrs. Adams and Wilson have presented some experimental results, apparently proving that in small smooth pipes, *i. e.*, 2-in. seamless brass tubing, this ratio is more nearly 0.80, and they advance the theory that it is related to the diameter of the pipe. They show in Table No. 79 (Traverses Nos. 23 to 37, inclusive) fifteen traverses made in such pipe under normal conditions, wherein the ratio, $\frac{V_m}{V_c}$, ranges from 0.776 to 0.832, with a mean value of 0.798. These traverses were made with an instrument, Tube *E*, Plate IX, Fig. 2, having only a point opening, the other reading being taken from the ring piezometer. In Table No. 75, Messrs. Saph and Schoder present six traverses, Nos. 64, 66, 65, 9, 10 and 11, made under similar conditions, with the same tube, wherein this ratio ranges from 0.821 to 0.836, with a mean of 0.825, and they also give eleven traverses similarly made with Tubes *F* and *G*, wherein the point is a surface of revolution, as specified by Mr. White. These are marked *Fr* and *Gr* in Column 1 of Table No. 75, and show this ratio to range from 0.795 to 0.833, with a mean value of 0.820. Regarding this difference of values obtained by different observers with the same instrument, it is to be explained that Messrs. Adams and Wilson divided the cross-section of the pipe into concentric rings of equal area, as did the writers, while Messrs. Saph and Schoder, who were more interested in the form of the velocity curve, divided it into concentric rings of equal width. In computing the discharge from these observations, where the center velocity is used but once, it will be easily seen that on account of the longer chords between some points of observation, the former will give a lower value for the mean velocity than the latter, wherein, because of the shorter chords, only a very small correction is required to provide for the areas outside of the polygon of velocities. This explanation is borne out by the fact that, whereas the coefficient of Tube *E*, as obtained by the method of the former observers, is 1.039, by that of the latter it is 1.017, whence the value of V_m, and, consequently, its ratio to V_c should be increased by about 2% to reduce the work of the former to the same basis as that of the latter, which is more nearly that of the writers; this would make the value of the ratio $\frac{V_m}{V_c}$, by the former observers, 0.815. Messrs. Saph and Schoder have also presented, in Table No. 75, the record of the same eleven traverses with Tubes *F* and *G*, where the velocity is computed from the point and pressure readings, and these show the ratio to range from 0.788 to 0.831, the mean being 0.817. Considering also the values of the ratio in the 7½-in. pipe of Darcy (page 262) the smaller rectangular conduit of Darcy and Bazin, and

the 1⅛-in. pipe of Freeman, it seems that the contention of Messrs. Adams and Wilson is borne out, and that the ratio is less in small than in large pipes, but, nevertheless, the values derived when V_m is obtained by the method used by the writers fall within the limits set by them, even in these extreme cases. Messrs. Williams, Hubbell and Fenkell.

In establishing an accurate value of this ratio, however, there is still another point to be considered. The work of Messrs. Adams and Wilson (page 327) shows that when water passes from a larger to a smaller pipe the arrangement of velocities is disturbed, and, while the distribution remains symmetrical with respect to the axis of the pipe, the side velocities are increased and the ratio $\frac{V_m}{V_c}$ consequently increased. This is confirmed by the traverses of M. Bazin at Shaft *A* (page 256), and by those of John R. Freeman,* M. Am. Soc. C. E., wherein a ratio of 0.970 was obtained at the outlet of a conical nozzle. Messrs. Saph and Schoder show, in Fig. 125, and in Column 7 of Table No. 75, that vertical traverses, through a pipe in which the water has recently been deflected horizontally by a curve, show similar characteristics, to the extent of a high value of $\frac{V_m}{V_c}$. So far as the writers are aware, there is no information relating to the distribution of velocities where the stream has flowed from a smaller into a larger pipe, except in the case of a Venturi meter, where the law of the contraction holds; but such cases are comparatively rare in practice. The disturbances ordinarily occurring are produced by contractions, by curves, or by openings in the pipe wall, the effects of which latter the writers (see page 67) have found to be similar in nature to those due to curves. It is evident, therefore, that if the ratio, $\frac{V_m}{V_c}$, is abnormal from any of these causes in a symmetrical traverse, its value will be high, and it follows that the normal value of it is to be looked for among the lower rather than the higher results. Thus, considering all the data, the writers believe the value, 0.84, to be sustained as an approximation within the established limits, but would add that for absolutely normal flow they would expect it to fall below this figure, even as low as 0.81, and, in cases of symmetrical distortion, to have any value from this to unity. The values to be expected in cases of unsymmetrical distortion are shown in Table No. 75 for locations near the curves. It also appears, from Mr. Mills' experiments and those of Darcy, that roughness of the bounding surface decreases this ratio, and in this may lie the cause of small pipes showing lower values than larger ones, as the roughness of a small smooth pipe, relatively to its diameter, may be greater than that of a large rough pipe.

Transactions, Am. Soc. C. E., Vol. xxi, pp. 416 and 417.

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RESULT *H*.

The Velocity Curve.—In Conclusion *H* was presented the derivation of the ellipse as the approximate form of the normal curve of velocities in straight circular pipes. This conclusion is strongly supported by the analysis presented in the discussion by M. Bazin, and Mr. Tutton has deduced, from certain assumptions involving the molecular motions of the water, an equation leading to the ellipse as this curve, but, unfortunately, based upon a flow in straight-line filaments.

The conclusion of the writers was based upon the ratio, $\frac{V_m}{V_c}$, as established above, and the additional observed facts that the minimum velocity was about half of the maximum, and that when plotted the curve conformed closely to the ellipse, wherein, as pointed out, with the minimum velocity equal to half of the maximum, this ratio would be $0.83\frac{1}{3}$. However, as the value of this ratio may be actually between 0.81 and 0.82, a dome generated by revolving half a cycloid about its directrix, seems to fit the case fully as well, and, for the above conditions, gives $\frac{V_m}{V_c} = 0.8125$. The cycloid, moreover, can be accounted for without involving the assumption of rectilinear flow or the coefficient of viscosity. It seems quite probable that, whatever curve exists in pipe, it is the same as that in open channels under normal flow—a rare condition in the latter—and since the former is very evidently not a parabola, it seems doubtful that the latter can be. To those interested in the flow in open channels, the studies of the effect of disturbances in pipes, contained in the last two discussions, together with M. Bazin's presentation of the effect of the free surface upon the velocity curve, should prove highly valuable.

RESULT *G*.

Pressure Variation.—Conclusion *G*, that the pressure in straight pipe is not always the same at all points around the circumference of the same cross-section, is based first upon the experimental evidence in the 16-in. pipe investigation, discussed on page 158 and shown in Fig. 77. No discussion has been submitted containing direct additional evidence upon this particular point. All the Pitot-tube work shows that the components of velocity parallel to the axis of the pipe vary from point to point throughout the cross-section, and that changes of that component of velocity take place at each individual point under distorting influences. If these components represent, or are proportional to, the total velocity in each part of the fluid, it follows from the Law of Conservation of Energy, since all the water in the cross-section was originally started on its course under the influence

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of the same head, that if velocity and consequently velocity head be increased, pressure must be decreased, unless the elevation of the stream above datum be decreased, whether a single particle or the whole stream be considered, and, therefore, that the pressure must vary throughout the cross-section, a conclusion which is confirmed by the most obvious interpretation of a large number of experiments. The experiments of Messrs. Adams and Wilson, and Saph and Schoder, as well as some of those of the writers, prove pretty conclusively that a properly formed point combined with a ring piezometer gives readings the summation of the deduced velocities of which amount to the true discharge, on the theory used by the writers that $h = \frac{V^2}{2g}$, without introducing a coefficient, as has been demonstrated in the case of a jet and in open channels by Mr. White.* This being true, the acceptance of varying pressure throughout the cross-section, which pressure varies inversely as the velocity head leads to some complications, if one undertakes to maintain that the point effect is $\frac{V^2}{2g}$. Another interesting circumstance is the reading given by two conical points, one receiving the impact of a current in a closed pipe and the other directed down stream. Theoretically, it might be expected that the difference between their readings would be $\frac{2}{2}\frac{V^2}{g}$, and this alleged fact is the basis of the claim for a patent granted for such a device some years ago, but, experimentally or practically, nothing of the kind happens with this form of apparatus.

Regarding the question of the proper formula for the Pitot tube, the writers are not yet entirely satisfied, and while fully appreciating Mr. Frizell's efforts to clear the matter up (page 242), they yet prefer to leave the question where they left it before, that, practically, it makes little difference in the reduction of results, whether a tube reading actually is $\frac{V^2}{2g}$ or $\frac{V^2}{g}$. The discussion presented by Mr. Seddon gives excellent reason to believe that there is a large account of internal forces to be balanced before the laws of flow will be fully understood, and until those forces have been more fully investigated, too rigid laws had best not be laid down.

Result *I.*

Extent of Effect of Disturbances.—All the facts brought out in the discussion bearing upon Conclusion *I* corroborate the experiments of the writers in demonstrating that the effect of a disturbance extends many diameters beyond the point of interference.

* *Journal of the Association of Engineering Societies*, August, 1901, page 57, *et seq.* See, also, in this connection, Encyc. Brit., IX Ed., article "Hydromechanics," pages 516 and 517.

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The actual distance required for an abnormal distribution of velocities to become again normal varies greatly with the nature and form of the obstruction which causes the distortion. Messrs. Adams and Wilson show, by their own work on 2 and 5-in. pipe, and by M. Bazin's experiments on 31½-in. pipe, that the effect of a contraction, either cycloidal or with square, sharp edges, extends only about 35 diameters down stream, and at this distance is independent of the velocity. Traverses *A*, *B*, *C* and *D* (Table No. 8) were made by the writers in a 12-in. pipe, 30 diameters down stream from a 16 × 12-in. reducer (about 2 ft. long), and the ratio of mean to center velocity is approximately 0.80, indicating that the distribution of velocities is nearly if not quite normal.

Regarding the distortion produced by curves, the writers concluded that it disappeared in less than 295 diameters in 30-in. pipe, and Messrs. Saph and Schoder, in their discussion (pages 302 and 317), show that such distortion extends for about 180 diameters in 2-in. pipe. Reasoning by analogy, from the apparent non-effect of diameter of pipe in the case of symmetrical distortion, it appears that this length would be substantially the same in large as well as in small pipes.

Result *K*.

Effect of Curvature.—The final Conclusion *K*, may be re-stated in the following more explicit language:

In a given length of pipe consisting of two tangents joined by a curve of 90°, the loss of head will decrease as the radius of the curve is decreased, to a limit of about 2½ diameters, and will increase as the radius is increased above that limit, the total length remaining the same.

It may be extended as follows, for the range of these experiments, at least:

To unite two points on two tangents intersecting at 90°, and equally distant from their intersection, by a pipe line consisting of portions of the two tangents and a curve of 90°, the line of least hydraulic resistance will be one in which a curve of about 2½ diameters radius is used. The latter is the practical problem, and from it, it appears at once that the saving of head due to the shorter lines when a long-radius curve is used is more than compensated by the increased resistance, due to the longer curve. This conclusion is based upon experiments upon 30-in. and 12-in. pipe, with curves of radius varying from 1.08 to 24 diameters. Against this conclusion, the following criticisms have been presented:

By Mr. Henry: That it is foolish and illogical and contradictory to preceding experiment.

By Professor Church: That the results are modified by variations of diameter at the piezometer openings whence the losses of head are obtained.

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By Mr. Sherman and Mr. Kuichling: That the effect of the number of joints has not been considered, and that the locations of the piezometric openings were such as to render them likely to gather air.

By Mr. Tutton: That the piezometric indications may be affected by the distortions of the internal velocities which may not have become readjusted to normal conditions at all the piezometric openings.

By Mr. Campbell and Mr. Gould: That the shorter length of the long curve line compensates in part for curve loss, and practically and economically may be the best line.

Regarding the criticism of foolishness and illogicalness offered by Mr. Henry, the discussions of Mr. Wisner, Mr. Campbell, Mr. Brinckerhoff and M. Bazin, are, it seems to the writers, sufficient refutation. As to the contradictions in former experiments, some comparisons will be presented later.

Diameters.—In the matter of the diameters at piezometric orifices, the writers regret that at the time the line was laid, the importance of this subject was not appreciated, for it would not have been difficult, in the case of the 30 in. pipe, to have obtained actual measurements, though it could not have been so easily accomplished with the

TABLE No. 85.—CALIPERINGS OF 30-INCH PIPES.

	Pipe No. 1.		Pipe No. 2.	
Position.	Calipering.	Mean.	Calipering.	Mean.
Hub	$30\frac{1}{16} \times 30$	30.031	$29\frac{3}{4} \times 29\frac{3}{4}$	29.687
1 ft. from hub	$30 \times 29\frac{13}{16}$	29.906	$29\frac{1}{2} \times 29\frac{15}{16}$	29.531
2 " " "	$30 \times 29\frac{15}{16}$	29.969	$29\frac{7}{8} \times 29\frac{5}{8}$	29.563
3 " " "	$30 \times 29\frac{7}{8}$	29.938	$29\frac{15}{16} \times 29\frac{5}{8}$	29.594
4 " " "	$30\frac{1}{8} \times 30$	30.062	$29\frac{15}{16} \times 29\frac{15}{16}$	29.562
5 " " "	$30\frac{1}{16} \times 30\frac{1}{16}$	30.094	$29\frac{1}{2} \times 29\frac{11}{16}$	29.594
6 " " "	$30\frac{1}{16} \times 30\frac{1}{16}$	30.062	$29\frac{5}{8} \times 29\frac{3}{4}$	29.687
7 " " "	$30\frac{1}{8} \times 30$	30.062	$29\frac{11}{16} \times 29\frac{5}{8}$	29.656
8 " " "	$30\frac{1}{16} \times 30$	30.031	$29\frac{3}{4} \times 29\frac{3}{4}$	29.750
9 " " "	30×30	30.000	$29\frac{5}{8} \times 29\frac{3}{4}$	29.687
10 " " "	30×30	30.000	$29\frac{5}{8} \times 29\frac{3}{4}$	29.687
11 " " "	30×30	30.000	$29\frac{11}{16} \times 29\frac{11}{16}$	29.687
Spigot	30×30	30.000	$29\frac{3}{4} \times 29\frac{3}{4}$	29.750
Mean of all measurements		30.0119		29.6489
" " eight end measurements		29.9843		29.6638
Maximum mean diameter		30.094		29.750
Minimum " "		29.906		29.531

16 and 12-in. pipes. In Tables Nos. 51, 52 and 53 accompanying the discussion by Professor Church there are presented computations of the percentage errors in the results for the several

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experimental sections, based upon the assumption that the diameter at the piezometric orifice is the mean of the eight diameters measured in the length of pipe where it is situated, from which it appears that for the curve sections no error so great as 1% is introduced, and, as the extreme limits of diameters that are given were in many cases in the specials themselves, while the straight lengths conformed more closely to the mean, the writers believe the results in these tables to be a close approximation to the truth. As to how closely the mean of the eight measurements may be expected to coincide with the mean diameter, determined by caliperings at right angles of thirteen sections, 1 ft. apart, in 30-in. pipe, the writers present the data in Table No. 85, obtained from two pipes left over from the 30-in. line, they being all that remained of that lot when the point was raised.

Piezometric Opening.—In reference to the location of the piezometric opening, whether it be a single point at the zenith of the section or elsewhere, or be four or more openings symmetrically distributed about the circumference, the writers have no doubt that the last is the preferable form for straight-pipe work, but it is also considerably more expensive than the single opening, and at the time this work was planned, the writers never having had the benefit of the direction of anyone more experienced in this line of experiment than themselves, did not foresee all the conditions that were to be encountered, and therefore adopted what then seemed to be both an economical and a satisfactory construction. It was not until near the completion of the 30-in. experiments of 1898 that the observations upon the 16-in. pipe, presented in Fig. 77, proved conclusively that in straight pipe a considerable difference of pressure might exist at the two extremities of the same horizontal diameter. That a difference of pressure amounting to 0.025 in. of water would exist at the two sides of a 16-in. pipe which was straight for 213 ft. preceding, when there was no following contraction of the cross-section, was certainly unexpected. As soon as the distortion of the velocity distribution, caused by curvature, was discovered, which, fortunately, was on the very first day of the 30-in. experiments, it was realized that this distortion would very probably be accompanied by a change of pressure conditions throughout the cross-section. The preparations for the experiments did not include provisions whereby the extent to which these distortions were carried beyond the curves could be determined, but it was evident that certain piezometric openings were within the range of the disturbance, though it was also surmised that, since the disturbance of the velocity was probably greatest along the horizontal diameter, it would be least along the vertical, and, therefore, the pressure in the layer of water next to the wall at the extremities of the vertical diameter would be affected very little, if at all, by the

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disturbances caused by the curves, which were in every case horizontal. That this last reasoning was not entirely correct appears from the work of Messrs. Saph and Schoder, who find a considerable acceleration of the velocities near the walls in a plane normal to the direction of curvature, but, from the same experiments, it seems doubtful whether the location of the opening proposed by Mr. Sherman, 45° from the vertical, would have given more reliable results than the one at the zenith used by the writers; and, further, it seems even doubtful if much better results could have been obtained from four orifices, however they might have been located. The general principle underlying the use of piezometric orifices, in a stationary pipe or channel wall, is that only readings of such orifices can be compared directly for loss of head when they are so situated that the absolute velocity of the particles of water in the film adjacent to them is the same at all the points which are thus compared. The criticism of Professor Church was based upon this principle, and the same or similar inaccuracies may be introduced by irregularities of the distribution of the velocities as are due to actual changes of mean velocity caused by variations of diameter.

Air.—The presence of air in the gauge connections caused more or less trouble throughout the work, but great care was taken to detect its presence and remove it. By always having at least three instruments the indications of which were related by known laws, and by making comparisons at each change of head, it was usually possible to detect and locate errors promptly, and the connections were at once blown off and shaken out whenever the pressure of air was indicated. The log of the experiments of August 13th and 14th, 1898, presented on Plate VI, shows instances when air interfered with the observations and was removed by shaking out.

Joints.—When it is attempted to consider the effect of the number of joints in the several curves, the indications are complicated by the presence of gates and specials, and by the possible errors of indication of pressure due to distortion.

Column 9 of Table No. 21 shows the number of joints in each experimental section of the 30-in. pipe, from which it appears that, excepting the last curve, the number ranged from 21 to 27 in the 200-ft. curve sections, and it seems probable, therefore, that the relative change due to them would be small. Similar data for the 16 and 12-in. lines are given in Tables Nos. 29 and 33. In the 12-in. pipe line the two 3-ft. radius curve sections, the 2-ft. radius, and the 1.08-ft. radius curve sections each contained ten joints, and the results, with the single exception discussed in the original paper (page 182), checked the 30-in. work very well. While they would not argue that the number of joints has no effect on the loss of head, the writers do feel warranted in expressing the opinion that the effect of joints upon the relative values of the curve resistance is insignificant.

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Distortion Effects.—Returning again to the effect of distortion upon the piezometric indications, it is to be remarked that distortion seems to be produced most naturally by an increase of velocity in the particles of the stream, since they have to move transversely as well as longitudinally to produce it, the mean velocity in the direction of the axis of the pipe remaining constant, and hence, as absolute velocity must increase, pressure must decrease and an indication by a ring piezometer, at a point where distortion is changing, will be low. This is confirmed by the experimental data presented by Messrs. Saph and Schoder in Columns 7 and 12 of Table No. 78, wherein are given the observed excess losses of head over an equal length of straight pipe for the several sections, and it will be seen that the excess for the section *B J*, which includes all the others, is less than that for any of the preceding ones, therefore proving that the piezometers within the range of distortion were reading low by varying amounts. Commenting on these results, the observers expressed the opinion that a length of 200 diameters of pipe is necessary to eliminate the distortion produced by a curve, and they argue that therefore the writers' results are more or less in error. Turning to Figs. 124 and 125, it is seen that the distortion, both in the curve and in the following tangent, passes through different and quite distinguishable phases, the phases occurring in inverse order in the tangent from that in which they are produced in the curve. The conclusion of the observers evidently refers to the case of the phase or degree of distortion represented at the outlet of the curve, which, as it seems to be almost the same as at the boss 15° up stream, may be called complete distortion, and is somewhat of the nature of that shown in Fig. 11, but quite different from that of Fig. 12, which resembles much more closely the condition at Boss 2, near the inlet of the curve. Now, the condition of Boss 2, by the position of the point of maximum velocity, coincides with that at about Piezometer No. 6 on the tangent, which is 17 diameters from the P. T. of the curve. So it would be expected that if 200 diameters are required to eradicate the more complete distortion of Fig. 11, only 183 diameters, or 457 ft., would be required in the case of the distortion of Fig. 12, and as the former represents the distortion of a 40-ft. radius curve, and the latter of a 15-ft. radius curve, the distances required for the distortions produced by the small compound, the 10-ft. radius and the 6-ft. radius curves would probably be still less.

Distortion, and Reversing Curves.—The effect of reversed curves is also to be considered in dealing with the writers' piezometers. It is quite easy to conceive that if immediately after leaving the curve, with the distortion shown in Fig. 124, the stream had passed into a second exactly similar curve, of proper length, turning in the opposite direction, the water would have emerged from the latter without any distortion whatever, and experiments with the Venturi meter immediately

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beyond a curve showed the distortion due to curvature to be almost entirely eliminated after passing the meter. Referring now to the 30-in. line of the writers, as shown in Fig. 23, it is seen that there are three reversals of direction with quite short intervening tangents, these being made by the first six curves, and it seems quite safe to suppose that the water, on leaving the second of each pair of curves, is not distorted to such a degree as to require 200 diameters for what distortion is left to disappear. These three curves are followed by 129, 144 and 122 diameters of straight pipe, respectively, before the inlet piezometer of the following curve is reached, and, in view of the fact that in the last two cases the second curve is of somewhat less curvature than the first, the writers believe that the error of the piezometric indication due to distortion will be quite insignificant in these three cases, except perhaps in the first, though they admit that the so-called down-stream piezometers of the 200-ft. curve sections were in each case affected; but, for the shorter curves, the effect was probably less than for the longer ones. This at once assists in rectifying somewhat the plotting of Fig. 90.

Corrections to Fig. 90.—It is to be assumed that the inlet piezometer of the first large compound curve read correctly, but the outlet undoubtedly read low, so that the apparent loss in this curve was too great, and, consequently, Point 4 should be brought nearer to Point 3. The inlet of the second large compound curve read low, as did also its outlet, by a greater amount, so that Point 3 is somewhat too high. The inlet of the 60-ft. radius curve probably read very nearly right, but the outlet read relatively very low, so that Point 1 on the diagram should be lowered considerably. The inlet of the 40-ft. radius curve, being only 5 ft. from the P. T. of the 60-ft. radius curve, read much lower than the down-stream piezometer of this section, so that Point 2 should be raised. As the 40-ft. radius and long compound curves are of almost exactly the same length, while the former contains a gate, it follows that Point 2 should fall above Point 3. The inlet to the 25-ft. radius curve probably gave a correct indication, while the outlet was low, as was also the inlet of the small compound curve, by a greater amount than was its outlet, so that Point 5 should be lowered and Point 7 raised. The indications of the remaining 30-in. curves are probably high, so that Points 6*a*, 8 and 14 should be lowered slightly, the effect of all these changes being to drop the portion of the locus near the origin a little and to cause the right-hand extremity to run more nearly parallel with the axis of abscissas, all the points except No. 4 (which is affected by specials) now coinciding quite closely with the line. If, in this connection, the computations presented by Professor Church in Table No. 54 be worked out for the second large compound curve, using the value of loss of head originally given, the quantity in Column (f) of that table becomes 1.6433, which, as the curves are of the same length, may replace the value for the 40-ft.

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radius curve, and is seen to conform more closely to the other values than does that of 1.478 obtained from the latter, thus showing the 40-ft. radius curve resistance, as originally given, to be very probably low, and confirming the foregoing assumptions to that extent.

Applying similar reasoning to the 16-in. curve experiments, the piezometer at the inlet of the 5-ft. radius curve, which was preceded by 473 ft. of straight pipe without specials, undoubtedly gave a correct indication, while the outlet read low, thus giving this curve, like the first on the 30-in. line. a high apparent resistance. For the compound curve, the readings of the inlet and the outlet were probably about equally affected, being at nearly equal distances from the outlets of curves of nearly the same mean radii, so that the observed loss is probably nearly correct, but the inlet to the Army tangent reading low, while its outlet, being over 260 diameters from the curve, read correctly, makes the loss in this section, to which the curves are compared, too low, so that their excesses are high and Points 13 and 10 on Fig. 90 should be lowered, bringing them into closer coincidence with the 30-in. experiments. The specials in the Porter tangent, and the additional fact that its inlet was probably affected by a 16 x 12-in. Y, about 6 ft. up stream, renders it impossible to deduce from the 16-in. experiments any more concrete results.

On the 12-in. line, the chances seem to be, as claimed by Messrs. Saph and Schoder, that the piezometers at the outlets of all the curves gave low indications, and that those at the inlets of the 4-ft and 1.08-ft. radius curves also were affected, on account of the preceding gates, the effect of the latter being possibly to increase the readings on account of the retardation of the velocity at the top of the pipe. From this it would appear that the observed losses for all these curves are too high, and that the error may be greater in the cases of the 4-ft. and the 1.08-ft. radius curves, so that Points 9, 11, 12, 15 and 16 on Fig. 90 should be lowered.

Confirmation from other Experiments.—Regarding the locus presented by Mr. Horton in Fig. 112, the writers are unwilling to admit that portion of it beyond Point 14, as its passing through the origin involves the assumption that a right-angled deflection, as formed by two intersecting cylinders, would give no excess resistance over a straight pipe. On this point the experiments of Weisbach, all of which were made upon curves of less than 2 diameters radius, including the right-angled elbow, confirm the indications of the writers' experiments that from a radius of 0 diameter to one of about $2\frac{1}{2}$ diameters the resistance decreases as the radius increases. Weisbach's formula, as stated by Mr. Campbell, shows that the curve of increase of resistance becomes so flat after passing the 2-diameter point that little is to be gained by using a radius greater than 5 diameters, and the conclusion is that, for practical purposes, the line soon becomes asymp-

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totic to the axis of abscissas. Had Weisbach's experiments been extended to curves of greater radius, he probably would have found that instead of an asymptote he had a minimum with a branch curving upward. The form of the writers' curve for the higher radii confirms the result of the experiments upon 2½-in. hose, made in 1888 by John R. Freeman,* M. Am. Soc. C. E., and presented to this Society in 1889, wherein it was found that decreasing the radius of curvature from 14.4 diameters to 12 and then to 9.6 diameters caused the excess resistance over straight pipe to decrease continuously.

Recomputation of Curve Excesses.

From the foregoing, however, it appears that a further investigation of the writers' experiments is necessary to establish more reliably the quantitative results. For this it is possible to select, from the various points at which piezometer connections were located, some which are preceded by such considerable lengths of straight pipe as to render probable the existence there of nearly, if not quite, normal conditions of flow, whence their indications may be considered as reliable.

Conditions.—The connections so located are shown in Table No. 86.

TABLE No. 86.—Piezometers Under Normal Flow.

No.	Outlet of—	Inlet of—	Preceding Tangent.		Nearest Specials.		
			Feet.	Diameters.	Kind.	Distance up stream.	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	
						Feet.	Diameters.
1...	.....	Experimental line.	1 970+	788	None.	.....	
2...	Hale tangent....	60-ft. radius curve.	321.83	129	"		
3...	Rowena "	25-ft. " "	360.81	144	30-in. gate.	345	= 138
4...	Brady "	15-ft- " "	304.40	122	None.		
5...	S. Rivard "	N. Rivard tangent.	756.08	302	30 x 6-in. tee.	192.3	= 58
6...	N. Rivard "	10-ft. radius curve.	940.18	376	30 x 6-in. "	126.5	= 48
7...	E. Alexandrine tangent.....	Hastings special...	737.65	295	None.	.....	
8...	W. Alexandrine tangent.....	6-ft. radius curve..	1 170.16	468	30 x 10-in. Y.	410	= 164
9...	N. Alley tangent..	.....	880.19	352	30 x 6-in. cross.	377.9	= 150

Of these nine points, all but three seem to be beyond question, so far as effects of distortion are concerned. The three possibly doubtful ones are Nos. 2, 3 and 4, but any effect at these points must be very nearly the same in amount at all, so that, if present, the result

* *Transactions*, Am. Soc. C. E., Vol. xxi, page 362.

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will be high losses from 1 to 2 and low losses from 4 to 5, the losses from 2 to 3 and 3 to 4 being approximately correct, as the connections at each end would be similarly affected.

The eight divisions of the line made by these connections will be designated by the first eight letters of the alphabet. From them it is possible to write eight equations between the various factors causing loss of head, and the observed losses of head in the component experimental sections of each division, the latter being taken from the experiments of 1899, because only then were all the sections experimented upon

The factors entering these equations are:

1. The loss of head per foot in straight 30-in. pipe under normal flow.

2 to 10. The excess loss of head caused by nine curves.

11. " " " " " a 30-in. gate.
12. " " " " " a 30-in. Y.
13. " " " " " a 30 x 10-in. Y.
14. " " " " " a 30 x 8-in. cross.
15. " " " " " a 30 x 6-in. cross.
16. " " " " " a 30 x 6-in. tee.
17. " " " " " a joint.
18. " " " " " changes in roughness of the pipe wall.
19. " " " " " changes in diameter.

It thus appears that there are nineteen unknowns to be obtained from eight equations, and a rigid solution becomes impossible. As the variations in diameter and in roughness are comparatively small, these items may be rejected, and as the effect of the joints has already been examined and found negligible, it will also be dropped. It is at once evident that a 30 x 6-in. tee may fairly be assumed to introduce one-half the excess resistance introduced by a 30 x 6-in. cross, and as in these specials the 6-in. ones had a bell-mouth, while the 30 x 8-in. cross did not, it also seems fair to assume that the latter cross would cause very nearly the same excess resistance as one of the former. The 30 x 10-in. Y will also be assumed to cause the same excess resistance as the 30 x 6-in. cross. All these specials had the branch at the top or bottom, not at the center of the 30-in. pipe, and on this account the last assumption becomes tenable.

The division of the line into the eight sections is such as to include two curves in each of the first three, so that they can only be treated in pairs, and thus there remain six curve unknowns, the gate, the 30-in. Y, and the small specials, besides the loss of head in straight pipe, making ten unknowns to be derived from eight equations. Table No. 87 presents the data from which the equations may be written.

TABLE No. 87.—LENGTHS AND DISTURBANCES.

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Division. (1)	Sections. (2)	Length, in feet. (3)	Curves. (4)	Gates. G. (5)	30-in. Y. Y. (6)	Small specials. Z. (7)	Observed H_f in terms of observed losses in Brady tangent. (8)
A...	I, II, III, IV.....	653.59	2 long compounds. 2 p..	1	1	1	4.2308
B...	V (— VI), VII, VII..	644.35	60-ft. and 40-ft. radius. $r + s$.....	1	0	2	3.9486
C....	IX (— X), XI, XII...	593.08	25-ft. radius and small compound. $t + u$.	0	0	1	3.2889
D...	XIII, XIV.....	867.86	15-ft. radius. v.....	1	0	7	4.5292
E...	XV.....	184.10	.....	1	0	3	0.9660
F...	XVI, XVII.....	845.59	10-ft. radius. w.....	0	0	0	4.1474
G...	XVIII, XIX.....	432.51	.....	1	0	2	2.0993
H..	XX, XXI, XXII.....	984.90	6-ft. radius. x.....	1	0	2	4.8180

Equations.—Letting the loss of head per foot of 30-in. straight pipe under normal flow $= a$, and the observed loss of head in the Brady tangent $= b$, the following equations are written:

(a)..... $2p + G + Y + z = 4.2308\ b - 653.59\ a;$

(b)..... $(r + s) + G + 2\ z = 3.9486\ b - 644.35\ a;$

(c)..... $(t + u) + z = 3.2889\ b - 593.08\ a;$

(d)..... $v + G + 7\ z = 4.5292\ b - 867.86a;$

(e)..... $G + 3\ z = 0.9660\ b - 184.10a;$

(f) ..... $W = 4.1474\ b - 845.50a;$

(g)..... $G + 2\ z = 2.0993\ b - 432.51a;$

(h)..... $x + G + 2\ z = 4.8180\ b - 984.90a.$

Determination of $\frac{b}{a}$.—To simplify the second member, a relation must be established between a and b. For this there may be utilized the observed losses in eight tangents which contain no specials, but three of them are so short that they would be discarded for this purpose, even if they were not so located as to be very probably affected by preceding curves. The others are shown in Table No. 88.

Of these, the Brady and the East Alexandrine tangents seem most entitled to credit, and as the experiments show the curve preceding the former to cause more resistance than that preceding the latter, and as, moreover, the latter is longer and hence proportionally less affected by the resistance extending into it from the preceding curve, and the imperfect reading of its inlet piezometer (if that is imperfect), the relation between a and b will be established from the East Alexandrine tangent as a first approximation. Using the observed results from Table No. 28, this gives:

$$645.50\ a = 3.0678\ b;\ \text{whence}\ b = \frac{645.50\ a}{3.0678} = 210.41\ a.$$

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TABLE No. 88.—TANGENTS.

Name. (1)	Length, in feet. (2)	Inlet distant from curve. Feet. (3)	Inlet distant from curve. Diameters. (4)	Ratio of observed H_f per 200 ft. to observed H_f in Brady tangent. (5)
Hale Street	234.92	86.91	35	0.8611
Rowena Street	292.23	68.58	27	0.8598
Brady Street	212.25	92.25	37	0.9420
East Alexandrine Avenue	645.50	92.15	37	0.9504
West Alexandrine Avenue	409.00	737.65	295	0.9416
	(Inlet preceded by Hastings specials.			

Equations (e) and (g) furnish a ready opportunity of verifying the accuracy of this relation; whence is obtained

$$\text{(e)} \ldots . G + 3z = (203.26 - 184.10)\ a = 19.16\ a$$
$$\text{(g)} \ldots . G + 2z = (441.71 - 432.51)\ a = \ 9.20\ a$$
$$z = \qquad\qquad 9.96\ a$$

This value of z is evidently absurd, for it requires that G have a negative value, which seems highly improbable, as all the traverses taken near and down stream from gates show the flow to be retarded on the side next the bonnet, with considerable distortion of the velocities. To give G a positive value, a higher value for b in terms of a is required, or the ratio of the tangent resistance to the Brady tangent must be decreased. This confirms the earlier conclusion, that a portion of the resistance caused by the curves was carried on into the following tangent beyond the down-stream piezometer of the curve section. Such being the case with the Alexandrine tangent, which follows one of the two curves producing the least resistance, it is a fair conclusion that the other tangents will be affected even more, and hence there is no reason to select any particular one of the relations given by the five tangents above, and it becomes necessary to establish the relation between a and b between limits as the only remaining method of procedure. Evidently, the least value which G may consistently have would be $G = z$, and the greatest would result when $z = 0$. From Equations (e) and (g) it is possible to deduce a value for $\frac{b}{a}$ on each of these assumptions, which will therefore be limiting values of this ratio. On the first assumption, that $G = z$

$$\text{(e)} \ldots . G + 3z = 0.9660\ b - 184.10\ a = 4\ G = 4\ z$$
$$\text{(g)} \ldots . G + 2z = 2.0993\ b - 432.51\ a = 3\ G = 3\ z$$

Whence $\frac{3}{4}\ (0.9660\ b - 184.10\ a) = 2.0993\ b - 432.51\ a,$

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or $0.7245\ b - 138.075\ a = 2.0993\ b - 432.51\ a$,

or $294.435\ a = 1.3748\ b$, and $\frac{b}{a} = \frac{294.435}{1.3748} = 214.16$;

and, on the second assumption, $z = 0$, there results, from (e) and (g), $2.0993\ b - 432.51\ a = 0.9660\ b - 184.10\ a$, or $1.1333\ b = 248.51\ a$, and $\frac{b}{a} = \frac{248.41}{1.1333} = 219.19$.

From the ratios $\frac{b}{a} = 214.16$ and 219.19, the ratios of the loss in 200 ft. of normal tangent to the observed loss in the Brady tangent become 0.9337 and 0.9124, respectively, which are the limiting values, and it is seen, by reference to Column 5 of Table No. 28, that none of the observed values of tangent resistance falls within these limits, some values being greater and some less. This may at first seem anomalous, but it must be remembered that the effect of distortion is to cause an increase of resistance, while the flow is readjusting, and that this readjusting, or change of condition, causes the piezometric indication to be low, as already described. It is quite possible, therefore, that the increased resistance may be so great as to cause the apparent resistance to show a part of the increase, even though the piezometric indication at the inlet is affected in opposition.

H_f per 1 000 Ft.—The loss per 1 000 ft. of 30-in. tangent, by comparison with the Brady tangent reductions given in Table No. 28, Columns 7 and 8, becomes, as a maximum:

$$\frac{0.9337}{0.9421} \times 0.1192\ V^2 = 0.1181\ V^2;$$

and as a minimum $\frac{0.9124}{0.9421} \times 0.1192\ V^2 = 0.1154\ V^2$.

This shows the variation between the extreme values of the tangent resistance to be slightly less than $2\frac{1}{2}\%$, or, if a mean value be taken, it cannot be more than $1\frac{1}{4}\%$ in error.

Taking as such a mean value of H_f per 1 000 ft. of straight 30-in. pipe, $0.1170\ V^2$, for which c, in the Chezy formula, is 116.94, the proper values of G and z are determined from Equations (e) and (g), as follows:

For H_f per 1 000 ft. $= .1170\ V^2$, $a = 0.46202$ Brady, or $\frac{b}{a} = 216.44$.

(e) $G + 3\ z = 0.9660\ b - 184.10\ a = (209.08 - 184.10)\ a = 24.98\ a$

(g) $G + 2\ z = 2.0992\ b - 432\ 51\ a = (454.37 - 432.51)\ a = 21.86\ a$

Subtracting $z =$ $3.12\ a$

Adding $2\ G + 5\ z =$ $46.84\ a$

$$G = \frac{46.84 - 15.60}{2}\ a = 15.62\ a.$$

Attention is to be called to the fact that these values of a, G and z, the losses in straight 30-in. pipe, in a 30-in. gate, and the small

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30-in. specials, are determined from sections wherein no piezometric error due to distortion is to be expected. No correction has been applied for possible difference of diameters at the piezometer points, because the best evidence at hand indicates that there was no difference in the case of the shorter length, the North Rivard tangent, 184.10 ft. long, and that in the other section made up of the Hastings specials and the West Alexandrine tangent, the length being 432.15 ft., the necessary correction for this effect, as may be deduced from Table No. 51, would be less than ½ of 1 per cent.

Excess Resistances.—Applying these values to Equations (a), (b), (c), (d), (f) and (h), the results in Table No. 89 are obtained:

TABLE No. 89.—EXCESS RESISTANCE IN 30-INCH CURVES AND SPECIALS.

Mean Value H_f per foot of Straight 30-In. Pipe, with Normal Flow $= 0.000117\ V^2$.

Disturbing cause. (1)	Total Excess Hf Measured in Feet of Straight Pipe. By last reduction. (2)	By Fig. 90. (3)	Apparent Error in Fig. 90. No Corrections for G or z. Feet. (4)	Percentage. (5)
30-in. gate	15.62	.....	.....	.....
30 x 10-in. Y, or 30 x 6-in. cross, or two 30 x 6-in. tees	3.12	.....	.....	.....
6-ft. radius = 2.4-diameter radius curve	36.06	31.40	−4.66	−14.9
10-ft. " = 5 " " "	52.16	34.60	−17.56	−50.8
15-ft. " = 6 " " "	74.99	83.00	+8.01	+9.6
25-ft. radius = 10-diameter radius curve with small compound = 5.7-radius curve, reversing and joined by 136.79 ft. of tangent	125.65	155.00	+29.35	+18.9
60-ft. radius = 24-diameter radius curve, with 40-ft. radius = 16-diameter radius curve, reversing and joined by 73.58 ft. of tangent	188.43	303.40	+114.97	+37.9
Two long compound = 16-diameter radius curves, reversing and joined by 193.59 ft. of tangent, and one 30 x 30-in. Y in first curve	243.39	382.60	+139.21	+36.4

As is shown in the last column of Table No. 89, while the qualitative results in the original paper are sustained, the quantitative results are altered materially, and Column 2 may be taken as giving the most reliable values of excess resistance obtainable from the experiments. The only source of error in the last computations seems to lie in the assumed relation between 30 x 6-in. crosses and 30 x 10-in. Y's, which error, if it exists, will produce but a small change in the final values, as the total amount of the loss z is only 3.12 ft., and if it be assumed

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that the Y causes twice the resistance due to the cross, the greatest change occurs when the value for the latter remains 3.12 ft., the Y becomes 6.24 ft., and the gate is reduced to 9.38 ft., an amount considerably below what judgment would ascribe.* Whatever errors may exist in the piezometric indications at Points 2, 3 and 4 can only affect the last four determinations of Table No. 89, and, as already said, probably do not affect materially those of either the 60-ft.-40-ft. radius, or the 25-ft. radius-small compound sections. These final results establish a much more rapid increase in the excess between $2\frac{1}{2}$ and 4 diameters than did the direct observations, which increase in the latter case was more than sufficient to counteract any gain due to the shorter lines connecting two points, as a simple computation will show.

To connect two points 100 ft. from the P. I. of tangents at right angles:

	$R = 6$ diameters.	$R = 4$ diameters.	$R = 2.4$ diameters.
Tangent length	188.00	192.00	195.2
Curve length	9.42	6.28	3.77
Total	197.42	198.28	198.97
Loss, as straight pipe	197.42	198.28	198.97
Curve excess	74.99	52.16	36.06
Total	272.41	250.44	235.03

The resistance is here represented in units of the resistance in 1 ft. of straight pipe.

The next phase of the question to be considered is that of the cost of the several lines.

Cost of Construction.—The question of the relative cost and advantages of long *versus* short curves, in practical work, has been alluded to by Mr. Campbell, and, later, Mr. Gould (page 272), "thinks that, other things being equal, the practical decision will always be in favor of the shortest line," a proposition that all will grant, but, in this case, other things are by no means equal, as will be shown.

That no misunderstanding may exist, the following is taken from page 3 of the paper:

"The writers do not pretend by any means to have exhausted the subject, and, as these investigations cover only curves of 90°, and do

* With this assumption the remaining excesses become: 6-ft. radius = $42.29a$; 10-ft. radius = $52.16a$; 15-ft. radius = $68.74a$; small compound + 25-ft. radius = $125.65a$; 60-ft. radius + 40-ft. radius = $191.54a$; two large compounds + 30-in. Y = $249.62a$; and $b = 214.81a$ to $219.19a$, or approximately as before, the principal changes being in the 6-ft. and 15-ft. radius curves, and none appearing in the 10-ft. and the small compound sections.

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not reach the curves of very long radius occasionally used, there is still a very interesting and extensive field left untouched."

The use of curves of extremely long radius, such as are formed by deflecting straight-length pipe, is generally confined to places where, for topographical reasons, it is desirable to keep at or near the hydraulic grade line, or closely conform to contour lines. In general water-works construction, however, curves are located in streets and alleys, where the width of available right-of-way is limited, and the radius needs to be short enough to allow the pipe to have plenty of clearance at the street corners.

Without considering the relative frictional loss in curves of various lengths, discussed above, Table No. 90 shows the relative length and cost of a line of water pipe containing two tangents and one 90° curve; the beginning and end of the line fixed, and, in all cases, 100 ft. from the P. I. of the tangents. As the length of the curve increases, the lengths of the tangents decrease somewhat.

The cost of straight pipe is taken at $1 per foot, and that of curves at from $1 to $5 per foot.

For a practical example of these relative costs, it may be stated that at Detroit, in 1901, 7 000 tons of 42-in. pipe cost about $23 per ton, and the necessary specials cost $65 per ton, or curve castings cost 2.83 times the equal weight of straight pipe. Because of the additional weight, number of joints, and expense in laying, the curved pipe when set cost approximately 3.13 times that of an equal length of tangent.

It appears, therefore, that, without regard to frictional resistance, unless the cost for curves laid is the same or lower than that for straight pipe, which it seldom if ever is, the shorter the radius of the curve, the less the cost of the work.

Conclusion.—In the light of the foregoing, the writers believe Conclusion *K* to be established.

As to the correctness of the statement that "the theories and practices regarding curve resistance, as set forth in the hydraulic treatises of all nations up to the present time, are absolutely incorrect and the diametric opposite of the true conditions," it is unnecessary and impossible to cite all the evidence accumulated. An examination of the various treatises on hydraulics, contained in one of the most complete libraries in America, made before this paper was written, failed to show in an Italian, Spanish, French, German, English, Canadian or American hydraulic textbook or manual, a single statement to indicate that curve resistance ever decreased with the radius, and likewise failed to show one in which it was not either directly stated or clearly intimated that long easy curves offer less resistance to the flow of water than those of shorter radius. Although Mr. Freeman's experiments on curves, referred to herein, have been in print more than ten years, the writers have yet to find a single citation of them in any hydraulic textbook.

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TABLE No. 90.—Relative Length and Cost of Line of Water Pipe Containing Two Tangents and One 90° Curve; The Beginning and End of the Line Fixed, and in All Cases 100 Ft. from the Point of Intersection of Tangents.

Radius of curve, in feet. (1)	Length of curve (90°), in feet. (2)	Length of tangent, exclusive of curve, in feet. (3)	Total length of line, in feet. (4)	Total cost of line (length given in Columns 2, 3 and 4), assuming tangents to cost $1 per foot and the curves at different prices. Cost of curves per foot. $1.00. (5)	*$1.27½. (6)	$1.50. (7)	$2.00. (8)	$2.50. (9)	$3.00. (10)	$3.50. (11)	$4.00. (12)	$4.50. (13)	$5.00. (14)	Number of feet line has been shortened. (15)
0.....	0.00	200.00	200.00	$200.00	$200.00	$200.00	$200.00	$200.00	$200.00	$200.00	$200.00	$200.00	$200.00	0.00
1.....	1.57	198.00	199.57	199.57	200.00	200.35	201.14	201.92	202.71	203.49	204.28	205.06	205.85	0.43
2.....	3.14	196.00	199.14	199.14	200.00	200.71	202.28	203.85	205.42	206.99	208.56	210.13	211.70	0.86
3.....	4.71	194.00	198.71	198.71	200.00	201.06	203.42	205.77	208.13	210.48	212.84	215.19	217.55	1.29
4.....	6.28	192.00	198.28	198.28	200.00	201.42	204.56	207.70	210.84	213.98	217.98	220.26	223.40	1.72
5.....	7.85	190.00	197.85	197.85	200.00	201.77	205.70	209.62	213.55	217.47	221.40	225.33	229.25	2.15
6.....	9.42	188.00	197.42	197.42	200.00	202.13	206.84	211.55	216.26	221.07	225.68	230.39	235.10	2.58
7.....	10.99	186.00	196.99	196.99	200.00	202.49	207.98	213.48	218.97	224.49	229.96	235.46	240.05	3.01
8.....	12.57	184.00	196.57	196.57	200.00	202.86	209.14	215.43	221.71	227.10	234.38	240.57	246.85	3.43
9.....	14.14	182.00	196.14	196.14	200.00	203.21	210.28	217.35	224.42	231.49	238.56	245.63	252.70	3.56
10.....	15.71	180.00	195.71	195.71	200.00	203.57	211.42	219.28	227.13	234.99	242.84	250.70	258.55	4.29
15.....	23.56	170.00	193.56	193.56	200.00	205.34	217.12	228.90	240.68	252.46	264.24	276.02	287.80	6.44
20.....	31.42	160.00	191.42	191.42	200.00	207.13	222.84	238.55	254.26	269.97	285.68	301.39	317.10	8.58
25.....	39.27	150.00	189.27	189.27	200.00	208.91	228.54	248.18	267.81	287.45	307.08	326.72	346.35	10.73
30.....	47.12	140.00	187.12	187.12	200.00	210.68	234.24	257.80	281.36	304.92	328.48	352.04	375.60	12.88
35.....	54.98	130.00	184.98	184.98	200.00	212.47	239.96	267.45	294.94	322.43	349.92	378.41	404.90	15.02
40.....	62.83	120.00	182.83	182.83	200.00	214.25	245.66	277.08	308.49	339.91	371.32	402.74	434.15	17.17
45.....	70.69	110.00	180.69	180.69	200.00	216.04	251.38	286.73	322.07	357.42	392.76	428.11	463.45	19.31
50.....	78.54	100.00	178.54	178.54	200.00	217.81	257.08	296.35	335.62	374.89	414.16	453.43	492.70	21.46
60.....	94.25	80.00	174.25	174.25	200.00	221.38	268.50	315.63	362.75	409.88	457.00	504.13	551.25	25.75
70.....	109.96	60.00	169.96	169.96	200.00	224.94	279.92	334.90	389.88	444.86	499.84	554.82	609.80	30.04
80.....	125.66	40.00	165.66	165.66	200.00	228.49	291.32	354.15	416.98	479.81	542.66	605.47	668.30	34.34
90.....	141.37	20.00	161.37	161.37	200.00	232.06	302.74	373.43	444.11	514.89	585.48	656.17	726.85	38.63
100.....	157.08	0.00	157.08	157.08	200.00	235.62	314.16	392.70	471.24	549.78	628.32	706.86	785.40	42.92

* $\frac{4}{\pi} = 1.27323.$

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Conclusions from the Discussions.

In the discussions of this paper many points have been brought out in addition to the conclusions of the writers, and some of the most important will be considered.

Curves Other than 90 Degrees.—One of the most interesting questions, particularly in connection with Conclusion *K*, is involved in the experiments presented by Messrs. Saph and Schoder, upon curves of 180°, in which the radii varied from 9.58 to 3.84 diameters, the pipe line being of 2-in. seamless brass tubing.

On first examination, the results obtained appear to flatly contradict those of the Detroit experiments, for neither with the Pitot tube nor with the loss-of-head gauges was there found any indication that either the distortion or H_f was influenced by the radius of curvature. The explanation, however, is obvious. If, as concluded by the writers, the resistance due to curves is largely controlled by the distortion produced, and, as is shown, the short curve produces less distortion than a long one, it is to be expected that a curve of any radius might be made sufficiently long to produce the same distortion as one of greater radius but less angular deflection. This conclusion is borne out by the traverses of the observers in the 9.58-diameter curve, Fig. 124, whereby it is shown that very little distortion takes place in the second quadrant, and explains the experiments of Weisbach, who found that adding 90° to a 90° curve of the same radius only increased the resistance about 40 per cent.*

It appears, therefore, that, in these 180° curves, for all radii, the water received its full distortion before leaving them, and, consequently, the resistance of all was substantially the same. This also accounts for the right-hand portion of the locus on Fig. 90, when properly corrected, as indicated on page 353, becoming asymptotic to the axis of abscissas, and it follows that, after a certain radius is reached for any degree of curvature, no, or very little, additional loss will be introduced by lengthening either the angular deflection or the radius, but that the loss will be perceptibly diminished if either is decreased. Consequently, short radii should be used for small deflections. Weisbach found that when a tangent was introduced between two curves turning in the same direction, the loss of head added by the second curve was increased, which is as would be expected, for as soon as the water reaches the tangent the distortion begins to reduce itself and the water not only has to be re-distorted in the second curve, but energy is absorbed in the readjustment in the tangent.

Combinations of Curves.—In the light of this, it is by no means impossible that the plan for joining two tangents, proposed by Mr. Gould on page 271, by using two 45° curves with a tangent between, might cause a greater loss of head than the longer line with a single

* See *The Technic*, University of Michigan, 1899, p. 53.

90° curve. Evidently, in the case of two curves turning in the same direction, the nearer together they are placed the less will be the excess resistance introduced after the second comes into the region of influence of the first, because the total work of distortion is reduced as they are brought closer together. When the curves turn in opposite directions, however, some further considerations are introduced. If the loss of head due to curvature were wholly made up of the resistances produced by distortion in the interior of the stream, it would then make no difference whether curves reversing direction were near together or far apart, but it seems quite evident, from Fig. 122, that the average velocity next the wall in curve-distorted flow is greater than in normal flow, and hence the loss of head due to friction on the pipe wall must be greater, so that while, so far as the internal work is concerned, the energy expended in a pair of reversing curves would be the same whether they were near or wide apart, the loss due to this added frictional resistance is reduced when the slow rearrangement of velocities in the tangent, with its prolonged distorted flow, is cut out by placing the curves close together. This conclusion seems to be borne out by the experiments of the writers on the pairs of reversing curves, as presented in Table No. 89, where it is seen that the losses for the 25-ft. radius and the small compound curve are not twice those for the 15-ft. radius curve, though the radius of the small compound is 14.15 ft. It might be assumed, perhaps, that this meant that the 25-ft. radius curve was of a radius such that the loss of head was decreasing and might continue to do so for larger radii thereafter, were it not that the data for the next pair of similarly situated curves, the 60-ft. and the 40-ft. radius ones, show the loss to be still increasing as the radius increases. The first pair of curves in the 30-in. line is complicated by the presence of the 30 x 30-in. Y, with branch pointing up stream, it being placed there to feed into, not out of, the experimental line. In this position it would be expected to produce a maximum effect, and it is possible to approximate that effect from the data. The mean radius of the two large compound curves is the same as that of the 40-ft. radius curve, and if it is assumed that the resistance due to the 60-ft. radius is the same as that due to the 40-ft. radius, by subtracting the excess loss in the second pair of curves from that of the Y and the two curves in the first section, a limiting value of the excess due to the Y is obtained. This value is seen to be equal to the resistance of 54.96 ft. of straight pipe, or about the same as that of the 10-ft. radius curve, or of three gates and four 30 x 10-in. Y's, according to the values in Table No. 89. The 60-ft. radius curve probably does introduce more resistance than the 40-ft. radius curve, and other complications tend to make it quite evident that this value is somewhat too great for the 30 x 30-in. Y.

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Relation of Curve Resistance in Large and Small Pipes.—Were observations available on the intermediate tangents of the 12-in. pipe line

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of the writers, it would probably be possible to trace the relation between the diameter of the pipe and the curve resistance, but, as the curve results on the 12-in. pipe are somewhat in error, although no criticism stands against the tangent, it seems unwise to attempt it. To undertake to go from the 90° curves in 30-in. pipe to the 180° curves in 2-in. pipe without an intermediate step, seems to be too wide a jump.

The relations used in Fig. 90, based on a reduction of all resistances to an equal length in diameters, the writers now regard as misleading to some extent, and, therefore, in this closing discussion, they have adhered to the simple excess resistances found in each case, which, when properly measured, should be the same, whatever the length used after the full effect has been realized.

Mr. Brinckerhoff, page 272, presents a theory of curve effect which is about as wide of Mr. Tutton's assumed conditions of flow in straight pipe as two propositions well could be, and, while the writers are not very strongly inclined to the latter, neither are they quite ready to accept the former. At one time, one of them was disposed to explain certain phenomena on the theory of a spiral motion, but in spite of attempts to locate such motion, no direct experimental evidence of its existence has yet been obtained, and, though the writers consider that Mr. Brinckerhoff's theory is ingenious and possibly correct, they are not prepared to indorse it.

Piezometer Errors.—Of all the points presented in this paper or its discussions, probably none is more important, from the experimental standpoint, than the erroneous indications of piezometers under conditions of abnormal flow. This was suggested by one of the writers in connection with the Ogden Experiments,* and was further discussed by him in an article† in which it was shown that Darcy's famous pipe experiments were thus affected at some points to such an extent as to call down upon them the severe criticism of the late Hamilton Smith, Jr., M. Am. Soc. C. E.,‡ but that, had the observations simultaneously taken at the middle of the pipe line, where normal conditions existed, been utilized, no such criticism could have been made. Many experiments have doubtless been affected similarly, and this is undoubtedly the cause of the variable action of the Venturi meter in the experiments already discussed (page 338). The increased resistances due to the distortions, thus exhibiting themselves at pipe entrances, quite probably may account for part of the difference in the results quoted by Mr. Mills regarding a 12-in. curve experiment, from those of the writers. Mr. Mills (page 205) shows that in his experiment the losses were greater than are shown by the writers, while all corrections that seem to be warranted in the writers' data would reduce their excesses still further. The curve used by Mr. Mills was near the inlet of his

* *Transactions*, Am. Soc. C. E., Vol. xliv, page 65.

† *Journal of the Association of Engineering Societies*, March, 1901, page 177.

‡ "Hydraulics," page 258.

line, and, very probably, may have given a high resistance on account of the distortion of the stream entering it. Messrs. Williams, Hubbell and Fenkell.

Temperature Effects.—The data regarding the influence of temperature, presented by Messrs. Saph and Schoder, page 313, open up a very interesting subject. Examining the 12-in. experiments of the writers, part of which were made in November, and part in June, evidence of the influence of temperature is found, though the data are insufficient to determine its extent. In another investigation, one of the writers, in experiments involving discharges of water amounting to 200 cu. ft. per second, has encountered what was interpreted as an effect due to temperature, and its influence upon the discharge of small weirs has been investigated and demonstrated. Such effects may even be traced in some of the weir experiments of the late James B. Francis, Past-President, Am. Soc. C. E The writers, therefore, agree with the observers that this element cannot be always safely overlooked.

Capillary Effects in Differential Gauges.—The discussion presented by Mr. Fisher, page 288, in reply to Mr. Wisner's question regarding the influence of the diameter of glass tubes upon piezometric indications, is another valuable contribution to experimental data. The question is one that has frequently arisen in practice, and caused the writers considerable speculation before their experiments were completed. The results obtained by the experimenter are fully sustained by so much of the writers' work as bears upon the subject.

Critical Velocities, Internal Motion and Pulsations.—The discussions of this paper contributed by Messrs. Thrupp, Seddon, Tutton, Bazin, Horton, Henry and Brinckerhoff, leading as they do into the realm of molecular forces, require for their proper consideration a treatment far transcending the limits permissible here.

Professor Osborne Reynolds* has shown that when water flows through pipes of small diameter, up to $\frac{1}{2}$ in., at least, at very low velocities, the loss of head varies as the first power of the velocity, and that then the flow takes place in straight lines, but that, at a certain more or less constant velocity, this filamental flow breaks down and the loss of head thereafter varies as a higher power of the velocity than the first. The experiments of Poiseuille, Hagen, Jacobson and Hazen show that when water flows through capillary tubes or fine sands, where it is prevented from taking up internal motions, because the area of the cross-section of the stream is almost molecular, that H_f varies very nearly as the first power of V. All reliable experiments on record show that as the diameter decreases the exponent of V, in $H_f = m\ V^n$, decreases, as has been shown for the lines investigated in this paper: 30-in., $H_f = m\ V^2$; 16-in., $H_f = m\ V^{1.86}$; 12-in., $H_f = m\ V^{1.78}$; and 2-in. brass, $H_f = m\ V^{1.76}$, from a possible limit of V^2. In other

* *Philosophical Transactions*, London, 1883.

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words, the more the chance for internal resistance, the higher the exponent of V. To the writers, then, the variation of the exponent of V is an index of the character of the flow, and when that becomes greater than unity, straight-line flow is over, or, the critical velocity of Professor Reynolds is past. If, then, these internal motions are capable of so increasing the rate of loss of head, it is evident that in them the controlling conditions of the laws of flow are to be looked for, rather than in the surface resistances. But, beyond this first critical velocity, there appear to be others where peculiar phenomena appear, for it has been observed that with the 2-in. brass pipe used by the writers, below certain velocities, piezometers near the inlet gave astonishingly low and variable readings, as compared with those farther down the line, and yet the exponent of V for the down-stream section was apparently the same as at higher velocities where there was no evidence of erratic performance. The location of this critical velocity might be anywhere from 0.3 to 0.7 ft. per second. At a certain other velocity, about 2.6 ft. per second, there has been observed a violent pulsation of the stream in this pipe, apparent in Pitot tube gauges, particularly when connected to point and ring, but not so noticeable on resistance gauges, both sides of which were connected to ring piezometers. At this velocity it is very difficult to observe with the tube, but at slightly lower and slightly higher velocities the disturbance ceases. The supply of this line comes from a tank, the inlet of which is controlled by a float-valve, and there exists no means of transmitting a vibration from the mains, which are themselves fed from a reservoir, to the experimental line. The pulsation evidently originates in the line itself, probably, in much the same manner as a tone is produced in an organ pipe.

The writers have often felt that the subject of Hydraulics has not yet attained to the dignity of a science, but is really little more than an art, and that it is only by the most conscientious experimenting, and the most scrupulous presentation of the facts thus obtained, that it can be advanced along the proper lines. They entered upon this investigation with no theories to either sustain or demolish, and were simply looking for truth. It may be of interest, in connection with Mr. Hering's remarks on the analogy between the results brought out by the writers and the experience with railroad curves, to state that these experiments were first suggested to the writers by the chapter on curve resistance in Wellington's "Railway Location," and though the suggestion had to lie dormant for fully ten years, the determination to make the experiments only awaited the opportunity.

In closing, the writers wish to express their appreciation of the thoroughly scientific spirit in which this paper has been received and discussed, as well as of the many encouraging sentiments of commendation, and particularly of the valuable experimental data which have

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been added to it. Some, no doubt, will criticise them for their departure from the hitherto recognized methods of reduction and presentation of such data, but to such they would reply that the formula, $H_f = m\ V^n$, is a perfectly logical one, and about the simplest that can be applied to the flow of water; for those who wish Chezy coefficients, Tables Nos. 37 and 38 have been prepared. Others may wish that the theoretical side had received more extensive treatment, but, bearing in mind the following precept of an honored instructor of one of them: "No matter how rational a theory may appear, no matter how many phenomena it may account for, no matter how perfectly it may harmonize with any number of observed results, if there remains one fact whose existence cannot be harmonized with it, the theory must give way, for the fact will not," they have felt that until the theories they would propound were at least beyond collision with facts with which they themselves were familiar, they should confine their statements as closely as possible to a simple presentation of experimental results.

A word of caution they would add: The observations herein presented are facts; the interpretations are necessarily theories; the latter may change, and must be used with caution, and the former may be only true for the particular conditions and limits under which they were observed, though the writers see no reasons at the present time for thus limiting them.

AMERICAN SOCIETY OF CIVIL ENGINEERS.

INSTITUTED 1852.

TRANSACTIONS.

Paper No. 912.

CURRENT METER AND WEIR DISCHARGE COMPARISONS.*

By Edward C. Murphy, Assoc. M. Am. Soc. C. E.

With Discussion by Messrs. Charles H. Miller, Rudolph Hering, Morris R. Sherrerd, L. J. Le Conte, J. B. Lippincott, E. E. Haskell and Edward C. Murphy.

This paper gives the results of a series of experiments made at the Hydraulic Laboratory, Cornell University, for the Hydrographic Branch of the United States Geological Survey, to test the accuracy of velocity measurements obtained with the current meter, used by different methods. One series, only, is used, in order that the paper may be short. The fifth series is selected on account of the wider range of velocities embraced in it, and also because two meters at a time were used in the same discharge section, thus giving a comparison of simultaneous meter velocity and discharge, as well as meter and weir discharge.

* Presented at the meeting of October 16th, 1901.

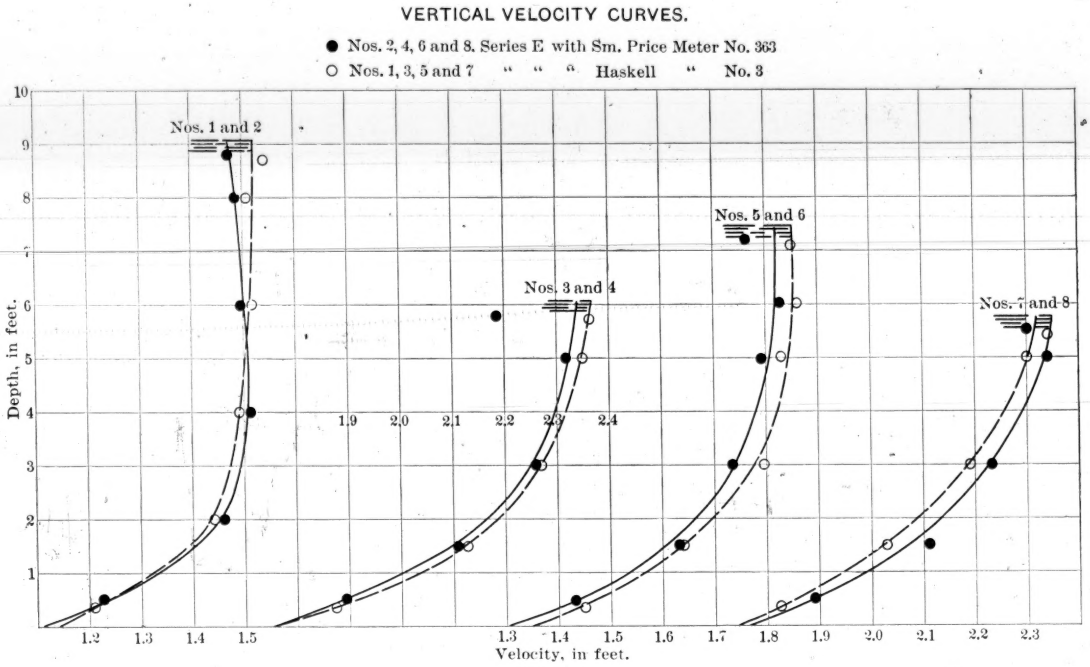

VERTICAL VELOCITY CURVES.
● Nos. 2, 4, 6 and 8. Series E with Sm. Price Meter No. 363
○ Nos. 1, 3, 5 and 7 " " " Haskell " No. 3
Nos. 1 and 2
Nos. 3 and 4
Nos. 5 and 6
Nos. 7 and 8
Depth, in feet.
Velocity, in feet.

FIG. 1.

The writer has two objects in view in presenting this paper. The first is to show engineers and others the degree of confidence that may be placed in results obtained with the current meter, under favorable conditions; the other is to bring out in the discussion the reasons why some engineers look with disfavor on the current meter.

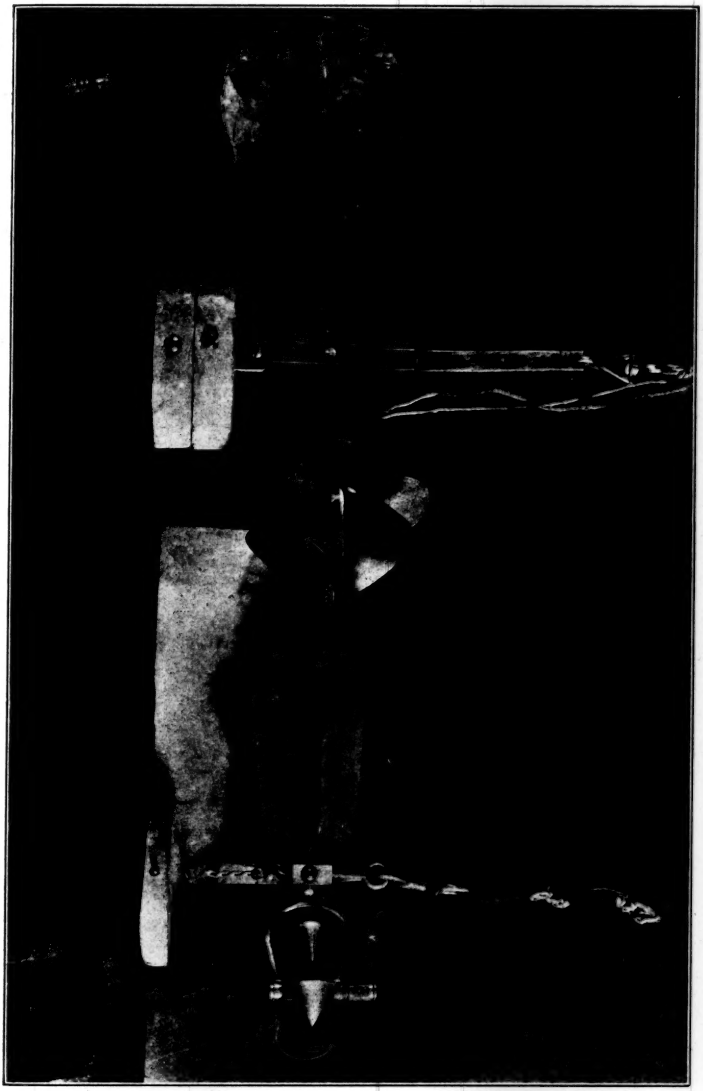
The Hydraulic Laboratory of Cornell University has been described elsewhere * and it will be sufficient, for the purpose of this paper, to describe only the canal in which the meter measurements were made. This is a rectangular channel having a concrete bottom and sides, 415 ft. long, 16 ft. wide and 10ft. deep, with a bottom grade of 1 in 500. There are six rectangular gates at its upper end and four at the lower end for regulating the water supply. Across the canal, and 59 ft. from the upper end, there is a bulkhead, on top of which is fastened a 3.5 x 5-in. steel angle-iron forming a sharp-crested or standard weir, 16 ft. long, and 11.1 ft. above the bottom of the canal. There are baffles above the weir for quieting the water after it passes under the gates, also one below the weir for a similar purpose.

In this series of experiments the elevation of the water surface was measured with a portable hook-gauge at a point 21¼ ft. up stream from the weir, and at two points 164 and 304 ft., respectively, below the weir. Six readings were taken at each of these points before passing to the next point.

Three current meters were used: Small Price meters Nos. 351 and 363, and Haskell meter No. 3. They were held with an insulated wire cable and 12-lb. sinker, except in Experiments Nos. 5 and 7, in which the Haskell was held with a rod, with freedom to tip. Two meters at a time were used in the same discharge section. One was started on the south side, and progressed toward the north side, the other started on the north side and moved toward the south side. Thus three simultaneous discharge measurements were obtained, two with the meters and one with the weir.

Plate XI shows Haskell meter No. 3 and Small Price meter No. 363. The Haskell meter is of the screw or propeller-wheel type, the four helicoidal blades revolving in a vertical plane. The Small Price meter is of the anemometer type, the five conical cups revolving in a horizontal plane. The total length of the Haskell meter is 21 ins. and of the Price meter 14 ins. The former is made by Ritchie, of Boston,

* *Transactions*, Am. Soc. C. E., Vol. xliv, p. 285, and *Engineering News*, March 2d, 1899.



"HASKELL" AND "SMALL PRICE" CURRENT METERS.

TABLE No. 1.—Data for Current Meter and Weir Comparisons.
Series E.

No. of Experiment.	Date.	Depth, in feet.	Meter velocity, in feet.	Weir discharge, $Q w$, in cubic feet.	Meter discharge, Q, in cubic feet.	$\frac{Q w - Q}{Q w}$, percentage.	Name of meter.	Percentage of difference in meter discharge.
	1900.							
1....	April 1st.	9.075	1.472	204.41	214.08	−4.73	Haskell No. 3.....	
2....	" 1st.	9.075	1.456	204.41	212.33	−3.87	Small Price No. 363..	0.86
3....	" 2d.	6.079	2.208	208.88	216.28	−3.54	Haskell No. 3.....	
4....	" 2d.	6.079	2.187	208.88	214.22	−2.55	Small Price No. 363..	0.99
5....	" 2d.	7.453	1.744	200.81	208.85	−4.00	Haskell No. 3.....	
6....	" 2d.	7.453	1.710	200.81	204.77	−1.97	Small Price No. 363..	2.03
7....	" 2d.	5.742	2.137	197.64	197.02	−0.31	Haskell No. 3.....	
8....	" 2d.	5.742	2.177	197.64	200.15	+1.27	Small Price No. 363..	1.58
9....	" 5th.	7.409	1.869	222.67	222.68	0.00	Haskell No. 3.....	
10....	" 5th.	7.409	1.883	222.67	224.35	−0.75	Small Price No. 363..	0.75
11....	" 5th.	6.295	2.137	220.33	216.00	+1.96	Haskell No. 3.....	
12....	" 5th.	6.295	2.177	220.33	220.03	+0.14	Small Price No. 363..	1.82
13....	" 5th.	5.628	2.377	219.38	214.82	+2.08	Haskell No. 3.....	
14....	" 5th.	5.628	2.438	219.38	220.33	−0.43	Small Price No. 363..	2.51
15....	" 5th.	5.294	2.611	221.96	222.02	−0.03	Haskell No. 3.....	
16....	" 5th.	5.294	2.602	221.96	221.27	+0.31	Small Price No. 363..	0.34
17....	" 5th.	4.892	2.764	222.81	217.32	+2.46	Haskell No. 3.....	
18....	" 5th.	4.892	2.827	222.81	222.27	+0.24	Small Price No. 363..	2.22
19....	" 5th.	4.617	2.936	224.75	218.32	+2.86	Haskell No. 3.....	
20....	" 5th.	4.617	3.019	224.75	224.50	+0.01	Small Price No. 363..	2.85
21....	" 6th.	4.808	2.985	230.36	230.66	−0.13	" " 351..	
22....	" 6th.	4.808	3.006	230.36	232.30	−0.84	" " 363..	0.71
23....	" 10th.	8.744	1.670	208.75	234.68	−1.70	" " 351..	
24....	" 10th.	8.744	1.657	208.75	232.85	−0.99	" " 363..	0.71
25....	" 10th.	7.874	1.846	229.28	233.61	−1.89	" " 351..	
26....	" 10th.	7.874	1.862	229.28	235.64	−2.77	" " 363..	0.88
27....	" 10th.	6.889	2.108	229.91	233.35	−1.49	" " 351..	
28....	" 10th.	6.889	2.073	229.91	229.38	+0.23	" " 363..	1.68
29....	" 11th.	6.129	2.340	227.16	230.25	−1.36	" " 351..	
30....	" 11th.	6.129	2.320	227.16	228.28	−0.50	" " 363..	0.86
31....	" 11th.	5.483	2.596	225.01	228.59	−1.60	" " 351..	
32....	" 11th.	5.483	2.604	225.01	229.30	−1.91	" " 363..	0.31
33....	" 11th.	5.185	2.725	224.06	226.98	−1.30	" " 351..	
34....	" 11th.	5.185	2.758	224.06	229.73	−2.53	" " 363..	1.23
35....	" 11th.	5.003	2.816	222.60	226.37	−1.69	" " 351..	
36....	" 11th.	5.003	2.834	222.60	227.82	−2.34	" " 363..	0.65
37....	" 11th.	9.256	1.522	223.40	226.38	−1.33	" " 351..	
38....	" 11th.	9.256	1.538	223.40	228.71	−2.38	" " 363..	1.05
39....	" 12th.	9.344	1.486	224.63	223.11	+0.68	Haskell No. 3.....	
40....	" 12th.	9.344	1.541	224.63	231.37	−3.00	Small Price No. 351..	3.68
41....	" 12th.	8.934	1.613	221.39	231.58	−4.60	" " 351..	
42....	" 12th.	8.934	1.543	221.39	221.53	+0.07	Haskell No. 3.....	4.67
43....	" 12th.	8.297	1.646	221.39	219.50	+0.81	" " 3.....	
44....	" 12th.	8.297	1.700	221.39	226.70	−2.40	Small Price No. 351..	3.21
45....	" 12th.	7.806	1.775	221.49	222.69	+0.54	Haskell No. 3.....	
46....	" 12th.	7.806	1.803	221.49	226.19	−2.07	Small Price No. 351..	2.61
47....	" 12th.	7.266	1.879	221.21	219.34	+0.85	Haskell No. 3.....	
48....	" 12th.	7.266	1.943	221.21	226.81	−2.53	Small Price No. 351..	3.38
49....	" 12th.	6.439	2.132	221.03	220.43	−0.27	Haskell No. 3.....	
50....	" 12th.	6.439	2.482	221.03	225.61	−2.07	Small Price No. 351..	1.80

Mass.; the latter is made by W. and L. E. Gurley, of Troy, N. Y., and is described in their 1900 catalogue.

The meters were operated by the ordinary point method. They were held at from 5 to 8 points in the 8 verticals, 1, 3, 5, 7, 9, 11, 13 and 15 ft., depending on the depth and the revolutions of the meter wheel counted for two consecutive periods of 25 seconds each at each point. The points in a vertical where the meter was held are indicated by the centers of the little circles in the vertical velocity curves of Fig. 1. This gave from 40 to 50 observations of 50 seconds each, or one to each 2.3 sq. ft. of discharge area, for each discharge measurement.

The field force consisted of five men: Two meter observers, one meter recorder, one hook-gauge observer, and one hook-gauge recorder. The head on the weir was obtained by subtracting the zero reading of the hook from the mean hook reading during the experiment. The weir discharge was then found from a chart prepared from Bazin's formula for sharp-crested weirs, which gives the discharge in cubic meters for head in centimeters.

The depth at the meter station was obtained from the mean surface and bottom elevations. The area for each foot of depth is known from previous measurement.

The mean velocity was obtained from the vertical velocity curves. The average number of revolutions per second in the 8 verticals at each of the distances above the bottom was computed and converted into velocity from a rating table. These are plotted, using the velocities as abscissas, and the distances above the bottom as ordinates, and a smooth vertical velocity curve sketched among them. These curves, for the first 8 experiments, are shown in Fig. 1. The mean abscissa of each curve was then computed, which is the mean velocity for the whole cross-section. The discharge is the product of the cross-sectional area and the mean velocity.

The water surface at the meter station was quite rough for the higher velocities, being a succession of waves, without eddies or "boils," and it was difficult to measure its position accurately with the hook-gauge. To quiet it, somewhat, 2 x 12-in. planks, 15 ft. long, were suspended from their ends across the canal to float on its surface. In Experiments Nos. 9 to 22 two of these were used, and in Experiments Nos. 23 to 50 three were used 66 ft. above the meter station. These quieted the surface somewhat, but distorted the vertical velocity curves.

The results of the computations of these 50 discharge measurements are given in Table No. 1. The headings of the columns are self-explanatory. It is to be noted that the sign before the numbers in Column 7 is the sign of the correction to be applied to the meter discharge to reduce it to the corresponding weir discharge. Column 7 shows a comparison between corresponding meter and weir discharges, and Column 9 a comparison between corresponding meter discharges. In Experiments Nos. 1 to 20 the comparison is between the Small Price meter No. 363 and the Haskell meter. In Experiments Nos. 21 to 38 the comparison is between the two Small Price meters, and in Experiments Nos. 39 to 50 it is between the Small Price meter No. 351 and the Haskell meter.

It is seen that, in general, the difference between any two corresponding meter and weir discharges is greater than that between the corresponding meter discharges. This is to be expected, as errors in the calibration of the weir, in the velocity observations, in the rating of the meter, and in measuring the head and depth, affect the former; while only errors in observing velocity, and in the rating table, affect the latter. In no case does a meter discharge differ from the corresponding weir discharge by 5%, and in no case is the difference between corresponding meter discharges 5% of the discharge.

The results of these comparisons have been put in the condensed form shown in Table No. 2.

TABLE No. 2.—Condensed Results of Comparison of Meter and Weir Discharge. Series E.

No. of Experiment.	Name of Meter.	Percentage of Difference Between Meter and Weir Discharge.				Percentage of Difference Between Simultaneous Meter Discharges.			
		Max.	Min.	Mean.*	Range.	Max.	Min.	Mean.	Range.
1 to 20	Haskell No. 3.....	—4.73	0.00	—0.33	—4.73 to +2.86	.....	....	.....	....
1 to 20	Small Price No. 363	—3.87	+0.01	—0.76	—3.87 " +1.27	2.85	0.34	*0.29 / †1.60	2.51
21 to 38	" " 351	—1.89	—0.13	—1.38	—1.89 " —0.13	.....	....	.....	....
21 to 38	" " 363	—2.77	+0.23	—1.56	—2.77 " +0.23	1.68	0.31	*0.18 / †0.90	1.37
39 to 50	" " 351	—4.60	—2.07	—2.78	—4.60 " —2.07	.....	....	.....	....
39 to 50	Haskell No. 3.....	+0.85	—0.27	+0.40	+0.85 " —0.27	4.67	1.80	*3.22 / †3.22	2.87

*Having regard to the sign.
†Without regard to the sign.

The mean "having regard to the sign" is obtained by summing the positive and negative differences and dividing the differences of these sums by the number of positive and negative differences.

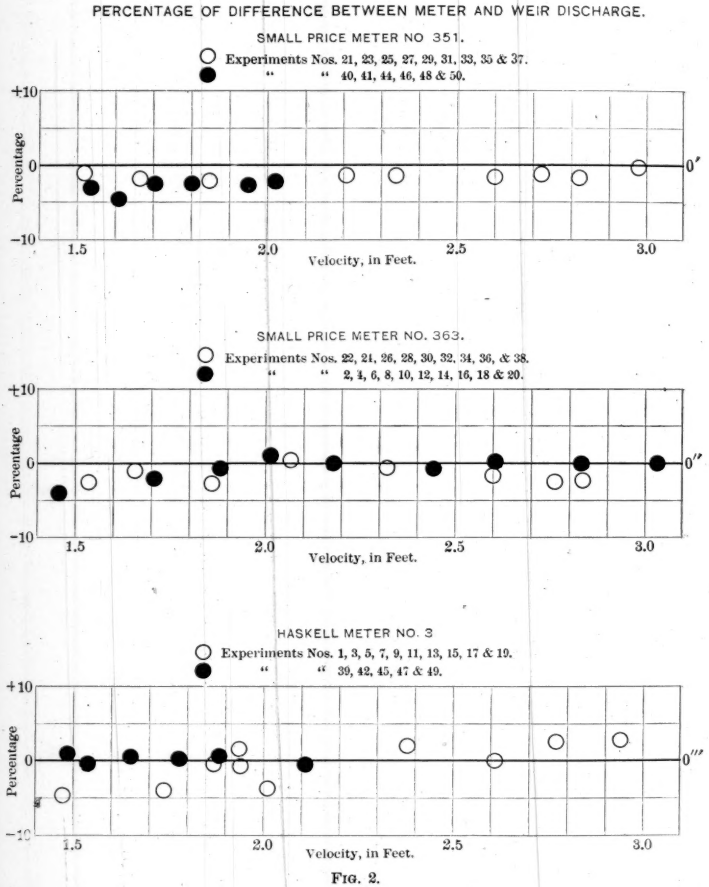


FIG. 2.

There appears to be a constant error in the Small Price meter work of Experiments Nos. 39 to 50, on April 12th. The velocities shown by it on that day appear to be about 2% larger than they should be. This could hardly be due to a reduction of friction in the meter, nor could

it be due to error in the time, as the watch used was keeping good time before and after this date, and the apparent error extends through the whole day's work.

Omitting farther consideration of Experiments Nos. 38 to 50 in this comparison, for the reason above stated, it is seen that the mean difference between the meter and weir discharge is 0.33% for the Haskell, and 0.76% for the Small Price meter in Experiments Nos. 1 to 20; and 1.38% for the Small Price No. 351, and 1.56% for the Small Price No. 363 in Experiments Nos. 21 to 38. The mean difference between two corresponding meter discharges is 0.29% for Experiments Nos. 1 to 20 and 0.18% for Experiments Nos. 21 to 38.

Limiting the comparison still farther to the two Price meters only, in Experiments Nos. 21 to 38 it is seen that in eighteen consecutive discharge measurements of this canal made with the two Price meters, the mean velocity ranging from 1.5 to 3.0 ft., and the depth from 4.8 to 9.2 ft., the greatest difference between any two simultaneous meter discharge measurements is 1.68%, the mean difference is 0.9%, and the mean difference having regard to sign is 0.18 per cent. Is it not a reasonable conclusion to draw from these tests that discharge can be measured with a Small Price meter, under favorable conditions, with an error of not more than 1 per cent.?

The degree of accuracy of ordinary river discharge measurements made with a meter is necessarily less than in this canal, and varies with the character of the discharge section. A good discharge section, in which the bed is smooth, permanent, and without obstructions, which has a good measurable velocity in all parts of it, at low as well as high stages, has no shallow parts at low stage, and is far enough away from river bends, mill dams, and other disturbing causes to be out of their influence, is difficult to find. When it is found it may not be selected on account of cost of maintenance.

In river gauging it is the accurate measurement of the volume flowing per day and per month that is desired. Single discharge measurement is only one of the factors on which this depends. Daily flow is found from a discharge curve giving the relation between discharge and river stage, and on the daily fluctuations in the stage. The accuracy of the discharge curve depends on the accuracy of the individual discharge measurements, the permanence of the river-bed, the position of the gauge, and the accuracy of the gauge readings. The

accuracy of river stage fluctuation measurements depends on the kind of instrument used for this purpose, its position, and the frequency and accuracy of the observations.

It is thus seen that the accuracy of the current meter discharge measurements plays only a comparatively small part in the accuracy of measurement of stream flow. Is it fair, then, when the monthly flow is found to be in error, to blame the meter for it and say it is "no good"? The only fair test of the accuracy of current meter work is one similar to that made in this canal. When it is seen that in eighteen discharge measurements made on three days, by two persons with two of these meters, with a considerable range of depths and velocities, the greatest difference between any two made at the same time is less than 1.7%, and the mean difference is less than 1%, the great value of this instrument for measuring velocity is evident.

In carrying on this investigation the writer desires to acknowledge his indebtedness to F. H. Newell, M. Am. Soc. C. E., Chief Hydrographer of the U. S. Geological Survey, for his hearty cooperation, and his permission to use in this paper some of the data obtained; to Professor E. A. Fuertes, M. Am. Soc. C. E., Director and Dean of the College of Civil Engineering, Cornell University, for the use of the Hydraulic Laboratory, chronograph, computing machine, current meter, and electric register, and for his kindly interest in the work; to Professor Gardner S. Williams, M. Am. Soc. C. E., in charge of the Hydraulic Laboratory, for many valuable suggestions, and for the use of a chart for converting head on the Cornell University standard weir into cubic meters; to Mr. C. E. Torrance, Scholar in Civil Engineering; to the members of the Senior Class in Civil Engineering, and to Mr. C. D. Cass, Assistant Mechanician of the College, for cheerful assistance.

DISCUSSION.

CHARLES H. MILLER, M. Am. Soc. C. E. (by letter).—It was with much interest that the writer read this paper, and it is to be hoped that it will have the effect desired by the author, namely, to bring out the reasons why current meters are believed to be unreliable. The writer, after a wide experience, firmly believes that, under the conditions usually existing in river-discharge work, and with the proper amount of care, results are obtained that are within 5% of the true values. After fruitless efforts to account for sudden changes or apparent errors in the workings of current meters from day to day, it is gratifying to note that, under the favorable conditions stated in the paper, "there appears to be a constant error in the Small Price meter work * * * on April 12th." This difference, without a doubt, is due to some change in the meter itself, and not to any error in the work of observing; and is something that may be expected at any time. Had a rating of this meter been made on that day, the change might have been indicated; but it is quite likely that it would not have shown a sufficient difference from previous ratings to account for all of the change, thus leaving the same unexplained. Very often, a meter will work this way for several days or even a week, and then suddenly drop back to its old standard. Mr. Miller.

The writer had charge of some experimental work on the Mississippi River at New Madrid, Mo., about a year ago, having two Price meters and one Haskell meter. These were rated and worked in pairs at all times, thus presenting a good opportunity for comparisons. Having left the service before fully completing the reductions, and not having any of the reports or figures on hand (the same not having been published as yet), this discussion will be but general in character. The final figures in several instances will show slightly greater differences in discharge than 5%, but it is believed that this is mainly due to the fact that the observations were taken from a skiff, which, it was afterward found, was not of sufficient stability to insure results from which an accurate comparison could be made, many of the observations having been taken during windy weather. To find what effect the movement of the skiff would produce, it was securely anchored, during calm weather, in a current running between 2 and 3 ft. per second. The Haskell meter was lowered from one side of the skiff and one of the Price meters from the other side, both to the same depth (about 5 ft.). The registrations or revolutions of each, for periods of from 5 to 10 minutes, were noted; then, for similar periods of time, the skiff was caused to rock from one side to the other at short intervals, and the registrations recorded, the movement of the skiff giving to the meters a vertical motion of from 1 to 2 ft.

Mr. Miller. Under this motion the revolutions of the Price meter increased, while those of the Haskell meter decreased, but in a less proportion than the increase of the Price meter, showing that the error due to instability of support is greater in the Price than in the Haskell meter. Lack of time prevented more extended observations, and in different velocities of current, which probably would have evolved some fixed difference due to the different construction of the meters in question.

During the observations mentioned, all possible care was exercised to eliminate errors, as far as the comparison between the meters was concerned. By means of an automatic register for the Haskell meter, an ordinary relay (the ticks of which were counted) for the Price meter, and a stop watch, it was possible for one observer to do the work, and this was adhered to at all times.

It was while making these observations that the writer noted the difference in the workings of the meters, and especially of the two Price meters when side by side, for it could then be noted at once. They would often vary as much as 3% without any apparent cause whatever. Frequent ratings were made with great care, but without discovering the cause for such changes.

With regard to the unaccountable differences in discharge between different stations on the same river, having different observers, the writer believes that it is partly due to the different methods in rating the current meters, or to the lack of care in conforming to a uniform method. Along the Mississippi River it is often difficult to obtain a proper base for rating a meter, and at times it is done in water of insufficient depth, causing the meter to be affected by the "drag." Full and careful rules for all ratings should be formulated and strictly adhered to.

Another point which has often been noted, and which is again brought out by the experiments mentioned in the paper, is that discharges by current meter will generally average a little more than those obtained by other methods.

"A good discharge section" cannot be found on the Mississippi River, for the conditions at any section that may be selected will vary very much for different stages of the river; and this factor, more than any other, causes the differences in discharges between different stations. They are not due to the meters being "no good."

Mr. Hering. RUDOLPH HERING, M. Am. Soc. C. E.—It is a little surprising to the speaker that, as a general proposition, there should have been serious doubt expressed regarding the efficiency of the current meter. This instrument, in its simplest form, was invented one hundred years ago, by Woltman, a hydraulic engineer, in Hamburg. It has been used continually ever since, and has been gradually improved so as to be-

come more and more convenient in handling and in reading the results. No other instrument has been used as much in determining directly the velocity of streams. Mr. Hering.

Like all measuring instruments, it has its limitations as to degree of accuracy and range of applicability. Its value, therefore, should be sought only within such limitations, which are, for instance, in a water free from suspended or floating matter, as the latter may interfere with the movement of the wheel or of its axle, or which may change its rating. The accuracy of this meter, therefore, is naturally reduced when gauging sewage or water carrying much fibrous matter; yet, under such conditions, the accuracy of weir measurements must also be discounted, to a certain extent. Further, when a stream is shallow, so that the wheel is not entirely submerged, or when it is influenced by local currents, produced by an irregular bed, the accuracy of the wheel suffers, and better methods of gauging are preferred. Also, when the velocity of the water is very slight, a current meter is not the proper instrument to use, because the force impelling the wheel, due to the velocity of the passing water, does not differ much from that which is necessary to overcome the frictional resistance of the apparatus. This force, therefore, is less distinguishable, and the velocity less capable of accurate measurement, by the revolutions of the wheel.

For very shallow streams and for those in which the velocity is very slight, that is, not more than about 6 ins. per second, and where the water carries much suspended matter, the current meter should not be used. Under these conditions, a weir measurement would, unquestionably, give more reliable and more accurate results. For deeper and larger streams, and where the current is swift, a meter will give a high degree of accuracy, and the expense of measuring water by its aid is usually much less than by building a weir. Where existing dams are used, it is also questionable whether current meters will not, under some conditions, give more accurate results.

It might be added that weir measurement is a more recent practice than the use of the current meter. Not until the late James B. Francis, Past-President, Am. Soc. C. E., established suitable coefficients, at Lowell, nearly fifty years ago, could a weir formula be used under proper conditions for cases demanding a high degree of accuracy. Since then, there have been developed still other accurate methods of measuring the quantity of flowing water, particularly in pipes, each method having its special sphere, within which it has advantages over the others.

Morris R. Sherrerd, M. Am. Soc. C. E.—Within the limits mentioned by the author, viz., 5%, the speaker has had quite implicit faith in the accuracy of current meters when carefully used. No doubt, the reason why weir measurement is so much oftener adopted is because of the ease with which a river can be measured at different Mr. Sherrerd.

Mr. Sherrerd. stages by its use. Obviously, if the current meter is used (as is often done to advantage where no dam exists), it is necessary to make constant observations, which take much time and great care in order to get anything like the accuracy obtained from weir measurements made on a dam having a fairly level crest; and, as the form of the cross-section of any stream changes with the rise and fall of the water surface, numerous additional observations would be necessary to obtain the flow by the use of a current meter, while only one reading for each change would be required on a dam or weir.

In the velocities recorded in Nos. 7 and 8, in Fig. 1, a peculiar result seems to have been obtained with two meters when used near the surface of the water. It will be noticed that the two velocity curves cross each other at a point less than 1 ft. from the surface; that is, according to the diagram, the Haskell meter gives a much higher velocity near the surface, while the Price meter seems to give a much lower one. Possibly the construction of the meters accounts for this variation. The upper portion of the vanes of the Haskell meter may have been just above the surface of the water, while the other meter, revolving in a horizontal plane, could be kept entirely submerged, the axis of each meter being at the same elevation.

The author says:

"The mean abscissa of each curve was then computed, which is the mean velocity for the whole cross-section. The discharge is the product of the cross-sectional area and the mean velocity."

It would seem to be necessary to divide the cross-section into horizontal layers and take a mean velocity for each horizontal layer to get the quantity. Perhaps that is what the author means, using horizontal layers of uniform thickness. These horizontal layers, however, should be quite numerous in order to insure accurate results.

Mr. Le Conte. L. J. Le Conte, M. Am. Soc. C. E. (by letter).—The writer is an admirer of the accuracy of current-meter work, within proper limits. This admiration is based on 20 years of observation and experience with these instruments, under all kinds and conditions of river and tidal-current work. Long experience has naturally brought out the strong and weak points, as adapted to every-day practice. It is proper to state that the writer's experience has been largely confined to the propeller types of meters, with two and four blades.

The good results shown by the author's experiments prove conclusively the value of meter work, as he says, "under favorable conditions," all of which is perfectly true; but, exactly at this point, the engineer's troubles begin to develop. A very large percentage of all the river gauging he has to make is not under favorable conditions. During low stages of the river the flow is quiet, the stream lines are practically parallel and are at right angles to the cross-section, the meter work is generally good and the results inspire complete confidence. Unfortunately, when the flood waters begin to come down,

the whole character of the stream flow changes; so much so, that the general down-stream velocity, which is desired, is indescribably mixed up with other turbulent velocities, which are, of course, entirely independent and of no particular interest or value for gauging purposes. The meter, by nature, cannot discriminate, and, consequently, makes a record which may be called a heterogeneous mixture of all the velocities coming to it, no matter what their direction may be with reference to the cross-section. Inasmuch as the engineer wants absolutely nothing but the down-stream velocity at right angles to the cross-section, the question arises: "How can he get it from such a meter record?" This important question has been on the *tapis* for some time past, and, apparently, is destined to stay a while longer. Mr. Le Conte.

The writer has given much time and thought to trying to devise some simple method of determining, approximately, the changes in the coefficient due to different degrees of turbulence, or "boiling" action, of the flowing waters. Some standard of comparison had to be adopted, and the loaded-pole float was selected as the best device for turbulent stream flow. By simultaneous observations with meters and pole-floats there can be obtained a series of modified coefficients, which are adapted to any particular site for all stages of river discharge. The results thus obtained are not altogether satisfactory, but, nevertheless, are decidedly better and more rational than those obtained by the general use of the standard coefficients based on small-scale experiments or canal currents.

If the engineer is to be confined to the use of the canal coefficients, then it is evident that the current meter will be very limited in its adaptability.

J. B. Lippincott, M. Am. Soc. C. E. (by letter).—Having had occasion during the past four years to use current meters of various types, the writer has made an effort to test their accuracy whenever practicable, in order to determine whether their rating had changed, and to form some general idea as to the amount of dependence that can be placed upon them. Numerous tests have been made by others with current meters under conditions that were ideal, or nearly so, to determine their accuracy, but, in the ordinary practice of an engineer, the conditions encountered are frequently far from perfect. Mr. Lippincott.

The following tests were made under such conditions as were encountered at various places in California. All tests are given, except one, on which occasion known errors existed.

Where the meter is compared with a weir, and shows a difference from the volume determined by the weir computation, it does not at all necessarily follow that the weir is correct and the meter wrong. Under perfect conditions, the weir measurement is believed to be

Mr. Lippincott. better than the meter measurement, but it has been the writer's experience that there are relatively few perfect weirs in the Southwest. Usually, where a weir is located in a canal or river, the bay rapidly silts up, and the inner depth below the lip becomes insufficient. A high velocity of approach in the water results therefrom, and has to be taken into consideration in the computations. In the writer's judgment, when this velocity of approach exceeds 1 ft. per second, the theoretical head corresponding thereto, which has to be added to the measured head, as used in the Francis formula, gives a result below the truth. When this velocity reaches 2 ft. per second, the error is serious, frequently exceeding 10 per cent. It has been observed that the meter measurements nearly always show slightly more water than the weir measurements, particularly where small volumes are measured. It is believed that this is because the slowest film of water around the perimeter of a channel cannot be measured by meter. It is believed that for small volumes of water the smaller meter gives a more accurate result than the large meter, as it more nearly can be placed in this slowest portion of flow.

Generally speaking, where the meters check each other, under favorable conditions, or check good weir measurements, within 5%, the work has been considered sufficiently accurate. Where the error exceeds 5% the meter has been re-rated. It is believed that, ordinarily, the meter measurements will check good weir measurements within 3%, and that the meters hold their rating for several years without change, provided they are handled carefully and oiled.

It is quite customary, in measuring a stream, to raise and lower the meter through the section measured. With meters of the Haskell make, with propeller-shaped blades, this can properly be done, and is the better method; but, with meters having conical-shaped cups, the vertical movement given to them by this process accelerates the revolution, and gives erroneous results. This can easily be demonstrated by raising and lowering a meter of this class in still water.

1.—January 10th, 1897. Azusa-Duarte Division Box, on San Gabriel River, California.

Large Price Electric Meter No. 67	19.93	second-feet.
Weir; Francis formula; velocity of approach, 1.6 ft. per second	19.97	" "
Weir; Bazin formula	21.50	" "

The meter measurement was made in the weir bay, which is a rectangular flume. The floor of the bay was covered with sand. The depth of water was about 2 ft., and velocity observations were taken at mid-depth only, so that the meter measurement probably shows slightly too much water.

At the weirs, the depth below the lip to the sand was 0.5 ft. The head on the weirs was 0.75 ft., the velocity of approach being 1.6 ft.

per second. The Francis formula was corrected for this. The side contractions of the weirs were poor, but were allowed for. Mr. Lippincott.

2.—June 9th, 1896. Kern River, California, at First Point of Measurement.

Large Haskell Meter;	A. K. Warren,	observer...	3 221	second-feet.	
" " "	F. McCray,	" ...	3 280	"	"
" Price "	J. B. Lippincott,	" ...	3 004	"	"

The river at this point is 345 ft. broad, with a shifting sandy bed. Measurements were made synchronously from a foot-bridge, each observer having previously rated his own meter. The first two observations were made by raising and lowering the meters, and the third by taking the top, middle and bottom velocities. The Price meters cannot be raised and lowered without accelerating their revolutions.

3.—February 10th, 1897. Caloway Canal, on Kern River.

Large Haskell Meter;	A. K. Warren,	observer....	317.4	second-feet.	
" Price "	J. B. Lippincott,	"	314.0	"	"

Bottom of channel sandy, but not shifting. Same methods used as in No. 2. This is a better gauging station than No. 2.

4.—May 24th, 1897. Vineland Tunnel, at San Gabriel River, California.

Large Price Electric Meter No. 67.....	1.88	second-feet.
Weir; Francis formula; velocity of approach, 0.8 ft. per second.....	1.90	" "

Meter measurement in rectangular flume above weir. Weir good.

5.—November 19th, 1896. Weir of West Los Angeles Water Company, near Los Angeles, California.

Large Price Electric Meter No. 67.....	7.29	second-feet.
Small " " " " 12.....	7.60	" "
Weir; Francis formula; velocity = 1 ft. per second.	7.47	" "
" Bazin formula.....	7.38	" "

Meter measurement in flume above weir.

6.—Buena Vista Reservoir, Los Angeles, California. Test of Large Price Electric Meter No. 67.

A canal in earth discharges into a small rectangular concrete reservoir with vertical sides at this point. The discharge of the canal was determined carefully by volumetric measurements, and the meter measurements were compared therewith. The volume was about 10 cu. ft. per second.

Mr. Lippincott.

May 21st, 1897.	Test, raising and lowering meter.	4.44%	excess.
" 21st, "	Top, mid-depth and bottom velocities	2.01%	"
" 26th, "	Test, raising and lowering meter.	3.17%	"
" 26th, "	Top, mid-depth and bottom velocities	4.05%	deficiency.

In the test of raising and lowering the meter, the rate of movement was ½ ft. per second. It was believed that this would give results in excess of the true flow. The section of the canal was free from weeds and large stones, but it was an earthen canal.

7.—June 6th, 1899. Measurements at weir "*I*," on Los Angeles River, California.

Small Price Acoustic Meter	18.56	second-feet.
Weir; Francis formula	18.64	" "

The weir was a good one, with the depth below the lip more than twice the head on the weir, and with a velocity of approach of less than ½ ft. per second. About 100 ft. below the weir the river was run through a rectangular wooden measuring flume, where the meter measurements were made.

May 18th, 1900. Measurements at the same place as No. 7.

Meter	16.92	second-feet.
Weir	16.75	" "

June 6th, 1900. Measurements at the same place as No. 7.

Meter	15.60	second-feet.
Weir	15.88	" "

8.—August, 1899. Meter measurements at the above-described measuring box compared with said weir.

Large Price Electric Meter No. 99	2%	excess.
" " " " " 74	6%	"
Small " " " " 137	2%	"

9.—April 21st, 1897. Verde River, at Fort McDowell, Arizona.

Large Price Electric Meter No. 67	1 336	second-feet.
" " " " " 69	1 328	" "

Channel broad and sandy.

10.—January 24th, 1898. Los Angeles River, at Bridge 1.

Large Price Electric Meter	31.64	second-feet.
Weir	31.41	" "

Meter measurement made in rectangular measuring flume. Weir fairly good.

11.—January 24th, 1898. Los Angeles River, at Bridge 2. Mr. Lippincott.

Large Price Electric Meter	55.45 second-feet.
Weir	55.92 " "

Same conditions as No. 10.

E. E. HASKELL, M. Am. Soc. C. E. (by letter).—The author has made the writer somewhat of an interested party to his paper, and, while he is greatly interested in the results exhibited, and in the subject in general, he does not feel like taking all the credit for the meter on Plate XI that bears his name. The only part of it that he can recognize as his own design, or that ever came from the shops of Messrs. E. S. Ritchie and Sons, of Brookline, Mass., the makers of his current meters, is the wheel. The remainder of it is of poor design, and, certainly, is not attractive in appearance. Mr. Haskell.

The first requisite of a good current meter is that it shall offer the minimum of resistance to the free flow of water; secondly, it must have strong directive properties; that is, a rudder large enough to make the wheel head instantly to a change of direction of current.

A close examination of Plate XI will show that the meter has been given, for a sinker, a lead or iron weight, the longest axis of which is forward of the line of support of the meter, where it acts in direct opposition to the rudder of the meter wheel. Again, this sinker has been placed within about 2 ins. of the wheel, where it will cause a disturbance of the water in close proximity to it. This sinker has been given no rudder, a feature which stands in striking contrast with the Price current meter (also on Plate XI), and also in striking contrast with the writer's meters, as shown in the authorized illustrations. The standard and the manner of making the supporting and electric connections are clumsy in appearance and clumsy in fact. The author, in presenting this meter in this connection, undoubtedly overlooked the fact that it had been remodeled. Had more careful attention been paid to details of construction in this remodeling, a better meter would have been the result.

The results of the author's comparisons of current-meter and wire discharges are of special interest from the point of furnishing a direct test of the accuracy of work done with current meters. In the majority of cases it is an expensive operation to apply a rigid test to work of this character, and, in consequence, one is seldom if ever made. It is the writer's firm conviction that, if more tests could be applied, the results would be to the credit of the meter; but the latter statement should be qualified by saying that the meter must be of good design, well made, well rated, well cared for, and used by a careful, painstaking observer.

If one wishes to make a careful study of the form of the discharge curve of a river, regardless of its size, the current meter offers the

Mr. Haskell. cheapest and, in the writer's opinion, the most satisfactory means for obtaining the data.

The writer has but little confidence in cup-wheel meters for general river gauging, except when used, as in the present case, from a firm support. It was his loss of confidence in them that led him to take a hand in the design and construction of one of the propeller-wheel type. Charles H. Miller, M. Am. Soc. C. E., in his discussion of this paper, points out their defect, and, to the writer, it is a matter of regret that Mr. Miller was not in a position to give the results of his experiments. The present forms of cup-wheel meters over-register if used from an unstable support, such as a boat. When tipped to a comparatively small angle with the direction of the current, they give a marked increase in the number of revolutions of their wheels for the same velocity of the water. The cause for this increase is not hard to find. The tilting of the meter changes the area and the form of the wheel which is presented to the current pressure, and, in consequence, its normal rating equation no longer applies. Mr. Miller states that the registrations of the Haskell meter decreased when used from the rocking boat, but in a less proportion than those of the Price meter. Propeller-wheel meters under-register when tilted, but in a very much less proportion than cup-wheel meters, for the same angle of tilt. Anyone who wishes can readily satisfy himself in regard to this point by rating simultaneously these two forms of meters. Attach each to separate vertical rods, far enough apart, so that they can in no way interfere with each other; rate them when left free to obey the action of their own rudders, and note the results. Next fasten each meter so that its longitudinal axis makes an angle of 5° with the horizontal—otherwise perfectly free—rate them, and note the results. Fasten them at angles of 10°, 15°, 20° and 25°, rating them at each position, and note the results. The writer is confident that this method will give all the data required for a thorough consideration of the subject.

The accuracy with which the discharge of a stream can be measured with current meters is, in the writer's opinion, dependent upon the size and character of the stream and the completeness of the programme of work. The author's results show the precision attained in his comparisons, with his programme, which was a comparatively simple one. A little more attention paid to a better determination of the velocity curves should readily reduce the residuals, for the measurement of the flow of so small a canal, to less than ½ of 1 per cent. The Niagara River, and similar streams of comparatively permanent regimen, can be measured, with residuals not exceeding 2% of their flow. The discharge of the Lower Mississippi River, with its ever-changing conditions, can be measured, with residuals not exceeding 5% of its flow.

Mr. Murphy. EDWARD C. MURPHY, Assoc. M. Am. Soc. C. E. (by letter).—The writer is pleased to learn that those who have used the current meter

for years believe that "under the conditions usually existing in river- discharge work, and with the proper amount of care, results are obtained that are within 5% of the true values." Mr. Murphy.

Among "favorable conditions" the writer would not include the shallow depths and low velocities, mentioned by Mr. Hering, or flood discharge, referred to by Mr. Le Conte. The writer's current-meter and weir-discharge comparisons for low velocities* show clearly that, for velocities of less than 1 to $\frac{3}{4}$ ft. per second, the meter discharge is less than the corresponding weir discharge, and this difference increases rapidly as the velocity decreases.

He is well aware of the fact that on some streams it is difficult to find a good discharge section, and also that a section may be a good one for some river stages, and a very poor one for others. Ordinarily, it is the extreme stages that give trouble; fortunately, these do not often occur. It frequently happens that two sections not very far apart can be used to advantage, one being favorable for discharge measurements at low river stages, the other for higher stages.

He has no means of knowing whether what he has called an error, in the Small Price meter work of April 12th, was due to a change in the rating of the meter on that day, as suggested by Mr. Miller, or to some other cause. If it was due to a change of rating, it serves to emphasize the fact that frequent ratings are necessary when a high degree of accuracy in current-meter work is desired.

Mr. Sherrerd has called attention to the fact, noticed by the writer about a year ago while using two kinds of meters at a time, that the Small Price meter indicated a velocity less than the true velocity when held near the surface of the water. The writer has measured the velocity near the surface simultaneously with the Small Price meter, the Haskell meter, surface-floats, and loaded 6-in. cubical floats. These experiments show clearly that when the Small Price meter is held with its center within 1 ft. of the surface, and the moving head wholly submerged, the velocity shown by it is less than the true velocity. The error is about the same, whether the meter is held with a cable and is free to tip, or is held rigidly with a rod.

Mr. Sherrerd inquires as to the necessity for dividing the cross-section into horizontal strips, and taking a mean velocity for each horizontal layer, to get the mean velocity in the section. This is not necessary, inasmuch as the cross-section is a rectangle.

Let F_1, F_2, F_3, etc., be the areas of horizontal strips, of equal width, of the total discharge area;

v_1, v_2, v_3, etc., the mean velocity in these strips;

F the total discharge area;

v_m the mean velocity in this area;

and Q the discharge, in cubic feet per second.

* To be published as "Water Supply and Irrigation Paper, No. 62," United States Geological Survey.

Mr. Murphy.

Then $Q = F_1 v_1 + F_2 v_2 + F_3 v_3 + \ldots + F_n v_n$.

As the strips are of equal width, and the canal section is rectangular, $F_1 = F_2 = F_3$, etc., and

$Q = F_1 (v_1 + v_2 + v_3 + \ldots + v_n) = F_1 n v_m = F v_m$.

That is, the discharge is equal to the product of the discharge area by the mean abscissa of the vertical velocity curve of that area.

In the closing paragraph of his discussion, Mr. Le Conte calls attention to the fact that in the use of the current meter the engineer is at the present time limited to canal coefficients. The tests of accuracy of current-meter work have thus far been made in canals where the conditions could be easily controlled by the experimenter, and the conditions affecting accuracy easily described. It is highly desirable, but very difficult and expensive, to make similar tests on rivers. The writer has under way, on a small scale, a study of the relative accuracy of current-meter methods as applied to streams of moderate and small size, and hopes in the near future to be able to throw some additional light on this subject.

The writer cannot agree with Mr. Lippincott, that "current meters hold their rating for several years without change, provided they are handled carefully and oiled." His experience is that, with the best of care, the small meters change their rating, and need frequent rerating. This change may be due to either increase or decrease in friction. The change in rating may not be very noticeable for the higher velocities, but is very noticeable for the lower ones.

The writer is pleased to have on record the meter and weir, and the meter and meter, comparisons made by Mr. Lippincott. He thinks, however, that a full description of all the conditions under which the comparisons were made (cross-section, greatest, least and mean velocity; distance of section from weirs, piers, bends or other disturbing influences) and methods used (to measure depths, surface fluctuations, revolutions of meter wheel, and for computing discharge) should be given. The value that an engineer will attach to such comparisons depends on his knowledge of the accuracy of measurement of each factor on which the discharge depends.

The writer's experience with the "integration method" is that it is not as accurate as the "point method." Some of his discharge measurements by the zigzag integration method check the corresponding weir discharge closely, while others do not. The vertical velocity-curve work shows that fluctuations in velocity near the bottom are greater than at any other place in a vertical, and hence the bottom velocity (with the center of the meter $\frac{1}{2}$ ft. above the bottom) is not a good function to use in measuring discharge.

The writer desires to disclaim any intention of making any person an "interested party" to his paper. He has avoided in this paper a detailed comparison of results by different meters. The meter referred

Mr. Murphy.

to by Mr. E. E. Haskell came to him under the name "Haskell meter," and he has referred to it by this name. It has been used suspended with a cable, with the revolving head near to the lead sinker, in order to obtain velocities as close to the bottom as possible. It has also been held with a rod, without a sinker, in which case the axis of the meter was parallel to the axis of the canal, independently of the rudder.

It was not the aim to use extreme accuracy in these tests. With greater care taken to maintain a constant depth, and more time spent in measuring the velocity in each vertical, a slightly greater degree of accuracy would have been attained. What was wanted was the degree of accuracy attainable by different methods of operating meters when operated quickly in a simple manner.

Mr. Haskell states that, "A little more attention paid to a better determination of the velocity curves should readily reduce the residuals for the measurement of flow of so small a canal to less than ½ of 1 per cent." In Mr. F. C. Shenehon's report to Mr. E. E. Haskell on his ratings of current meters, in connection with measurements of discharge of the Niagara River,* the following statement may be found:

"This latter was stretched over five days in an attempt to trace the cause of what has appeared elsewhere as a diurnal variation of the rating constants. The several days, however, failed to show any considerable change, and the observations almost deny what is clearly indicated throughout all the two-meter work, and what other ratings made by myself and other observers show to be a fact. This variation of 1 or 2% on either side of a mean rating line is baffling in the extreme."

If there is an uncertainty in observed velocity of 1 to 2%, due to daily changes in the meter constants of a genuine Haskell meter, there appears to be very little hope of reducing the residuals of flow of "even a small canal" to less than 1 or 2 per cent. If the meter work on the Niagara River "clearly indicates" a daily variation of 1 or 2% in the meter constants (as stated above), and a consequent possible error of 1 or 2 % in the measured velocity and discharge, from one source of error alone, how is it possible to reduce the residual errors to less than 2% when all sources of error are taken into account?

When an engineer sees, from his ratings of current meters, that there may be an error of 1 to 2% in his velocity measurements, from daily changes in the meter constants, is he justified in incurring the expense necessary to attain extreme accuracy in some details? Is "a chain stronger than its weakest link?" Why make some links very strong, when it is known that others are necessarily weak?

* Annual Report of Chief Engineer, U. S. Army, 1900, p. 5338.

AMERICAN SOCIETY OF CIVIL ENGINEERS.

INSTITUTED 1852.

TRANSACTIONS.

Paper No. 913.

A PROPOSED SOLUTION OF SOME HYDRAULIC PROBLEMS.*

By Charles H. Tutton, M. Am. Soc. C. E.

With Discussion by Messrs. Clemens Herschel, I. P. Church, Edward C. Murphy and Charles H. Tutton.

In the paper by Messrs. Gardner S. Williams, Clarence W. Hubbell and George H. Fenkell,† among other propositions, the theory is advanced that the true form of the velocity curve of the water flowing in a pipe is an ellipse. In the discussion which followed that paper, the writer presented a new theory of the viscous flow of liquids, and, assuming that the pressure, as measured by the difference in heights observed in two circumferential piezometers, could be considered as applying over the entire cross-section of the pipe, deduced therefrom that the true velocity curve is an ellipse.

* Presented at the meeting of December 18th, 1901.

† Page 1 of this volume of *Transactions*.

In the present paper the same theory will be applied to open channels, and an attempt will be made to solve determinately the much-discussed but never-answered question, of why the maximum velocity is found below the surface in such channels. The other discussion forms but a particular case of the present; therefore, even at the risk of being tiresome, some of the statements therein made will be repeated here, but they will be much more fully developed, in order to make the explanation complete. Others, again, must be modified.

As a preliminary step, let it be assumed that uniform motion exists in the stream, meaning by this that the motion along each fillet is uniform and parallel to the longitudinal axis of the stream, and that the total resistance to motion is composed of two entirely different, yet mutually dependent, classes of resistance, one being the so-called friction along the bed and sides of the stream, the other, the internal resistances in the liquid itself, or the so-called viscosity.

The method generally used in the textbooks of to-day, in developing a formula for the flow of a liquid in open channels, is to consider the mass of liquid as a body of invariable volume sliding down an inclined plane under the action of its own weight, this assumption being the foundation of the $v = c\sqrt{R\,S}$ formula. The usual interpretation of this supposition is so restricted that it gives correct results only in the case when the axis of the curve which represents the curve of mean velocity is found in the free surface. The maximum velocity would then be found in this axis, but, as a matter of observed fact, the maximum velocity is seldom or never found there. One of the objects of this paper is to prove that this is necessarily so, which can be done by extending the generality of interpretation of the above, or by an entirely independent method.

The First or Ordinary Method.—The usual method of treating this problem may be stated as follows: In a stream in which the surface $A\,B$, and bed, $C\,D$, are parallel (Fig. 1), let a mass of fluid of density ρ be flowing with uniform motion, then, if a prism be considered of width unity, length l, and depth z, its weight W will be $\rho\,g\,l\,z$. On such a prism, referred to axes parallel and perpendicular to the free surface, the end pressures p_o and p are equal, being respectively equal at equal depths, and hence

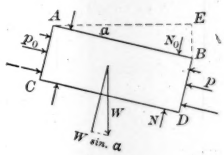


FIG. 1.

annul each other. The normal pressures, N_o on the surface and N on the bed, need not be considered, as fluid friction is supposed to be independent of normal pressure; therefore, if $B\ A\ E = \alpha$ be the angle which the surface makes with the horizontal,

$$W \sin.\ \alpha = W\ S = \rho\ g\ l\ z\ s$$

would be the component of the weight of this body of fluid in the direction of motion, which must be in equilibrium with the friction on the bed, or the viscosity between plane layers (modified by their motion) at a depth z from the free surface.

The writer objects to this statement, and, most emphatically so, to the limitation "from the free surface," which, although not always expressed, is always understood.* This "component of the weight" is an erroneous statement of the case, necessitated by the axes used, and it is the use of this expression which has caused confusion. The writer considers that this component applies, not to the aggregate mass as measured from the free surface, but to each individual and elementary fillet whatever its position; in other words, that this component represents a uniform pressure, $\rho\ g\ l\ s$, over the entire end, $A\ B$, of the prism, and is the same per unit of area over any fillet wherever located, and that $\rho\ g\ s\ l\ z$ results, simply because of this pressure being taken over an aggregate of area z, not because it is at any fixed depth below the free surface or below any other point, but because of the number of fillets considered.

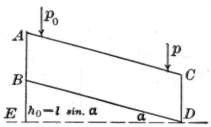


FIG. 2.

The Other Method.—Bernouilli's Theorem states that in any stream, such as represented in longitudinal section in Fig. 2 by $A\ B\ C\ D$, that at the two end sections, $A\ B$ and $C\ D$,

$$\frac{v_o^2}{2\ g} + \frac{p_o}{W} + h_o = \frac{v^2}{2\ g} + \frac{p}{W} + h,$$

provided the motion be uniform, the fluid be incompressible and friction be disregarded; in other words, the sum of the velocity, pressure and potential heads is constant. In the figure the arrows marked p_o and p represent the atmospheric pressure, which is constant, and therefore equal. The depths of liquid, $A\ B$ and $C\ D$, are equal, the top and bed slopes being parallel, hence the totals of the pressures developed by the weight of the water are equal. As, by supposi-

* Dupuit ("Mouvement des Eaux," p. 3), specifically states this. He also quotes Prony to the same effect.

tion, the velocity is uniform, therefore the velocity heads must be equal. In order, then, that Bernouilli's Theorem be true, as the velocity and pressure heads annul each other, the potential heads must be equal. Now, the potential head in this case would be the height of the bed of the stream above any arbitrary horizontal plane of reference, necessarily horizontal as gravity acts in vertical lines. Taking, then, this plane through the point D, h would be zero, but h_o, instead of becoming zero with it, becomes equal to $l \sin. \alpha$. Then, as h_o has not disappeared with h, it follows that resistances must exist, and also, in order that the motion be uniform, these resistances, as developed in the stream and along its beds and sides, must be in equilibrium with this head; in other words, the motive force is due to a column of water $l \sin. \alpha$ high,* acting over the end section, $A\ B$, of the stream, and it is this pressure, applied to the end of each and every individual fillet, which is in equilibrium with the viscous and frictional resistances. This pressure in the line of direction of motion would be $\rho\ g\ l \sin. \alpha$ over each individual fillet of the entire end section; or, if the fillet have an area a, it would be $\rho\ g\ l \sin. \alpha\ a$, immediately reverting to the form of the weight of the fillet, but showing in this case that the particular position of the fillet has no bearing on the question, but that $\rho\ g\ l \sin. \alpha$ is an end pressure, the equation taking the form of weight simply because it is multiplied by the area of the fillet.

There are other ways of proving this, but, believing it to be sufficiently demonstrated by the preceding, there results the following hitherto unnoticed law:

Law.—The cause of motion in a liquid is due to pressure on the end section, which pressure is that due to a head of liquid measured by $l \sin. \alpha$, uniform over the section, and normal in direction to the line of action of gravity, being also the same on a plane normal to the line of flow, by a well-known principle of fluid motion.

The above pressure applied over a given area a is equivalent to the weight of a mass of liquid of length l and area a; nevertheless, it is the pressure on, and not the weight of, the liquid fillet itself, that causes the motion. This is in strict conformity with the process used in the previous discussion, while the ordinary assumption is not.

* Particular attention is called to this, as showing that motion results from the surface slope, the bed slope having nothing to do with it. It is believed that this was first claimed in 1836 by M. Pierre Vauthier, Ingenieur en Chef des Ponts et Chaussées.

Viscous Resistance.—Next consider the resistances due to viscosity. In the previous discussion it was simply stated, "let it be assumed that what has been called the 'rate of distortion' of the liquid, varies not only in direct ratio to the distance of the filament from the center of the tube, but also with the velocity of that filament." This but partially expresses the general law, and a criticism by Mr. James A. Seddon* leads the writer to suppose that this statement proved to be misleading. To place the matter in a clear light, the following explanation† is given:

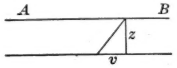


FIG. 3.

"Suppose a liquid to be flowing between two plates (Fig. 3), and F be the force necessary to move it, A being the area of the plate over which the force acts. Further, suppose the force to vary uniformly with its distance from one of the plates. Then $F = f A$, in which f is proportional to $\frac{v}{z}$, or if ε be the coefficient of viscosity, $f = \varepsilon \frac{v}{z}$, and f will be the 'rate of distortion' of the liquid."

Writing this in the form $f = \varepsilon \frac{d\,v}{d\,z}$, it is seen that $\frac{d\,v}{d\,z}$ is simply a ratio, and this is Navier's supposition for viscous resistance, adopted by all writers on capillary flow, as Helmholtz, Stokes, Lamb, Bassett and others, and which, developed, results in the parabolic theory of flow. The writer assumes that f varies also with the velocity of the filament, or $f = \varepsilon \frac{v\,d\,v}{d\,z}$, then, if $A\,B$ be a thread of maximum velocity, whether it be in the center of a cylinder, in the surface of a stream, or anywhere else in the stream, the stress on any other fillet, per unit of length, will be $\varepsilon \frac{v\,d\,v}{d\,z}$, in which z is the distance of the filament considered from the axis of maximum velocity. From the foregoing definition, this must be measured by $\rho\,g$.

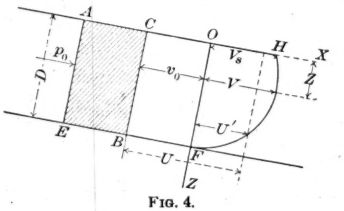


FIG. 4.

General Formula. — Let the shaded portion in Fig. 4 represent a vertical longitudinal section of a portion of a stream supposed to be of indefinite width, and let the thickness of the prism considered,

* *Transactions*, Am. Soc. C. E., Vol. xlvii, p. 238.

† Due, in part, to Messrs. Deeley and Wolff, in *The Engineer* of January 1st, 1897.

i. e., perpendicular to the plane of the paper, be taken as unity. Also assume co-ordinate axes, one of which lies in the free surface and the other perpendicular to it, the origin being taken at O. Suppose, further, that a thread or plane of maximum velocity exists at a depth Z from the surface of the stream. The distance of any filament from this axis may then be referred to the surface by writing $z - Z$ for z in the equations deduced with the axis of maximum velocity as one of the co-ordinate axes. The resultant pressure on any point of the end section of the stream has been shown to be that due to a column of water of height $l \sin. \alpha$, or $s\,l$ in the direction of motion, where s has the ordinary signification of difference in level between end sections divided by their distance apart; that is to say, the fall per unit of length. The pressure on the end of any prism will then be $\rho\,g\,s\,l$, and the area of an aggregation of such prisms being $z - Z$, as referred to the surface, as the width has been assumed to be unity, the total pressure will be

$$\rho\,g\,s\,l\,(z - Z) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

The viscous resistances will cause the consecutive prisms of fluid, parallel to the axis of maximum velocity, to slide upon each other independently of the sliding of the total mass upon its bed, in such a manner that the development of the motion per unit of time (usually a second), or what has been termed in the previous discussion the "velocity solid," can be represented by the lighter portion $C\,B\,F\,H\,O$, in which $C\,B\,F\,O$ represents the motion of the entire prism $A\,C\,B\,E$ along the bed of the stream, and which is resisted by friction on the bed only, and $O\,F\,H$ represents the excess of motion due to the liquid prisms sliding upon each other during that unit of time, or the variation in motion caused by the viscous resistances.

Velocity Curve.—Consider, first, the viscous resistances which cause the velocity curve.

Assuming that the viscous resistances between filaments per unit of area is proportional to $\varepsilon \dfrac{v\,dv}{dz}$, in which ε represents the so-called coefficient of viscosity, and which expresses that the viscosity varies both with the velocity and with the rate of distortion of the liquid as referred to the axis of maximum velocity; also, considering ε of the same nature as the frictional coefficient in so far that it may be represented as a force, and introducing a factor D^m to take into consideration the actual depth of the stream, the frictional resistance over the

length l between consecutive filaments at the distance z from the thread of maximum velocity will be $\rho g D^m l \varepsilon \frac{v}{2g} \frac{d(v_o + v)}{dz}$, or, as referred to the surface of the stream, and observing that $d(v_o + v) = dv$,

$$\rho g D^m l \varepsilon \frac{v}{2g} \frac{dv}{d(z - Z)}$$

Adding this to (1), the equation of equilibrium results

$$\rho g D^m l \varepsilon \frac{v}{2g} \frac{dv}{d(z - Z)} + \rho g l s (z - Z) = 0$$

whence $$v\,dv = -\frac{2gs}{\varepsilon D^m}(z - Z)\,d(z - Z).$$

Writing c for $\frac{2gs}{\varepsilon D^m}$, and integrating,

$$v^2 = -c(z - Z)^2.$$

But, when $z = Z$, v becomes the maximum velocity, V, or

$$V^2 - v^2 = c(z - Z)^2 \quad \ldots\ldots (2)$$

and when $z = D$, assuming no slip due to viscous resistances, $v = 0$, or

$$V^2 = c(D - Z)^2 \quad \ldots\ldots (3)$$

From this, $c = \frac{V^2}{(D-Z)^2}$, which value, substituted in (2), gives, after easy reductions,

$$\frac{v^2}{V^2} + \frac{(z - Z)^2}{(D - Z)^2} = 1 \quad \ldots\ldots (4)$$

showing the curve of velocity to be an ellipse, the semi-axes of which are maximum velocity and the distance of that maximum velocity from the bed of the stream.

This may be said to apply strictly only to that portion of the curve below the axis of maximum velocity. To simplify the discussion, as the actual resistance between water and air is unknown, it will be assumed as a liquid resistance, and that the same curve holds above the axis of maximum velocity.

To simplify the results, let $Z = n D$, thus expressing the depth of axis as a percentage of total depth, also let V_s = surface, V = maximum, $V_{\frac{1}{2}D}$ = mid-depth, and U' = mean, velocity in the elliptic head, z_u being the depth of the mean velocity.

Making $z = 0$ in Equation (2), v becomes V_s, and

$$V^2 - V_s^2 = c Z^2 = c D^2 n^2 \quad \ldots\ldots (5)$$

From (3) $$V^2 = c(D - Z)^2 = c D^2 (1 - n)^2 \quad \ldots\ldots (6)$$

Subtract (5) from (6) $$V_s^2 = c[(D - Z)^2 - Z^2] = c D^2 (1 - 2n) \quad \ldots\ldots (7)$$

$$\frac{V}{V_s} = \frac{1 - n}{\sqrt{1 - 2n}} \quad \ldots\ldots (8)$$

For the mean velocity in the elliptic head

$$U' = \frac{1}{D}\int_o^D v\,dz = \frac{1}{D}\int_o^D \sqrt{V_s^2 - c\,[(z-Z^2) - Z^2]}\,dz$$

$$= \frac{1}{D}\sqrt{c}\int_o^D \sqrt{(D-Z)^2 - (z-Z)^2}\,dz$$

integrating which

$$U' = D\sqrt{c}\,\frac{1}{2}\left\{ n\sqrt{1-2n} + (1-n)^2\left(\frac{\pi}{2} + \text{sin.}^{-1}\frac{n}{1-n}\right)\right\}. \quad (9)$$

which will be abbreviated by placing

$$\frac{1}{2}\left\{ n\sqrt{1-2n} + (1-n)^2\left(\frac{\pi}{2} + \text{sin.}^{-1}\frac{n}{1-n}\right)\right\} = M \ldots\ldots (10)$$

when it becomes $$U' = \sqrt{c}\,D\,M. \ldots\ldots\ldots\ldots (11)$$

The depth of this mean velocity is found by placing this value in the place of v in Equation (4), then substituting the value of V from Equation (6), and there results

$$Z_a = n \pm \sqrt{(1-n)^2 - M^2} \ldots\ldots\ldots\ldots (12)$$

For mid-depth velocity there is found in the same manner

$$V_{\frac{1}{2}D} = \frac{1}{2}\sqrt{c}\,D\sqrt{3-4n} \ldots\ldots\ldots\ldots (13$$

It must be remembered that all of that which precedes under this heading relates solely to the elliptic head.

Motion Along the Bed.—For the portion *C O B F*, in which the velocity is not only uniform, but equal in each fillet, and the motion of which is resisted by the friction on the bed alone, if the ordinary assumption be made that the resistances are proportional to the square of the velocity, the equation of equilibrium per unit of length will be, f being the frictional resistance expressed as a force,

$$\rho\, g f \frac{v_o^2}{2g} = \rho\, g\, D\, S,$$

whence $$v_o = \sqrt{\frac{2g}{f}\,D\,S} \ldots\ldots\ldots\ldots (14)$$

This equation would obtain if the mass of liquid to which it applies were considered as a body sliding down an inclined plane.

General Equations for Flow.—In the value of c in Equations (5) to (13) the different forms of equation arising from giving m values other than unity have been shown in the previous discussion. It may also be stated that a change in this value would only change the absolute

value of ε without affecting the area of the curve $O\,M\,F$ or the total volume in the elliptic head. Assuming then $m = 1$, replacing c by its value in Equation (11), and adding it to (14), there results, for total mean velocity in the vertical considered,

$$U = \sqrt{2\,g\,D\,S}\left\{\sqrt{\frac{1}{f}} + M\sqrt{\frac{1}{\varepsilon}}\right\} \quad \ldots\ldots\ldots\ldots \quad (15)$$

and similarly for the

$$\text{Total surface velocity} = \sqrt{2\,g\,D\,S}\left\{\sqrt{\frac{1}{f}} + \sqrt{1-2n}\sqrt{\frac{1}{\varepsilon}}\right\} \quad \ldots\ldots (16)$$

$$\text{Total maximum velocity} = \sqrt{2\,g\,D\,S}\left\{\sqrt{\frac{1}{f}} + (1-n)\sqrt{\frac{1}{\varepsilon}}\right\} \quad \ldots\ldots (17)$$

$$\text{Total mid-depth velocity} = \sqrt{2\,g\,D\,S}\left\{\sqrt{\frac{1}{f}} + \tfrac{1}{2}\sqrt{3-4n}\sqrt{\frac{1}{\varepsilon}}\right\} \quad (18)$$

$$\text{Depth of mean velocity} = n \pm \sqrt{(1-n)^2 - M^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (19)$$

Equations (15) to (19) comprise the fundamental formulas for the flow of water in a vertical section of a stream of indefinite width, and define the velocity curve. The values of the different elements will be found in Table No. 1.

Momentum and Kinetic Energy.—Consider next the change in momentum and kinetic energy of this stream as compared with the same had all the fillets the mean velocity U. This discussion, which is a model for clearness of presentation, is taken from Flamant's "Hydraulique."

In any stream let a be the area of the transverse section, $d\,a$ be an element of the same, u the mean local velocity corresponding to this element, which velocity is supposed to be parallel to the general direction of motion, or so nearly so that the deviation in direction may be neglected, and U the velocity in the whole section defined by the equation,

$$U = \frac{1}{a}\int_a u\,d\,a,$$

the integral being taken over all of the elements $d\,a$ of the section. Calling Q the total discharge per unit of time, and $d\,Q$ its element, there results

$$d\,Q = u\,d\,a,$$

and

$$Q = U\,a = \int_a u\,d\,a \quad \ldots\ldots\ldots\ldots\ldots\ldots (20)$$

TABLE No. 1.—Elements of Elliptical Head, also Giving Momentum and Kinetic Energy.

n. (1)	$1-n$. (2)	$\sqrt{1-2n}$. (3)	$\sqrt{3-4n}$. (4)	$\frac{1}{3}\sqrt{3-4n}-M$. (5)	z_u. (6)	z_u'. (7)	M. (8)	η. (9)	κ. (10)	$1+3\eta+\kappa$. (11)
—0.200	1.200	1.1832	1.9494	0.0827	0.6027	.....	0.8920	0.0892	—0.0369	1.2406
—0.175	1.175	1.1619	1.9235	0.0823	0.6042	.....	0.8795	0.0882	—0.0267	1.2378
—0.150	1.150	1.1402	1.8974	0.0820	0.6059	.....	0.8667	0.0872	—0.0266	1.2351
—0.125	1.125	1.1180	1.8708	0.0817	0.6077	.....	0.8537	0.0863	—0.0266	1.2322
—0.100	1.100	1.0954	1.8439	0.0815	0.6096	.....	0.8405	0.0853	—0.0266	1.2293
—0.075	1.075	1.0724	1.8166	0.0812	0.6117	.....	0.8271	0.0843	—0.0266	1.2262
—0.050	1.050	1.0488	1.7888	0.0810	0.6140	.....	0.8134	0.0831	—0.0265	1.2229
—0.025	1.025	1.0247	1.7607	0.0808	0.6164	.....	0.7995	0.0820	—0.0265	1.2195
0.0	1.000	1.0000	1.7320	0.0806	0.6190	.....	0.7854	0.0808	—0.0264	1.2158
0.025	0.975	0.9747	1.7029	0.0805	0.6218	.....	0.7710	0.0795	—0.0264	1.2121
0.050	0.950	0.9487	1.6733	0.0804	0.6249	.....	0.7563	0.0781	—0.0263	1.2081
0.075	0.925	0.9219	1.6432	0.0803	0.6283	.....	0.7413	0.0767	—0.0262	1.2038
0.100	0.900	0.8944	1.6124	0.0802	0.6319	.....	0.7260	0.0752	—0.0261	1.1994
0.125	0.875	0.8660	1.5811	0.0802	0.6360	.....	0.7103	0.0736	—0.0259	1.1947
0.150	0.850	0.8367	1.5492	0.0803	0.6404	.....	0.6943	0.0719	—0.0258	1.1898
0.175	0.825	0.8062	1.5166	0.0804	0.6453	.....	0.6778	0.0701	—0.0256	1.1846
0.200	0.800	0.7746	1.4832	0.0806	0.6507	.....	0.6610	0.0682	—0.0253	1.1792
0.225	0.775	0.7416	1.4491	0.0810	0.6567	.....	0.6436	0.0662	—0.0251	1.1735
0.250	0.750	0.7071	1.4142	0.0814	0.6634	.....	0.6257	0.0641	—0.0247	1.1676
0.275	0.725	0.6708	1.3784	0.0819	0.6710	.....	0.6073	0.0620	—0.0244	1.1615
0.300	0.700	0.6325	1.3416	0.0826	0.6795	.....	0.5882	0.0597	—0.0238	1.1553
0.325	0.675	0.5916	1.3038	0.0835	0.6890	.....	0.5684	0.0574	—0.0231	1.1492
0.350	0.650	0.5477	1.2649	0.0846	0.6999	0.0001	0.5478	0.0552	—0.0223	1.1434
0.375	0.625	0.5000	1.2247	0.0861	0.7122	0.0378	0.5262	0.0530	—0.0214	1.1377
0.400	0.600	0.4472	1.1832	0.0881	0.7263	0.0737	0.5035	0.0518	—0.0203	1.1350
0.425	0.575	0.3873	1.1402	0.0906	0.7424	0.1076	0.4795	0.0512	—0.0189	1.1346
0.450	0.550	0.3162	1.0954	0.0939	0.7607	0.1393	0.4538	0.0520	—0.0167	1.1393
0.475	0.525	0.2236	1.0488	0.0990	0.7826	0.1674	0.4254	0.0590	—0.0176	1.1595
0.500	0.500	0.0000	1.0000	0.1073	0.8095	0.1905	0.3927	0.0808	—0.0264	1.2158

Column 5, not explained in the text, is $V_{\frac{1}{3}D} - U$.

Comparing the momentum and kinetic energy of this stream with what it would be if all the fillets had, in place of their individual velocities, u, the mean velocity, U, and calling the weight of a unit of volume of the liquid δ, the mass flowing in a unit of time across the element $d\,a$ will be $\frac{\delta}{g}\,d\,Q$, and the momentum and kinetic energy corresponding to the liquid passing through that element will be, respectively,

$$\frac{\delta}{g}\,d\,Q\,u, \text{ and } \frac{\delta}{g}\,d\,Q\,\frac{u^2}{2}.$$

Replacing $d\,Q$ by its value, $u\,d\,a$, and summing these quantities over the entire section, the resulting momentum and kinetic energy will be

$$\frac{\delta}{g}\int_a u^2\,d\,a, \text{ and } \frac{\delta}{2g}\int_a u^3\,d\,a,$$

while, had these fillets the mean velocity U, the corresponding quantities would have been

$$\frac{\delta}{g} Q U, \text{ and } \frac{\delta}{2g} Q U^2,$$

or, placing for Q its value $U a$,

$$\frac{\delta}{g} U^2 a, \text{ and } \frac{\delta}{2g} U^3 a.$$

Comparing for momentum the quantities $\int u^2 d a$ and $U^2 a$, and for kinetic energy the quantities $\int u^3 d a$ and $U^3 a$, let u_1 be the excess, positive or negative, of the individual velocity u of any fillet over the mean velocity U, that is

$$u = U + u_1 \ldots\ldots (21)$$

Then

$$u^2 = U^2 + 2 U u_1 + u_1^2 \ldots\ldots (22)$$

$$u^3 = U^3 + 3 U^2 u_1 + 3 U u_1^2 + u_1^3 \ldots\ldots (23)$$

Now multiplying (21) by $d a$ and integrating over the entire section a

$$\int_a u d a = U a + \int_a u_1 d a,$$

which, from the definition of mean velocity (20), gives

$$\int_a u_1 d a = 0$$

Multiplying (22) and (23) by $d a$, integrating and substituting this result,

$$\int u^2 d a = U^2 a + \int u_1^2 d a,$$

$$\int u^3 d a = U^3 a + 3 U \int u_1^2 d a + \int u_1^3 d a,$$

consequently, the ratio of the actual momentum to that which corresponds to the mean velocity is

$$\frac{\int u^2 d a}{U^2 a} = 1 + \frac{\int u_1^2 d a}{U^2 a} \ldots\ldots (24)$$

and the ratio of the kinetic energies

$$\frac{\int u^3 d a}{U^3 a} = 1 + 3 \frac{\int u_1^2 d a}{U^2 a} + \frac{\int u_1^3 d a}{U^3 a} \ldots\ldots (25)$$

which will here be represented by

$$1 + \eta \text{ and } 1 + 3\eta + \varkappa, \text{ respectively} \ldots\ldots (26)$$

Flamant considers $\varkappa$ negligible, but here it will not be so considered. This discussion establishes the remarkable fact that on account of the inequality of velocity in the fillets, both the momentum and the kinetic energy of the stream are greater than they would be, had all the fillets the same velocity.

Applying these principles to the problem under consideration, and remembering that $m = 1$, it has been shown that

$$v = \sqrt{c}\ \sqrt{(D - Z)^2 - (z - Z)^2}$$

and

$$U_1 = \sqrt{c}\ M$$

whence

$$u_1 = v_o + v - (U_1 + v_o) = v - U_1 = \sqrt{c}\left\{\sqrt{(D - Z)^2 - (z - Z)^2} - M\right\}. \quad (27)$$

Substituting $Z = n\ D$, multiplying by $d\ z$, and changing the limits of integration from D and 0 to 1 and 0,

$$\int u_1^2\, d\ a = c \int_o^1 \left\{\sqrt{(1 - n)^2 - (z - n)^2} - M\right\}^2 d\ z$$

$$U^2\ a = (v_o + U_1)^2 = c\ (1 + M)^2, \text{ provided } f = \varepsilon.$$

It will be quite as general to consider the elliptic head only, at present, in which case $U^2\ a = c\ M^2$

then, by a simple integration,

$$\eta = \frac{1}{3\ M^2}\ (2 - 3\ n) - 1 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (28)$$

and similarly

$$\varkappa = \frac{1}{4\ M^2}\ (3\ n^2 + 6\ n - 5) + \frac{n}{4\ M^3}\ (1 - 2\ n)^{\frac{3}{2}} + 2 \ldots \quad (29)$$

$$1 + 3\ \eta + \varkappa = \frac{3}{4\ M^2}\ (1 - n\ )^2 + \frac{n}{4\ M^3}\ (1 - 2\ n)^{\frac{3}{2}} \quad \ldots\ldots\ldots\ldots\ldots \quad (30)$$

Application to a Stream of Indefinite Width.

Relation of f to ε.—The formula for mean velocity in a vertical section has been shown to be

$$U = \sqrt{2\ g\ D\ S}\left(\sqrt{\frac{1}{f}} + M\sqrt{\frac{1}{\varepsilon}}\right) \quad \ldots\ldots\ldots\ldots\ldots \quad (31)$$

in which M is dependent on the position of the axis of maximum velocity. Should the case exist when $f = \varepsilon$, it is readily seen that M expresses the ratio of the mean velocity in the elliptical head to the velocity v_o, this latter velocity being taken as 1.

Assume, as a first illustration, that the maximum velocity in the elliptic head is equal to the velocity v_o. Then the two portions of velocity in Equation (17) are equal,

$$\text{whence } \varepsilon = f\ (1 - n)^2$$

and the viscous resistances can only equal the frictional resistances when $n = 0$, or when the axis of maximum velocity is in the surface and $z_u = 0.619\ D$. This is the condition, and the only one, to which the ordinary assumption of a body sliding down an inclined plane is

applicable. All its formulas result from making $n = 0$ in Equations (15) to (19); and, from Equation (10), M would become $\frac{\pi}{4}$.

Suppose, as a second case, and a common case of observation, that the mean velocity in the head is two-thirds of that of v_o (which, by the way, is similar to the pipe formula), then, in Equation (31) there must exist the relation $\frac{2}{3}\sqrt{\frac{1}{f}} = M\sqrt{\frac{1}{\varepsilon}}$, whence $f = \frac{4\varepsilon}{9\,M^2}$.

If $f = \varepsilon$, then must result $M = \frac{2}{3}$. But, from Table No. 1, this value of M corresponds to a depth of axis at two-tenths nearly, and $z_u = 0.651\ D$.

If, as a third case, the maximum velocity lies at 0.1 depth, then, if $f = \varepsilon$, the mean velocity in the head is equal to $0.726\ v_o$, or, the maximum velocity in the head equals $0.9\frac{?}{?}\ v_o$, with $z_o = 0.632\ D$.

From these and similar results the following general deductions are apparent.

1.—Pulsation, or rise and fall of mean and maximum velocity in rivers, is caused by the variation of temperature in the water, which causes different degrees of viscosity between the filaments, and the pulsations themselves are due to the struggle for equality between the resistances due to viscosity, and those due to fluid friction in the bed and sides of the stream.

2.—It follows that the axis of maximum velocity is not always necessarily in the geometrical center of a pipe.*

3.—If viscosity be constant for a given temperature, it would follow that in a rough channel the elliptic head would be relatively longer than in a smooth channel, although the total motion would be less, not understanding, in this connection, roughness as creating eddies and absorbing energy, but rather one the coefficient of friction of which is higher.

Relation of Mean to Mid-Depth Velocity.—Considering $f = \varepsilon$, the three cases above-named give nearly the same result in

$$\frac{U}{V_{\frac{1}{2}D}} = \frac{1 + M}{1 + \frac{1}{2}\sqrt{3 - 4n}}$$

In Case 1, $n = 0$, $M = \frac{\pi}{4}$, Ratio $= 0.9568$

" 2, $n = 0.2$, $M = \frac{2}{3}$, " $= 0.9573$

" 3, $n = 0.1$, $M = 0.726$, " $= 0.9557$

* "Experiments at Detroit, Mich., on the Effect of Curvature Upon the Flow of Water in Pipes." *Transactions*, Am. Soc. C. E., Vol. xlvii, pp. 31 and 67.

Major Alexander Mackenzie, in his "Current Meter Observations in the Mississippi," in which the curves were determined by simultaneous observations on six meters in a vertical, found for his average result: Ratio = 0.958.

But, should f not equal ε, this ratio would be far from constant.

It follows that the 5-ft. depth rule of Humphreys and Abbot, and also all rules depending on single surface-velocity observations, are but approximate.

Application to Channels of Limited Width.—While not difficult to form the elementary equations, which, under the present theory, would take the general form

$$\varepsilon\left(\frac{v\,d\,v}{d\,z}+\frac{v\,d\,v}{d\,W}\right)=-c\,W(z-Z),$$

they would in general be very difficult to integrate, and, as the hydraulic radius can be substituted for D in the formulas as here given, within the limits of practice, and without exceeding the allowable error of observation, such development will not be here attempted.

Transverse Curves.—All transverse curves will evidently be of a general elliptic form, modified by the conditions in the last paragraph, usually having two different branches in natural streams, meeting tangentially over the curve of maximum velocity, and will be largely influenced by the figure of the bed, as has been announced by all careful experimenters. Major Cunningham, in his "Roorkee Experiments," considered them to be quartic ellipses.

Justification of the Elliptic Form by Experiment.—As it is easy to multiply comparisons, there will be but one curve figured here. This is taken at random from Major Mackenzie's "Current Meter Observations in the Mississippi," being his No. 24, which gave the result of reading six meters operated simultaneously for 27 consecutive minutes, registered electrically, and which, if the writer errs not, were undertaken as a result of the discussion on the paper* by the late James B. Francis, M. Am. Soc. C. E., entitled "On the Cause of the Maximum Velocity of Water Flowing in Open Channels being below the Surface."

The calculations involved in the foregoing are as follows: The maximum velocity is evidently around a depth of 3.6 ft. Assume $n = 0.175$ or 3.78 ft. Then $z_u = 0.6453\ D = 13.938$ ft. For $n = 0.175$ and $f = \varepsilon$,

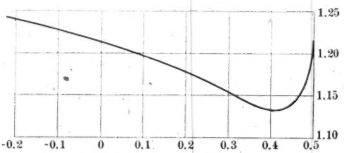
* *Transactions*, Am. Soc. C. E., Vol. vii, pp. 109, 168.

$U_1 = 0.678\ v_o$, and also V_{maximum} equals $1.825\ v_o = 2.401$, whence $v_o = 1.315$, from which $V = 1.086$. Placing this in Equation (4), with $D = 21.6$, and adding v_c to each result, the last column of Table No. 2 results.

TABLE No. 2.

D.	Experimental result.	Mackenzie's deductions.	Present theory.
0	.....	2.361	2.374
3.6	2.401	2.401	2.401
6.6	2.375	2.394	2.386
9.6	2.370	2.352	2.341
12.6	2.271	2.267	2.260
15.6	2.103	2.096	2.128
18.6	1.846	1.846	1.918
21.6	.....	0.94	1.315
Mean	.....	2.186	2.206
Mid-depth	.....	2.328	2.312

Mackenzie found $n = 0.2$, nearly, or 4.2 ft., with $z_u = 14.32$ ft. Humphreys and Abbot's rule gave, for value of U, 2.260 ft. Here, without trial curve or preparation, the calculation gives as close results as the more elaborate method, having for data simply the total depth of the stream, together with the value of the maximum velocity and its depth. Although the formula is a slope formula, the curve may be quickly determined from a few observations



CURVE OF KINETIC ENERGY IN A STREAM OF UNLIMITED WIDTH. ELLIPTICAL THEORY. "HEAD" ONLY.

FIG. 5.

Relation of Momentum and Kinetic Energy.—In Fig. 5 is plotted the curve of kinetic energy due to variation of velocity in the head only, for a stream of unlimited width, as calculated in Table No. 1, the abscissas being the depth of axis expressed in parts of depth, and the ordinates the values of the kinetic energy, $1 + 3\eta + \varkappa$.

This curve shows that the kinetic energy of the stream decreases as the axis lowers, and the position of this axis has been shown to depend upon the relation between viscosity and the bed friction, M expressing the ratio of the mean velocity in the head to that of v_o, assuming that the resistance at the surface is the same as that between the fluid fillets. As the kinetic energy is thus shown to de-

crease as the axis falls, it can be stated, although the limits are ill-defined, that:

On account of the principle of the least expenditure of energy, and under normal conditions as to temperature of water and bed friction, the axis of maximum velocity should be found below the surface, but should never exceed about 0.425 depth, the actual depth depending solely upon the relation of f to ε.

The value of η in Table No. 1 should be multiplied by the square of the ratio of the mean velocity in the head to the total mean velocity, and that of $\varkappa$ by the cube of the same, or, what is the same thing when $f = \varepsilon$, by $\left(\frac{M}{1+M}\right)^2$ and $\left(\frac{M}{1+M}\right)^3$, respectively, for the true values of η and $\varkappa$. Thus, in the second illustration, when $M = \frac{2}{3}$, taking $f = \varepsilon$, it is readily found, that

$n = 0.193$; $v_o = 0.5534$; $V = 0.4466$; $v_o + V = 1$; $U = 0.9223$; $U_1 = 0.3689$.

Then $\left(\frac{3689}{9223}\right)^2 = 0.16$ and $\left(\frac{3689}{9223}\right)^3 = 0.064$, whence, as from Table No. 1, $\eta = 0.0682$ and $\varkappa = -0.0253$, the correct values become, $u = 0.0682 \times 0.16 = 0.0109$; $\varkappa = 0.0253 \times 0.064 = -0.0016$; and $1 + 3\eta + \varkappa = 1.0311$ instead of 1.1793, as in the table, which is the least energy that could be developed under the conditions, as compared with that developed by the stream had all the fillets the mean velocity U. It is apparent from the curve of energy that this is less than it would have been were the maximum velocity in the surface.

General Remarks.—Whatever the form of the curve of velocity may be, and some of the preceding deductions are independent of this form, or of the writer's particular assumption as to viscosity, it seems as if it should possess two properties: Its branches should be tangent over the point of maximum velocity, and it should terminate tangentially to the bed. It would seem that a curve abruptly cut off at the bed should slip along it. It is hard to conceive the fluid torn apart as would be required by parabolas, hyperbolas, etc., although, on the supposition here made that $f = \varepsilon$, the resistances would insensibly glide into each other. The ellipse seems to be the proper curve, as other essential requisites are lacking in the circle. Navier's theory requires a parabola. The writer expressed the belief (but will not strongly insist upon it) that the Poiseuille coefficient of viscosity was based on an improper theory, for the reason that the experiments were

carried on with capillary tubes, and it was thought, when the remark was made, that the coefficient found was rather that of friction on the sides of the pipe than that of viscosity, capillary attraction practically annulling the "dome."

This theory and that in the previous paper can readily be made to apply to orifices and weirs, as well as to pipes and channels, avoiding the necessity of "coefficients of contraction." For instance, in a circular orifice with sharp edges, v_o would be necessarily zero. The volume in the elliptic head, and consequently Q, would then be, as a maximum, $Q = \frac{2}{3}\sqrt{2\,g\,h}$, the area being unity. This would be modified by ϵ only. Assuming a coefficient of velocity of 0.985, there is obtained $Q = 0.6566\sqrt{2\,g\,h}$, agreeing exactly with Boussinesq's deduction by an intricate method of approximation. v_o would not be 0 in the standard tube, hence Q would be greater. This, however, is foreign to the subject.

Final Summary.—The foundation of this paper is the discovery of the law, hitherto not recognized, that the motion of fluids is the result of pressure on the end section, and, consequently, each individual fillet moves under the action of a force which is equal to its weight, but is applied at its end, and owes nothing to the weight of the fillets surrounding it. Consequently, an axis of maximum velocity may be established at any point in the section around which, or parallel to which, the fillets may flow with the same velocity.

With definite air resistance, the curve might be modified above this axis, but, considering this resistance as a fluid resistance, the curve of velocity may be referred solely to viscosity and bed friction, with the results as hereinbefore given.

The assumption as to the law of viscosity gives a form of equation such that the laws of motion may be readily traced, and seems to agree well with experiment, and while many careless readers may see in the results nothing but the $v = c\sqrt{R\,S}$ formula, no similarity, except as to form, will be found if they will follow the line of reasoning.

It may be stated that in the case of a bed so smooth that f becomes 0, the first term of Equation (15) would be neglected as indeterminate, the viscous resistances still remaining as a resistance to flow, reverting to a case somewhat similar to a sharp-edged orifice in which the velocity lies entirely in the elliptic head and v_o is zero.

This discussion does not apply to floods where constant surface supplies are added to the stream.

It may be finally stated that Bernouilli's Theorem was fought bitterly before it was finally accepted as established. This may be, but the writer sees no other way to prove the actual, established fact, which all the theory in the world cannot controvert, that the axis of maximum velocity in an open stream is, in the great majority of cases, below the surface.

It will undoubtedly be asked, "If the motive force is not due to the weight of the liquid itself, where does it come from?" The reply to this is, that it is due to atmospheric pressure. It has been assumed that the resistance between the air and the moving liquid is of the nature of viscosity, that is to say, that the air in contact with the liquid surface moves with it, in contradistinction to the immovable bed. But it does not follow that the upper layers of air follow any such law. Replacing, therefore, the atmospheric pressure on the surface by the height shown by barometers of the same kind of liquid as that considered, the barometers at the two ends of the section would act independently, and the difference in pressure would be equal to the height of a column of the liquid considered, due to the slope of the surface. This considers a channel as a pipe with one fixed and one movable wall. The resistance overcome is measured by, but not caused by, the weight of water contained in it. The ordinary assumption would then, strictly, require the axis of maximum velocity to be always above the surface.

In one or two instances, the writer has simply said weight, when "component of the weight in the direction of motion" should have been used, but trusts the context will render the meaning plain, and avoid confusion.

DISCUSSION.

Mr. Herschel. CLEMENS HERSCHEL, M. Am. Soc. C. E. (by letter).—Civil Engineering has been defined, by what is generally accepted as the best definition extant, as "the art of directing the great sources of power in Nature for the use and convenience of man." Let this stated object of the art at any time disappear or become secondary; let an investigation cease to hold primarily in mind, and fixedly tend toward "the use and convenience of man," and the discussion at once passes from the domain of civil engineering to that of pure science, and from that of civil engineers to that of savants or physicists. This is not saying that the pursuit of science for science's sake, or as a form of recreation, has not a proper place in the world's development, but in this same world in which printing has now become so cheap or ready of accomplishment, and in which so much is in fact printed, it behooves civil engineers to stand ready to apply the touchstone of utility, or at least of the direct tendency toward proximate utility, to any article they may meet, if they wish to recognize and select their very own.

The hydraulic engineer has presumably offered him as much purely scientific dissertation as any other member of the greater profession of engineering, and yet stands in great need of work directly tending to advance his especial art, and it is as deploring a diversion of admirable forces from his own line of work to that of pure science, or of speculation, that he must view so much that is printed on hydraulic subjects.

These remarks have been instigated by the search among engineers just now for such things as "velocity curves" within pipes, and in open channels, when the real "use and convenience of man," demands rather more and better means for measuring quantities, and when it has been repeatedly shown that the search for curves of velocity does not lead to means of gauging streams of water. At the same time, means of measurement already devised are calling loudly for further experiment to perfect them, and the use and convenience of man would also welcome better such means to supplant those now practiced, if such can be devised. Savants there are a plenty, and there is no especial demand for engineers to encroach upon their specific domain. But if they do choose this line of work, let it be recognized as not being engineering.

Again: Whenever the engineer neglects to have direct and proximate utility in mind, it appears to tend to affect the correctness of his premises. Especially does this appear to be true in hydraulics, where for fifty years or more we have had "fillets," and "filaments," and "sliding friction," and "curves of velocity," dinned into the student of engineering, until the engineer has been led by the savants to live and think in and of a world of water that has no existence, and now cannot,

without the spur of an endeavor primarily to be useful to his fellow man, shake off the allurements of an intoxicating mere pursuit of yet more science. This may be illustrated from the paper under discussion, although the writer hastens to state that he thinks it no worse in this respect than very many others; yes, no worse than most others of the present day, and he wishes to apologize to the author of the paper referred to, for having selected his particular essay as the one to furnish the immediate object of the present adverse criticism. It is a most admirable discussion in itself, and would be a step toward progress, except that it lacks foundation, by reason of false premises, like the others spoken of. Mr. Herschel.

"As a preliminary step (says the author), let it be assumed that uniform motion exists in the stream, meaning by this that the motion along each fillet is uniform and parallel to the longitudinal axis of the stream, and that the total resistance to motion is composed of two entirely different, yet mutually dependent, classes of resistance, one being the so-called friction along the bed and sides of the stream, the other, the internal resistances in the liquid itself, or the so-called viscosity."

Upon which, the writer ventures to affirm that there is no use of reading any further. Any edifice built on such a foundation needs but a breath to cause it to crumble into the dust. No such described fluid exists on the face of the earth, and, conversely, no formula thus developed can be applicable (unless by empiricism) to fluids found on the earth. Water no more flows in lines parallel to the axis of the stream, no more slides along in layers represented by the customary "velocity curves" and having the resistance of viscosity between those layers, then would the contents of a feather bed floating down the street in a gale of wind. Of this we have recorded experimental proof, and anybody can repeat the experiment, or go out into the open and acquire conviction by looking at water flowing in a canal; or observe it, mixed with amber dust, flowing through a glass tube. Referring, in this connection, to the writer's treatment of the general subject now under consideration,* the following extract will describe the experimental proof above spoken of, that water flowing in a closed conduit cannot properly be described as flowing in concentric rings or layers.

"In the December, 1893, number of the *Transactions*, Am. Soc. C. E., is an article by G. H. Benzenberg, M. Am. Soc. C. E., on the 'Sewerage System of Milwaukee and the Milwaukee River Flushing Works.' This able engineer, after completing the discharge-sewer 12 ft. in diameter and 2 534 ft. long, for the latter-described works, was anxious to know its volume of discharge under various conditions. As both ends of it are submerged, he hit upon the expedient of measuring the velocity of the water through it by injecting suddenly two ounces of red eosine, dissolved in one quart of water, into the sewer at one end, and noting the discharge of this colored water half

* "115 Experiments on the Carrying Capacity of Large, Riveted, Metal Conduits," John Wiley & Sons, 1897, Chapter vii.

Mr. Herschel. a mile farther down-stream. He was enabled to do this with accuracy, because 'the color was readily perceptible at the outlet, and was never distributed over a length of more than 7 to 9 ft., being about $\frac{1}{3}$ of 1% of the length of the tunnel, the center of which was taken as the point observed. The compactness of the coloring matter showed that the velocity was practically uniform at all points in the cross-section of the tunnel, which again in itself was very uniform throughout the entire length of the tunnel.' Nevertheless, a 4 x 6 in. block 18 ins. long was found spiked to the interior on inspection of the tunnel the next spring.

Let us add that in these ten experiments of discharge the mean velocity ranged from 3.9 to 6.9 ft., and, that nothing might be lacking to prove the ordinary conditions of flow, that the computed coefficient in $v = c\sqrt{r\,s}$ ranged, with the velocity, from 122.7 to 137.3. Here we have the truth about flowing water, and a truthful representation of how it flows. A body of it equal in length to, say, half the diameter of the pipe stays together, though riddled and seething with internal motion, for the distance of half a mile, or for from 6 minutes 4 seconds to 10 minutes 52 seconds, and no doubt for a much longer space and time in actual practice in straight channels."

Suppose that there had been a "scale of velocities" and "velocity curves" in this sewer, such as are generally supposed to exist and are pictured in books and essays; and make, for purposes of illustration, the reasonable assumption that the maximum, mean and minimum velocities, as usually determined, had been 6, 5 and 4 ft. per second; then in passing over 2 534 ft. of conduit, the colored head of the center "filament" would have passed the down-stream goal in 422 seconds. At that moment the colored heads of the filaments of minimum velocity would, however, have traversed only 1 688 ft., and, consequently, we should have had a paraboloid of revolution 846 ft. long, colored by the red eosine, whereas, Mr. Benzenberg states that, "the color * * * was never distributed over a length of more than 7 to 9 ft."; that the coloring matter was compact, and that its observation, diffused throughout a mass of water of that length, served perfectly to determine the mean linear velocity of the mass.

Now then, investigators of hydraulic problems must either produce a stream of water long drawn out, after the fashion of molasses candy at a "candy-pulling," as they have represented them, or else they should stop talking about straight-line filaments and their accompanying "velocity curves," and, above all, should cease to put the idea of such filaments and curves into the premises of their investigations, and then use these premises as the basis for the determination of formulas. They should also cease to make these fancied curves the object of their investigations, when working as civil engineers, for it has been abundantly shown by 150 years of such work that such investigations are mere recreations, and do not lead to methods of gauging or to formulas for computing the discharge of water. They cannot be useful, because they are the mere products of fancy, and have no existence at any one moment of time.

Mr. Herschel.

The truth of the matter probably is, that a particle of water at one moment on the circumference of the tube section, is, a fraction of a second later, in the axis of the tube, and ½ second later may be on the circumference again, and these subsidiary currents of water constantly impinge against the walls of the tube, to be thence repelled, and again repelled elsewhere from the tube, the whole mass meanwhile moving in the direction of the current, and it is probably during or by reason of this process of the impinging of these subsidiary currents against the walls of the channel, that the varying retardation of different surfaces of tubes is produced. This would also explain why experiments made with geometric cylinders, are of but limited application to pipes such as are found in practice, with joints, branches, gates and similar irregularities, and why general laws cannot be derived from experiments conducted on irregularly formed conduits; and may produce in the mind the necessary conception, for purposes of grasping the whole situation, of the uniform averages of a multitude of chance effects having been mistaken for the uniform action of the physical law of flowing water.

Take 10 000 names from the New York City directory, and let their owners shoot at a target at 200 yds., and they will score a certain number of points. Then take 10 000 more such names, and the same process will result in practically the same score. But this does not establish any law of projectiles or of gunnery; only the law of averages of chance results. Similarly, the presence of nearly the same component of velocity parallel to the axis of the channel, at any point in the cross-section of a stream of water, at consecutive intervals of time, does not represent physical law, only the law of averages applied to chance effects. And "curves of velocity" are still less recognizable as symptoms of natural law.

Again, why seek for "curves of velocity" and pass from them to the determination of quantity, when quantity of discharge is the true aim and object, and may be sought directly? No one would think of torturing a §-in. pipe, and the flow of water through it, and writing voluminous reports of a season's operations of a party of engineers on such a pipe, to determine its discharge, but, instead, would put on a meter. Precisely the same thing can be done these fifteen years in the case of pipes of any diameter, and for a mere fraction of the cost incurred by any other process of determining such discharges*. A self-recording apparatus permits of keeping a perpetual, continuous record of such discharges; surely enormously better for all practical purposes—for any purpose tending to the use and convenience of man—than a solitary, few, pictured, imaginative gaugings by means of Pitot tubes or current meters.

* "115 Experiments on the Carrying Capacity of Large, Riveted, Metal Conduits," John Wiley & Sons, 1897, Note C. Merriman's "A Treatise on Hydraulics," John Wiley & Sons, 1895; Article 71. *Transactions*, Am. Soc. C. E., November, 1888.

Mr. Herschel. The hydraulic method used in the Venturi meter, above referred to, is also applicable to the determination of weir, and of orifice, discharges, and in the writer's opinion should be adopted, to the exclusion of all ordinary coefficients now used, to determine such discharges. An article* on this subject by the writer, entitled: "The New Hydraulics," is referred to, in connection with this discussion.

It argues for the abandonment of the mixture of hydrostatics and of hydraulics now followed in weir and orifice discharge gaugings, and for the substitution of measurements to be based wholly on the hydraulic data obtainable in each case. Thus: instead of seeking for a water height to be taken as the equivalent of "still water" up stream from a weir, or taking this water height at a designated point in a designated manner, and then correcting it by computation for "velocity of approach," let the effective water height be indicated hydraulically, or by water in motion, and already properly affected by the shape of the weir edge, etc.; in other words, let it be indicated by piezometer tubes set in the plane of water discharge, as has been done in a preliminary and tentative manner by Inspector General Bazin,† and commended by him for future experiments.

A set of experiments on the discharge of nozzles,‡ made by John R. Freeman, M. Am. Soc. C. E., is of similar character, and there, again, the observations were made on the hydraulic data of the case, not on its hydrostatics. All coefficients found by means of this method of observation, whether for the Venturi meter, by water-pressure tubes set in the body of the crest of a weir, or as in the case of nozzle discharge, have a very much smaller range of values, that is, are very many less in number, than usually found at the present day, when the hydrostatic conditions of the case are taken into account.

The same would no doubt be true for the cases of any and all forms of discharge through orifices, the computation of which is now nigh wholly impracticable, so numerous are the recorded coefficients, and so still more numerous, nevertheless, are the peculiar cases coming up for computation.

The determination of the new coefficients described in this article is, therefore, much to be desired, in the interests of an advance in the pursuit of hydraulics as a part of civil engineering, and promises to lead to many new discoveries.

The fact is, that the world to-day has a surplus of experiments recorded in it. After centuries of striving after the attainment of experimental knowledge, and for the introduction of the experimental

* *Engineering News*, November 10th, 1898.
† *Annales des Ponts et Chaussées*, 2d Trimestre, 1898.
‡ *Transactions*, Am. Soc. C. E., Vol. xxi, p. 303; Vol. xxiv., p. 492.

Mr. Herschel.

method in study, there has come an abuse of the experimental method of acquiring knowledge. Herbert Spencer has said somewhere that: "The value of an experiment depends upon the skill of the experimenter," and, he might have added also, upon his general and executive discernment and abilities. Unless a stop be put to the piling up of the records of experiments undertaken without correct aim, and with mistaken zeal in wrong directions, the facts of Nature will soon be much more readily arrived at, in any desired direction, by seeking them directly from Nature, than by looking for them in tomes of human lore. Indeed, that day has already come to many an earnest seeker for objects of practical worth; for many an engineer seeking to advance his art for the use and convenience of man.

To resume:

Engineers are the votaries of an art, which has for its object the furthering of things and methods that serve for the use and convenience of man.

They should make that object the aim of the direct tendency of their work, and the criterion of its fitness for the exercise of their energies. There is ample work of that kind waiting to be taken up.

Any diversion from it, in lines of purely scientific recreation, or of the satisfying of curiosity, belongs in the domain of physicists, and engineers are interlopers in it, and wasters of their much-needed energies in their own particular field.

The perfection of methods for the gauging of streams of water in pipes and channels is such a field that calls for the labors of the civil engineer.

It cannot be furthered by disquisitions on, or experiments on "curves of velocity" in such streams of water, and such curves of velocity do not exist as the result of physical laws.

The search for formulas of flow of water in channels, to be effective, must be conducted with a true foreknowledge of the nature of such flow, and too much care cannot be taken in basing such a search and such computations on premises that are absolutely unimpeachable.

In practical hydraulics, experiments with channels of geometrically perfect form yield results that are only approximately applicable to the more or less irregular channels that are met with in practice; and experiments made with irregular channels do not yield results that are closely applicable to channels of only slightly differing imperfections, nor to the development of general laws.

Mr. Church.

I. P. Church, Assoc. Am. Soc. C. E. (by letter).—The author has evidently adopted as the basis of treatment the "laminar hypothesis," or assumption of flow in plane layers parallel to the bed (in this case of uniform motion of water in a rectangular channel of great width), each layer moving parallel to the bed with a constant velocity, $v_o + v$; where v_o is the velocity of the bottom layer (or "filament") and v the

Mr. Church. excess of the velocity of any layer over that of the bottom layer. It is well known that this hypothesis gives but an imperfect representation of the reality, but it has, nevertheless, been adopted by many writers as a convenient rude approximation to the facts.

It is, of course, a rigorous consequence of this hypothesis, by the laws of mechanics, that the layer of maximum velocity must be at the surface of the water unless there is an up-stream wind (*i. e.*, up stream relatively to the surface layer, which is itself in motion); also that the internal fluid pressure is constant at any given depth, z below the surface, so that the end-pressures, parallel to the bed, on the two ends of a portion of a layer, of length l and width unity, are equal, leaving the component (parallel to the bed) of the weight of this portion to balance the (difference between) frictions exerted on this body by the adjacent layers. Since the motion of this portion of a layer is uniform and rectilinear, the components of all the forces acting on it neutralize each other, taken parallel to any axis whatever; but an axis parallel to the bed is most convenient, since it avoids bringing into play the pressures normal to the upper and lower forces of the body. Otherwise, mathematical elimination would be necessary.

As regards the viscous "friction" (or shearing action due to intensity of eddying) between contiguous layers, while Navier and others consider it as proportional to the angular velocity, $\frac{d(v_o + v)}{dz}$, or $\frac{dv}{dz}$, of a point in the upper surface of the layer with respect to a point just underneath in the lower surface of the same layer, the author assumes that it is not only proportional to that angular velocity but also to the (excess) velocity, thus following the line indicated by Bazin,* that it should be proportional to $K\frac{d(v_o + v)}{dz}$, where K is a quantity dependent on the velocity $(v_o + v)$.

Now, it is to be noticed that what the author calls the "first- or ordinary method" when applied to the equilibrium of the aggregation of layers (of width unity, and length l) situated between the depth, Z, of maximum velocity (where the friction is zero) and any other depth, z (where the friction is not zero) (it being assumed, for justification of the use of the laminar hypothesis, that there is an up-stream wind) gives rise to the equation:

$$-Fl + \rho g l (z - Z) s = 0 \ldots\ldots\ldots\ldots\ldots (32)$$

in which s = slope and ρg = weight of a unit of volume of water, while F denotes the friction per unit area on the under surface of the aggregation of layers in question, *i. e.*, at a depth, z, from the free surface of the water. This is the same as the initial equation of the author's treatment (on page 398) and is identical with an equation established by the "ordinary method," given on page 196 of Flamant's "Hydrau-

* As quoted by Collignon in his "Hydraulique," p. 307.

li ique," second edition, 1900. Flamant deals with Navier's assumption for F, while Mr. Tutton assumes F of the form already indicated, and proceeds with the mathematical treatment. Mr. Church.

It is also to be noticed that Unwin's treatment of the same problem* is practically identical with that of this paper (the ordinary method is used to establish a result for a single layer, which by integration is extended over any number of layers). Unwin uses the Navier form for viscosity, and at first places the maximum velocity below the surface, remarking later that if there is no resistance at the surface the maximum velocity will occur at the surface (that is, he calls attention to this rigorous outcome of using the laminar hypothesis).

Since, then, the only difference between the analysis of the author and that of the writers above quoted is the substitution by the former of a special viscosity factor, F, in place of the Navier form, the writer is quite unable to appreciate the alleged imperfection of the "ordinary method." In presenting what he calls "the other method," the author has brought out the familiar fact that the loss of head in the case of uniform motion in an open channel, between any two points of a given filament, is equal to the vertical distance, $l \sin. \alpha$, between the corresponding points in the surface of the stream; that is, it is equal to the "piezometric fall," since, for uniform motion in an open channel, the surface of the stream plays the part of a hydraulic grade line. He has also brought out the fact that the uniform motion of a filament or layer in the present problem is the same as it would be if its weight had no component parallel to the line of motion, and if (in addition) the pressure at its up-stream end parallel to the bed exceeded that at the down-stream end by an amount, $\rho g l \sin. \alpha$, per unit of end area. To such a statement as this no reader would take exception, and probably this is all that is meant by the "Law" on page 991;* but the author's language, taken literally, would seem to indicate that in his opinion there actually is more pressure at the up-stream end of a portion of a filament than at the down-stream end; which is quite at variance with the laminar hypothesis.

While the ellipse as the vertical velocity curve is the very interesting outcome of the special viscosity assumption made by the author, the writer can see no reason why the former curve does not stand on precisely the same footing as the parabola (which is the outcome of the Navier assumption), as regards any explanation of the fact that the maximum velocity is not at the surface of the water in those cases where it should be so located if the laminar hypothesis were strictly true. Such a case is presented, for example, when the wind is down stream and has a velocity equal to that of the surface layer; under which circumstances it seems to be quite generally admitted by hydraulicians (Unwin, Flamant, Bazin, Francis, etc., etc.) that experiment shows

* "Encyc. Brit.;" article, Hydromechanics, page 496.

Mr. Church. the maximum velocity (parallel to the bed) to occur below, and not at the surface. The conclusion is inevitable that a satisfactory explanation of this fact must involve some theory of internal motion differing radically from the laminar hypothesis. Attempts at such an explanation, based on the action of eddies thrown up from the bottom, vertical currents, etc., etc., have appeared in the pages of the *Transactions* of this Society; for instance, see the paper (referred to by the author) by Mr. Francis, in Vol. VII, page 109, and the discussion following it.

Mr. Murphy. E. C. MURPHY, Assoc. M. Am. Soc. C. E. (by letter).—The writer has read this paper with much interest. He has under way a study of vertical velocity curves of streams, from the experimental standpoint, and hoped that the author's "Proposed solution of some hydraulic problems" would explain satisfactorily some of the observed phenomena at variance with the common theory of laminar flow. In this, however, he is somewhat disappointed. Some of the author's statements are misleading, and the simple though approximate theory of laminar flow is made to appear quite complex.

The writer desires to call into question more particularly the statement on page 393.

"As a matter of observed fact, the maximum velocity is seldom or never found there (at the surface). One of the objects of this paper is to prove that this is necessarily so," etc.

The writer fails to see the proof of this statement. Admitting that the vertical velocity curve is in some cases an ellipse, it does not follow that its minor axis is below the surface. The vertical velocity curves obtained by the writer with current meters, and also some of those obtained by other experimenters, show that the maximum velocity in open channels is often at the surface.

Fig. 6 shows vertical velocity curves obtained by the writer in the Oswego River at Battle Island, N. Y. These curves are obtained from about 500 velocity observations, taken August 7th, 27th and 28th, 1901, with a small Price meter. The "point method" was used, the meter being held at points from $\frac{1}{2}$ ft. to 2 ft. apart, from $\frac{1}{2}$ ft. above the bottom to 1 ft. below the surface.

It is to be specially noted that the maximum velocity is at the surface in each of these curves. The change in the shape of the curve as the depth increases should also be noted.

The theoretical shape of the vertical velocity curve depends on the viscosity of the liquid, or the friction between the parallel laminæ. Fig. 8 shows a lamina at a distance z below the surface, l is its length, dz its thickness, b its breadth, P is the pressure at each end, w the weight per unit of volume, F the friction per unit area on the upper surface, and $F + d\,F$ the friction per unit area on the lower surface. The force producing motion is the weight of the lamina multiplied by

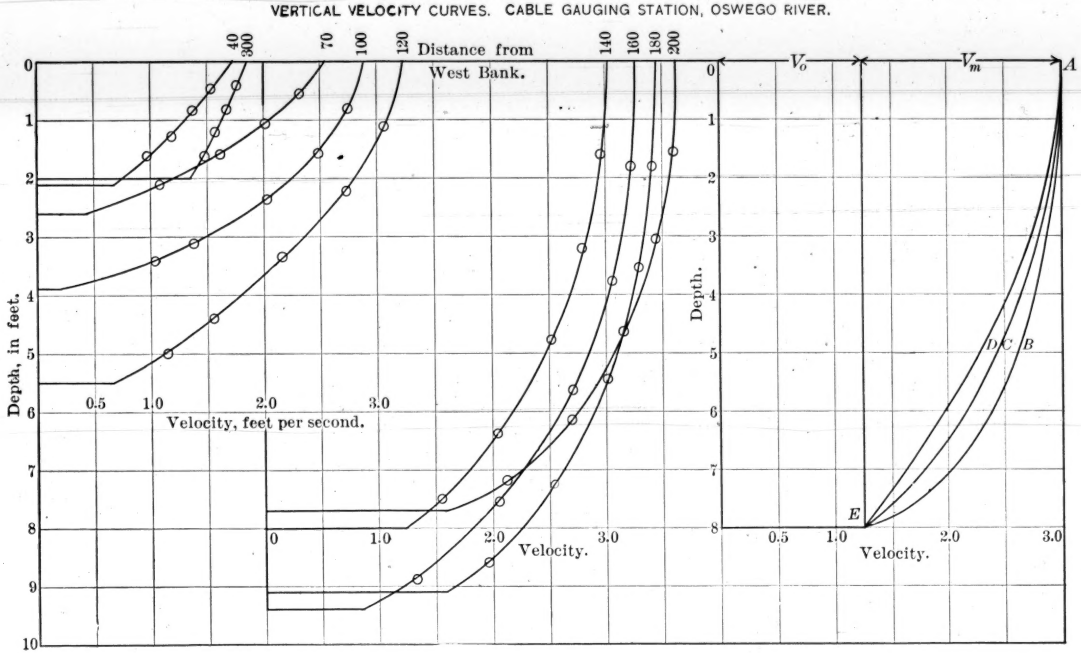

VERTICAL VELOCITY CURVES. CABLE GAUGING STATION, OSWEGO RIVER.
Distance from West Bank.
40
300
70
100
120
140
160
180
200
Depth, in feet.
0
1
2
3
4
5
6
7
8
9
10
0.5
1.0
2.0
3.0
Velocity, feet per second.
0
1.0
2.0
3.0
Velocity.

FIG. 6.

V_o
V_m
A
Depth.
0
1
2
3
4
5
6
7
8
D
C
B
E
0.5
1.0
2.0
3.0
Velocity.

FIG. 7.

Mr. Murphy. the sine of the surface inclination. (Whether this force be called the component of gravity, or a force produced by gravity, matters not.)

If the motion of the lamina is uniform, we have

$$w\ b\ l \sin.\ \alpha\ d\ z + F\ b\ l - b\ l\ (F + d\ F) = 0,$$

or

$$w \sin.\ \alpha\ d\ z - d\ F = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

If we assume $F = k \dfrac{d\ v}{d\ z}$ (k being a constant called the coefficient of viscosity), Equation (1) becomes:

$$w \sin.\ \alpha\ d\ z + k \frac{d^2\ v}{d\ z} = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Taking the anti-derivative of Equation (2) we have:

$$w\ z + k \frac{d\ v}{d\ z} + c = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Taking the anti-derivative of Equation (3), we have:

$$w \frac{\sin.\ \alpha}{2} z^2 + k\ V + c\ z + c^1 = 0 \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

Putting $z = 0$, and $V = V_s$ in Equation (4), we have:

$$-k\ V_s = c'; \text{ or } c' = -k\ V_s.$$

The velocity is a maximum $= V_m$ at the vertex of the curve, where

$$\frac{d\ v}{d\ z} = 0.$$

Hence, from Equation (4) we have:

$$k \frac{d\ v}{d\ z} + w \sin.\ \alpha\ z + c = 0;$$

and the distance, z', from the surface to the vertex of the curve is

$$z' = -\frac{c}{w \sin.\ \alpha} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

Substituting the value of z' in Equation (4), and simplifying, we have:

$$V_m - V_s = -\frac{w \sin.\ \alpha\ z^2}{2\ k} + w \frac{\sin.\ \alpha\ z'^2}{k} = \frac{w \sin.\ \alpha}{2\ k} z'^2 \ldots\ldots (6)$$

Subtracting Equation (4) from Equation (6) we have:

$$V_m - V = \frac{w \sin.\ \alpha}{2\ k} (z'^2 - z^2) - \frac{2\ c\ z}{2\ k} = \frac{w \sin.\ \alpha}{2\ k} (z' - z)^2 \ldots\ldots (7)$$

This curve is a parabola, the axis of which is parallel to the water surface, its vertex being z' from the surface. The parameter is $\dfrac{2\ k}{w \sin.\ \alpha}$. If D is the depth, when $z = D$, $V = 0$; and Equation (7) becomes $V_m = \dfrac{w \sin.\ \alpha}{2\ k} (z' - D)^2$; or $\dfrac{w \sin.\ \alpha}{2\ k} = \dfrac{V_m}{(z' - D)^2}$.

Equation (7) can now be written

$$V = V_m - \frac{V_m}{(z' - D)^2} (z' - z)^2 \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

Mr. Murphy.

If the viscous resistance per unit of area is assumed to equal

$$k\,V\frac{d\,v}{d\,z},$$

as the author has done, the result is Equations (2^1) to (7^1) which are derived from Equation (1) in the same way that Equations (2) to (7) are derived.

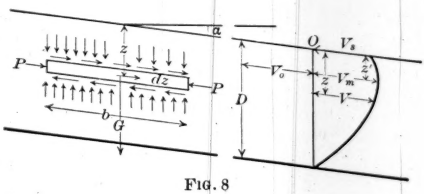


FIG. 8

$$w\,\sin.\,\alpha\,d\,z + k\,d\left(V\frac{d\,v}{d\,z}\right) = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots (2^1)$$

$$w\,\sin.\,\alpha\,z + k\,\frac{V\,d\,v}{d\,z} + c_1 = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots (3^1)$$

$$\frac{w\,\sin.\,\alpha}{2}z^2 + k\,\frac{V^2}{2} + c_1\,z + c'_1 = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots (4^1)$$

$$z' = -\frac{c_1}{w\,\sin.\,\alpha} \quad \ldots\ldots\ldots\ldots\ldots\ldots (5^1)$$

$$V_m{}^2 - V_s{}^2 = \frac{w\,\sin.\,\alpha}{k}\,z'^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots (6^1)$$

$$V_m{}^2 - V^2 = \frac{w\,\sin.\,\alpha}{k}\,(z' - z)^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots (7^1)$$

From Equation (7^1) we have

$$\frac{V^2}{V_m{}^2} + \frac{w\,\sin.\,\alpha}{k\,V_m{}^2}(z' - z)^2 = 1 \quad \ldots\ldots\ldots\ldots\ldots\ldots (9)$$

which is the equation of an ellipse having a semi-minor axis $= V_m$ and a semi-major axis $= \sqrt{\frac{k}{w\,\sin.\,\alpha}}\,V_m$.

When $z = D$, $V = 0$; hence, $\frac{w\,\sin.\,\alpha}{k\,V_m{}^2} = \frac{1}{(z' - D)}$, and Equation (9) becomes

$$\frac{V^2}{V_m{}^2} + \frac{(z' - z)^2}{(z' - D)^2} = 1;\ \text{and we have}\ V = V_m\sqrt{1 - \left(\frac{z' - z}{z' - D}\right)^2} \quad \ldots (10)$$

It can easily be seen that by substituting for F, $\frac{d\,v}{d\,z}$ and any function of V, the equation of a vertical velocity curve is obtained. If we make $F = k\,V^{\frac{1}{2}}\frac{d\,v}{d\,z}$, we obtain

$$V_m{}^{\frac{3}{2}} - V^{\frac{3}{2}} = V_m{}^{\frac{3}{2}}\frac{(z' - z)^2}{(z' - D)^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (11)$$

Now, which of these curves is the true one? Why is the ellipse the true one? They are all obtained on the basis of laminar flow.

For the vertical of the Oswego River 140 ft. from the west bank, shown in Fig. 6, we have $D = 8$ ft., $z' = 0$, $V_o = 1.25$ ft. and $V_m = 1.79$ ft.

Mr. Murphy.

Equation (8) becomes, for this vertical, $V_p = 1.79 - 0.028\, z^2$....(12)

Equation (10) becomes $V_E = 1.79 \sqrt{1 - \frac{z^2}{8^2}}$.....(13)

Equation (11) becomes $V_a^{\frac{3}{2}} = 2.39 \left(1 - \frac{z^2}{8^2}\right)$.....(14)

Table No. 3 gives the values of the velocity computed from Equations (12) to (14) for each fifth of the depth; also the corresponding values of V_c, the observed velocity.

TABLE No. 3.

Depth, in $\frac{z}{D}$	V_p, in feet.	V_c, in feet.	V_a, in feet.	V_E, in feet.
0.....	1.79	1.79	1.79	1.79
One-fifth.....	1.71	1.72	1.74	1.75
Two-fifths.....	1.50	1.54	1.60	1.64
Three-fifths.....	1.14	1.27	1.33	1.43
Four-fifths.....	0.64	0.81	0.91	1.07
Five-fifths.....	0.00	0.00	0.00	0.00

The values of V_E, V_p and V_c are plotted in Fig. 7, giving the curves $A\ B\ E$, $A\ D\ E$ and $A\ C\ E$, respectively. The parabola is nearer the observed curve than the ellipse in this vertical, and V_a is nearer the observed curve than the parabola. It can easily be seen that the curve obtained from $F = V^{\frac{1}{3}} \frac{d\,v}{d\,z}$ will more nearly coincide with the observed curve than the curve V_a.

It is seen from Fig. 6 that the vertical velocity curves at 160 ft. and 180 ft. from the west bank resemble closely that at 140 ft., and that the others differ from this one more and more, as the depth decreases and the banks are approached. Evidently, the shape of the channel, not only in the immediate vicinity of the gauging station, but for a considerable distance above and below it, modify the shape of the vertical velocity curves.

Mr. Tutton.

CHARLES H. TUTTON, M. Am. Soc. C. E. (by letter).—The writer wishes first to correct an error of statement into which he fell. It is stated (page 394, etc.) that the pressure in the end of any fillet (implying the upper end) is $\rho\, g\, l\, s$. This should have been $\rho\, g\, s$. Considering a filament, the end pressures upon it, due to exterior filaments, balance, as stated by Professor Church. In the fillet itself, when it is indefinitely shortened, $\rho\, g\, s$ may be considered a pressure applied to, as well as a component of weight of, the resulting molecule. At the lower end of a fillet, of length l, this would become $\rho\, g\, l\, s$. The resulting equations are not altered by this.

Mr. Tutton.

It may be stated that while pressure and weight are convertible terms, the writer considers weight to have no components except such as are vertical, and what by consent are called components of weight are in reality forces developed by it, which can only be defined by their effects. The idea of the paper is to confine attention to the fillet, and divest the argument of the idea of weight as measured from the free surface, which is claimed to be a necessary consequence of the laminar theory, involving the fact that the resistance at the surface must be zero.

That this latter assumption is false is shown by the fact that the maximum velocity is below the surface, even under the action of a down-stream wind. This has been shown by Boileau, Humphreys and Abbott, Cunningham, Ellis, Bazin, Price, and others. It follows, not that the atmosphere offers no resistance, but that we know very little of its character. It may be like the so-called skin of the water, and be very great as compared with the fluid resistance in the air immediately above it, but Nature shows that it exists. For this reason it has been assumed as a liquid resistance, which agrees very well with experiment. This allows an axis of maximum velocity below the surface, the depth of which depends on the relations between f and ε. In the illustrations used these were assumed equal. As a matter of fact, they are seldom equal, but whatever their relation, if they change other than in a constant ratio, the depth of axis will change with them.

Any molecule tends to sink vertically under the action of gravity, and its weight consequently develops a force or pressure in the direction of flow which may be considered as applied to it. All forces, then, necessary to overcome the resistances, are comprised in the molecule itself, and if its motion be uniform these resistances must be equal to the force developed in the direction of motion, whatever that direction be. As a logical sequence, in any river flowing with steady (not flood) motion, and having an irregular bed, the resistances in each section would be measured by the forces as above mentioned developed in that section, whatever the inclination of the bed. This s necessary in order that all molecules move instead of lying in istagnant pools, and is conceded in pipe flow. It will also be seen that pressure is required to produce this effect.

The writer's mode of expression on page 409 was unfortunate. It was not the intention to assert that atmospheric pressure, as distinct from gravity, produced motion, but to remove the hydraulic grade line, so that such grade line could be compared with that of a pipe, exterior to the liquid in the pipe.

To further justify the elliptical velocity curve, the experiments in Tables Nos. 4 and 5 are presented, in which, however, f does not equal ε. No effort has been made to find the most perfect fitting

Mr. Tutton. curve, but it is suggested that, developed on the laminar theory, they plainly show that the assumption of no resistance at the surface is not true, and that forces developed by weight are not necessarily weight itself.

TABLE No. 4.—VERTICAL VELOCITY CURVES AT CARROLLTON, MISS., 1883. W. G. PRICE.

See Chief of Engineer's Report, 1884, page 2878.

Depths, in tenths.	Average of 12 Verticals. Wind Down Stream.		Average of 16 Verticals. Wind Up Stream.		Average of 8 Verticals. Rising River.		Average of 33 Verticals. Mean of All.		Average of 21 Verticals Falling River.	
	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.
Surface.	5.34	5.33	5.72	5.65	5.96	5.99	5.49	5.44	2.72	2.67
0.1.....	5.42	5.41	5.80	5.74	6.09	6.10	5.60	5.54	2.75	2.72
0.2.....	5.44	5.46	5.79	5.80	6.14	6.16	5.60	5.60	2.75	2.75
0.3.....	5.48	5.48	5.82	5.82	6.18	6.18	5.62	5.62	2.75	2.75
0.4.....	5.48	5.46	5.75	5.80	6.13	6.16	5.60	5.60	2.68	2.72
0.5.....	5.42	5.41	5.74	5.74	6.12	6.10	5.55	5.54	2.68	2.67
0.6.....	5.36	5.33	5.76	5.65	6.03	5.99	5.53	5.44	2.59	2.59
0.7.....	5.28	5.18	5.67	5.49	5.93	5.83	5.43	5.28	2.50	2.47
0.8.....	5.10	4.98	5.48	5.16	5.79	5.60	5.24	5.05	2.44	2.30
0.9.....	4.66	4.67	4.92	4.92	5.23	5.23	4.70	4.70	2.04	2.04
1.0.....	3.05	3.81	3.05	3.97	3.64	4.22	3.07	3.72	1.35	1.34
n	.....	0.30	.....	0.30	.....	0.30	.....	0.30	.....	0.25

D. F. Henry's experiments on the Niagara, St. Lawrence, and St. Clair, are also referred to.*

Mr. Herschel's remarks, however appropriate to pipes, are scarcely applicable to rivers where current meters must be used, Venturi meters and weirs not being applicable. E. C. Murphy, Assoc. M. Am. Soc. C. E., in his paper on "Current Meter and Weir Discharge Comparisons,"† shows that these two methods of measurement agree remarkably well; yet, when using a current meter, velocity curves must be considered. He would be a poor engineer who could not use his instruments, and it is considered that the meaning of these velocity curves lies as much in the province of the engineer as in that of the physicist. Sewers have been tested in Buffalo by using a mixture of fluorescein and sodium hydrate, but without results comparable to those of Mr. Benzenberg, and if his experiment be considered conclusive, Mr. Herschel's subsidiary currents would appear to be without foundation.

* *Journal*, Franklin Inst., 1871.

† *Transactions*, Am. Soc. C. E., Vol. xlvii, p. 370.

TABLE No. 5.—Ellis's Experiments on the Connecticut.

United States Chief of Engineers' Report, 1875 and 1878.

Depth.	Mean of Meter Observations, below Average Velocity of 1.86.		Same, for Float Observations.		Mean of all Observations, below Average Velocity of 1.86.		Mean of Meter Observations, above Average Velocity of 1.86.		Same, for Float Observations.		Mean of all Observations, above Mean Velocity of 1.86.		Mean of all Meter Observations.		Mean of all Float Observations.		Mean of all Observations.	
	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.
0.0...	1.15	1.17	1.50	1.49	1.21	1.22	2.96	3.06	3.26	3.13	3.01	3.07	2.05	2.11	2.38	2.38	2.10	2.15
0.1...	1.19	1.20	1.52	1.51	1.25	1.25	3.08	3.10	3.24	3.17	3.11	3.11	2.12	2.14	2.38	2.37	2.17	2.18
0.2...	1.22	1.22	1.52	1.52	1.27	1.27	3.12	3.12	3.17	3.18	3.13	3.13	2.15	2.15	2.35	2.36	2.19	2.19
0.3...	1.22	1.22	1.53	1.51	1.27	1.27	3.06	3.10	3.17	3.17	3.08	3.11	2.13	2.14	2.35	2.31	2.17	2.18
0.4...	1.19	1.20	1.49	1.49	1.24	1.25	2.99	3.06	3.11	3.13	3.01	3.07	2.08	2.11	2.30	2.28	2.12	2.15
0.5...	1.14	1.17	1.46	1.45	1.19	1.22	2.91	2.98	3.01	3.06	2.93	3.00	2.01	2.05	2.23	2.22	2.05	2.09
0.6...	1.11	1.12	1.35	1.39	1.15	1.17	2.78	2.87	2.94	2.96	2.81	2.89	1.93	1.97	2.15	2.14	1.97	2.01
0.7...	1.03	1.05	1.29	1.31	1.07	1.09	2.62	2.71	2.80	2.83	2.65	2.73	1.81	1.86	2.04	2.04	1.85	1.90
0.8...	0.94	0.94	1.20	1.20	0.99	0.98	2.42	2.49	2.60	2.64	2.45	2.52	1.67	1.70	1.90	1.90	1.71	1.73
0.9...	0.79	0.79	0.97	0.93	0.82	0.82	2.16	2.16	2.35	2.35	2.20	2.20	1.47	1.47	1.66	1.71	1.50	1.50
1.0...	0.46	0.36	0.58	0.58	0.48	6.37	1.47	1.26	1.78	1.58	1.53	1.33	0.96	0.83	1.18	1.19	1.00	0.85
n	.....	0.25	.....	0.20	.....	0.25	.....	0.20	.....	0.20	.....	0.20	.....	0.20	.....	0.00	.....	0.20

AMERICAN SOCIETY OF CIVIL ENGINEERS.

INSTITUTED 1852.

TRANSACTIONS.

Paper No. 914.

THE WATER-WORKS OF GUANTANAMO, CUBA.*

By S. D. Rockenbach, Assoc. M. Am. Soc. C. E.

The town of Guantanamo is situated 15 miles in the interior from its port of entry, Caimanera, with which it is connected by rail. The bay is one of the finest in Cuba. It was used as a base by the United States Navy in the late war with Spain, and probably will be one of the naval stations to be selected under the Platt amendment.

The country surrounding Guantanamo is one of the largest sugar-producing sections, while the hills in view from the town were at one time dotted with the houses of coffee planters. The natural conditions point to a bright future for Guantanamo under a stable government. Its population is 7 137. The population within the limits of the municipality (immediate surrounding country) is 28 063.

At the time of the arrival of the American troops the town obtained its water from cisterns and from the Guaso River as it runs past the town. The river emerges from a cave in the hills about 10 miles from the town, and at an elevation of about 500 ft. above it. The water, from the cavern to the plain, is clear and cool, and ample to supply the town, but soon after reaching the plain it is badly polluted by the refuse from the sugar mills on its banks. The question of suitable water supply early occupied the attention of the United States Military Government.

* Presented at the meeting of November 6th, 1901.

The work was started in July, 1899, under the verbal orders of General Wood, then commanding the Department, by Lieut. R. L. Hamilton, 5th U. S. Infantry, Engineer Officer; A. S. Hobby, Jr., Chief Engineer, and J. D. Wilkes, Constructing Engineer. The plans were completed and an estimate for the work made on February 28th, 1900. This estimate amounted to $115 822.39. It contemplated a small masonry diversion dam situated in the narrow gorge of the Guaso, 42 600 ft. from the town, at an elevation of 250 ft. above it, and about 2 miles from the cavern, from which the river flows in a cañon, so that it is entirely free from contamination; a small settling basin at the mouth of the gorge; a 10-in. pipe line to the town, and a distributing reservoir on the edge of the town. On February 10th, 1900, Mr. C. A. Knowlton succeeded Mr. Wilkes as Resident Engineer of the work, and the writer succeeded Lieut. Hamilton as Engineer Officer on July 16th, 1900. The expenditure to July 1st, 1900, amounted to $111 447.18, distributed as follows:

Material	$56 772.74
Duty	2 299.84
Labor, including freight and hauling	52 374.60
Total	$111 447.18

For this expenditure the following work was done:

(*a*) The excavation for the dam foundation was completed.

(*b*) Below the dam, 700 lin. ft. of bench for the pipe line was blasted out and 70 lin. ft. of tunnel was driven, all through limestone.

(*c*) Between Stations 85 and 90, 500 lin. ft. of bench for the pipe line was blasted out.

(*d*) The main pipe line was laid, with the exception of 930 ft.

(*e*) Four river crossings (Nos. 1, 2, 3 and 5) were laid.

(*f*) The pipe line had been covered to a depth of 6 ins. for one-half its length; the remainder was fully back-filled.

A project of the size of the Guantanamo work required the most careful reconnaissance and investigation. The uncertainty of the status of the Island, and a Military control, did not allow the necessary preliminary work on which to base an accurate estimate. Machinery, tools and labor to make necessary borings for the investigation of the dam site were not obtainable at the time. The original plans con-

templated a small diversion dam at about 500 ft. from the mouth of the gorge. At that point the gorge is about 18 ft. in width, and the walls are best adapted for keying in the dam. To secure a solid foundation for the dam involved going to a depth of 21 ft. at one place. As the work developed, the gorge was found to be full of fissures, and very rotten, so that the plans under which the writer found the dam being built required the following:

Dam at Las Ninfas.—The dam at Las Ninfas is located at a point about 830 ft. up stream from where the gorge debouches into the valley, at an elevation of 200 ft. above the Barracks' Hill at Guantanamo, and 8.6 miles therefrom.

The foundation of the dam is 21 ft. below low water. From the foundation to the parapet crest the height is 32¼ ft., and the crest length from cliff to cliff is 50 ft.

In design it is ogee-faced, the curve being a reversed parabola calculated for a head of 4 ft. The section adopted was the fifth one designed, and was the third one submitted by Mr. Knowlton to the Chief Engineer.

The length of the low-water spillway is 28 ft. and its depth 1¼ ft. from the extreme crest of the parapet. With a depth of more than 1¼ ft. of water passing, the length of the spillway is 37½ ft.

Near the reversing point of the ogee, and above the deepest part of the stream bottom, the dam is pierced by a 30-in. circular waste sluice, closed with an iron gate-valve built in concrete against the back face.

On the west end of the dam is built a concrete pier rising 10 ft. above the parapet and projecting 2 ft. beyond the back face. From the top of this pier the valve of the intake is controlled by a direct rod to the hand-wheel above, and the sluice-valve is operated from the same point by shafts with bevel gears. This pier occupies a niche in the face of the cliff, so that there is little danger that it will prove to be an obstruction to debris.

The original plans for the control of the gate contemplated a shaft, 5 ft. in diameter and 22 ft. deep, through the east bluff. This bluff was found to be so rotten that a shaft could not be sunk without lining it, and the writer was opposed to the long rods and double set of bevel gearing which would have been required to control the sluice and the intake gate.

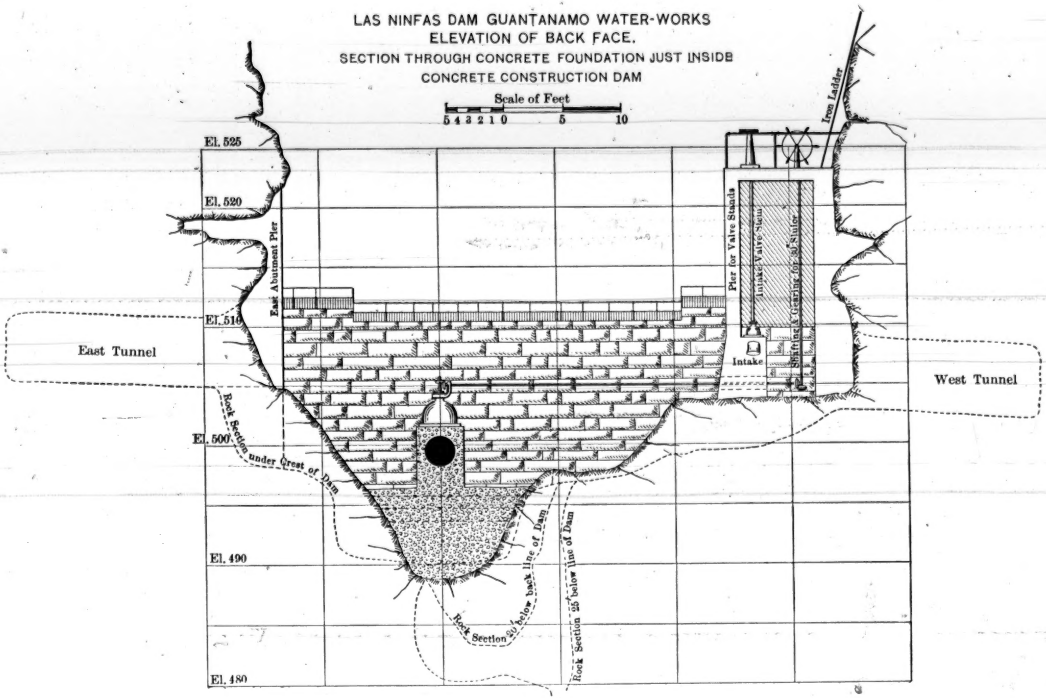

LAS NINFAS DAM GUANTANAMO WATER-WORKS
ELEVATION OF BACK FACE.
SECTION THROUGH CONCRETE FOUNDATION JUST INSIDE
CONCRETE CONSTRUCTION DAM
Scale of Feet
5 4 3 2 1 0 5 10
El. 525
El. 520
El. 510
El. 500
El. 490
El. 480
East Abutment Pier
East Tunnel
West Tunnel
Iron Ladder
Pier for Valve Stands
Sluice Valve Stand
30" Sluice
Intake
Rock Section under Crest of Dam
Rock Section 20' below back line of Dam
Rock Section 35' below line of Dam

FIG. 1.

At the east end of the dam, under a projecting ledge of rock, is a pier 8 ft. in height and flush with the cliff face down stream.

The dam is built of hard limestone, quarry-faced on the back, fine-pointed on the front and crandalled on the coping and the upper half of the spillway face. Vulcanite Portland cement was used throughout. A finely triturated gravel was used instead of sand in the lower portion of the dam and for concrete; on the upper portion a coral sand was used.

There were seventeen streams flowing from the rock within the limits of the foundation; these were either capped or led in pipes to discharge beyond the dam. At each abutment there were two large seams between the strata, partially filled with clay, nearly horizontal, and 5 ft. apart. Tunnels were driven 18 and 24 ft. into the cliffs and cutting the seams, and the tunnels were then filled with rubble masonry in cement. Fig. 1 is an elevation and Fig. 2 a cross-section of the dam. Down-stream and up-stream views of the dam are shown in Figs. 1 and 2, Plate XII.

The 12-in. intake passes through the dam near the west abutment, 4 ft. below the level of the spillway, and the pipe, in a heavy jacket of concrete, is laid on a bench blasted from the cliff, and through a tunnel 70 ft. in length which pierces a projecting spur (Fig. 1, Plate XIII). Below the tunnel the pipe lies in a rock trench blasted in the bench, and is covered with broken stone. This portion of the pipe rests on concrete saddles, 1 ft. square in plan, built under each bell.

The cost of the dam for the year ending June 30th, 1901, was $15 655.30, which is the total cost except for the foundation excavation and for materials used in the temporary work.

Sand Box or Settling Basin.—The sand box or settling basin is located on the river bank above the reach of freshets, and about 830 ft. down stream from the dam. It is a rectangular concrete structure, 10 x 29 ft. in plan inside and about 10 ft. deep, with three division walls of concrete 6 ins. in thickness (Fig. 2, Plate XIII). The water enters at the floor level of the north end, passes over the first partition, under the second, over the third, and is discharged through a bellmouth pipe about 6 ft. above the bottom.

Sliding doors are fitted in each partition, and the floor slopes from the sides to the center line, and to the supply pipe which can be used to blow out the deposit in the basin by the operation of two 12-in. valves 30 ft. up stream.

FIG. 1.—LAS NINFAS DAM: LOOKING DOWN STREAM, AND SHOWING THE CONSTRUCTION DAM.

FIG. 2.—LAS NINFAS DAM: LOOKING UP STREAM.

The supply pipe is fitted with a sliding automatic cut-off gate, and a 6-in. overflow pipe is also provided.

At the top are transverse beams of railroad steel which carry the flat covering of 2-in. pine plank, through which pass the rods for operating the flush-gates in the partitions. The foundation is on rock.

The total cost of the sand box (all for the fiscal year ending June 30th, 1901) was $2 874.71.

Blow-Off Valve Houses.—Eight blow-off valve houses are located on the pipe line, one at every low point and river crossing. These are of concrete construction throughout. They are 7 ft. square, and the walls are 7 ft. above ground, with a plain 4-in. cornice and a pyramidal roof. The walls are 6 ins. in thickness, and in each wall there are two panels 2 ins. deep, making the thickness of the panels 4 ins. The concrete roof is 6 ins. thick, and a bar of 1-in. square iron is built into the top of the wall all around. The only openings are a drain of 1-in. pipe, and the door, which is of ¼-in. iron plate with wrought-iron hinges, and is fastened with a brass padlock.

The total cost of eight valve houses was $3 963.18.

Air-Valve Boxes.—There are seven air-relief valves at the summits on the pipe line. Each valve is placed above the ground and connected with the main by a 2-in. wrought-iron pipe. It is protected by a rectangular concrete box covered with a sloping lid of ¼-in. iron plate which is hinged at the upper edge, and lifts like the lid of a trunk. They are fastened with brass padlocks. All padlocks used on the valve boxes and on the blow-off houses are similar, and open with one key. The principal expense in constructing the boxes was for the iron-work, which cost $35.18 each.

The total cost of seven air-valve boxes was $354.76.

Keeper's House at Las Ninfas.—The house for the gate-keeper at Las Ninfas is 25 x 30 ft. in plan, divided into four rooms. One of these is ceiled with dressed yellow pine, and is for the use of visiting inspectors. The other rooms are not ceiled, but all floors inside are of tongued and grooved yellow pine. The walls are of vertical spruce plank, battened; the roof is full sheathed with 1-in. plank, and covered with galvanized iron. A covered corridor or veranda 8 ft. wide on the front, and 6 ft. wide on the other three sides, extends entirely around the house. The veranda floor is of 2-in. pine; its roof is supported on 4 x 4-in. posts with a light railing between them. A ven-

tilator at the apex of the pyramidal roof and a 4-in. opening at the top of the outer walls under the veranda roof give good ventilation. The foundations are of short pieces of 12-in. cast-iron pipe filled with concrete.

The total cost of the house was $956.25.

Guantanamo Distribution Pipes.—Trenches for the distribution pipes in Guantanamo were opened on February 20th, 1901, and the

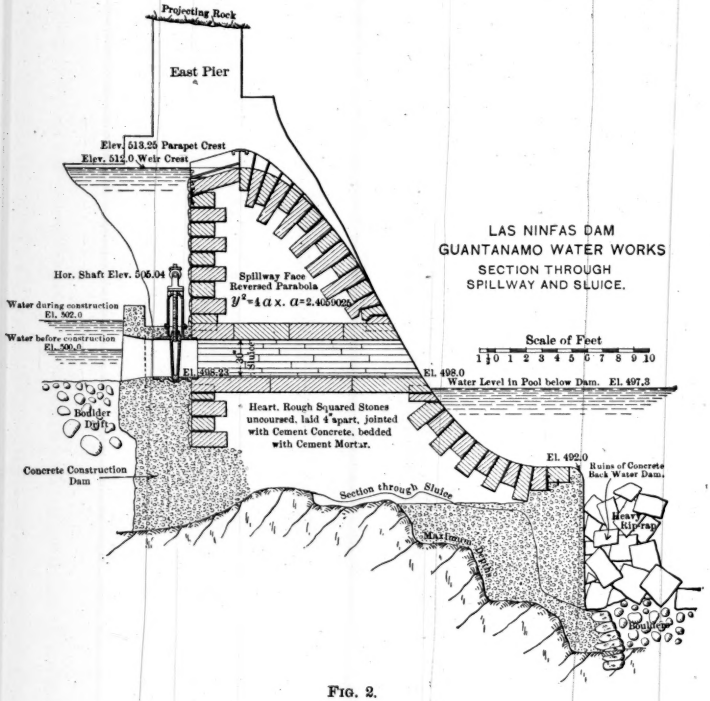


FIG. 2.

work was completed on May 15th, 1901. There were laid 7.6 miles of pipe, as follows:

4-in. pipe	16 419 lin. ft.
6-in. pipe	10 038 lin. ft.
8-in. pipe	8 398 lin. ft.
10-in. pipe	4 638 lin. ft.
14-in. pipe	625 lin. ft.

FIG. 1.—PIPE LINE AND TUNNEL: GUANTANAMO WATER-WORKS.

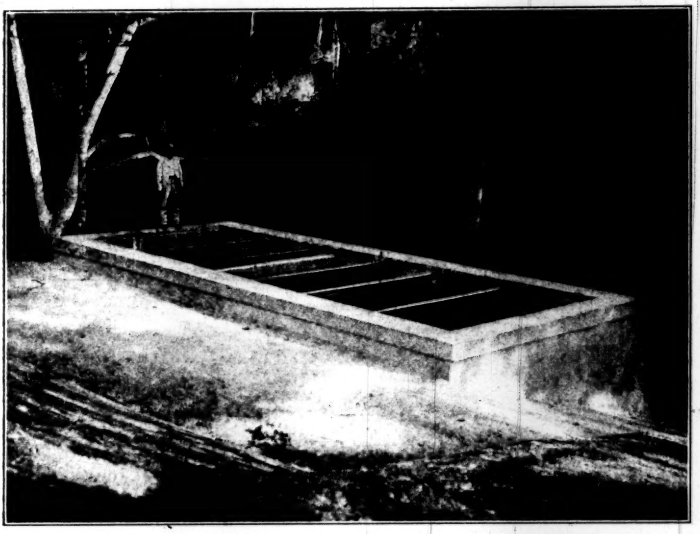


FIG. 2.—SETTLING BASIN: GUANTANAMO WATER-WORKS.

Thirty-six fire hydrants were put in.

The cost was as follows:

Total cost of materials	$34 725.20
Excavation and back-filling	10 126.41
Pay rolls to May 31st	10 096.02
Total	$54 947.63

In February, to avert a threatened epidemic, due to the dry season and the pollution of the stream, a 2-in. overhead hydrant was put in, and carts which formerly obtained water from the river to supply the town were allowed to get water free of charge. As soon as the mains were put in, house owners were allowed to make connections, under the supervision of the Resident Engineer and in accordance with the regulations of the Department: A $\frac{3}{4}$-in. connection, with an 18-in. goose-neck and a cock at the main, and $\frac{3}{4}$-in. galvanized-iron pipe with service box and curb-cock. To include June 30th, 1901, 368 houses had made connections, and it can safely be said that the entire town has been furnished with water for the past four months. A scheme for control and taxation is being drawn up and will be submitted as soon as possible.

The total cost of the work was	$206 304.98
Of this, the duty amounted to	7 090.81

The quantity of water supplied in 24 hours is 955 152 galls., or 133.8 galls. *per capita.*

The ideal plans on which this work was projected made the difficulties almost insurmountable. A cut-stone parabolic dam, built by native labor in Cuba at this date, means that the engineer had to lay out all forms, and watch the cutting and placing of each stone. Had he not been a stone mason, carpenter and plumber, as well as engineer, the work could not have been done. The works give not only a plentiful supply of water to Guantanamo, but have been a good mechanical school. Common laborers have become stone masons, carpenters, blacksmiths and plumbers, rising from $1 to $4 per day of ten hours, and have gone to sugar estates and towns where they command that wage. Two men, trained on the work, have opened a plumber's shop in Guantanamo.

Freight was excessive when the work was commenced, but owing to competition it was much reduced. For the pipe in the main line

the freight from New York to the dock at Caimanera cost $7.50 per ton. A year later the freight on the pipe for the distribution system was $5 per ton.

This work, from February 10th, 1900, was under the direct charge of Mr. C. A. Knowlton, as Resident Engineer, and great credit is due to him for the ability and energy he displayed in pushing it to its present state of completion.

A stand-pipe is needed to give a reserve supply and equalize the pressure in the town, and this and the extension of the pipe line to Caimanera will be taken up later.

This work, in its design, presents little that is peculiar or original, but in the execution there is much to be considered by those without experience in the Santiago Province of Cuba.

Labor costs 10 cents per hour, double the cost in Porto Rico, and the work performed may be estimated as worth $3\frac{1}{3}$ cents in the United States. As far as possible, work should be paid for by the piece or task. Time is worth nothing in Cuba, and contractors will find that the time allowed for a project requires more careful consideration than any other item.

Transportation, except on the rail and the few metalled roads, is practically impossible from June 15th to November 15th.

The main requisite of a constructing engineer in Cuba is sobriety and nerve, as new and unexpected difficulties, due to climate, native labor, and distance from market, will arise. It is absolutely necessary that he should have a workman's knowledge of all the details of his work. A nervous, excitable man swearing at the top of his voice will stop the work of every Cuban in hearing.

All things considered, concrete is the most economical construction in Cuba. The absence of frost, and a generally uniform temperature, allows a great variety of uses, and its strength is very great. A concrete construction dam 2 ft. thick, 15 ft. high and 24 ft. long successfully withstood 10 ft. of water flowing over its crest.

Duty and freight ran all imported material up 30%, and labor proportionately high, so that, taking the total cost of the work, $206 304.98, its cost in average cities in the United States should not have exceeded $140 000.

AMERICAN SOCIETY OF CIVIL ENGINEERS.

INSTITUTED 1852.

TRANSACTIONS.

No. 915.

SOME IMPORTANT PHASES OF CANAL NAVIGATION, ILLUSTRATED BY RECENT EXPERIMENTS IN GERMANY.*

By ELNATHAN SWEET, M. Am. Soc. C. E.

Some months ago the writer had occasion to investigate the subject of the resistance to be overcome in navigating the proposed 1 000-ton barge canal, then under consideration by the canal authorities of New York State, and in these studies he availed himself of the recent very extensive experiments and researches by M. Fernand DeMas, in France, and by Herr R. Haack, in Germany. The results of this investigation are published in the recent report to the Governor, by the State Engineer, on "The Barge Canal."

In that investigation attention was directed almost exclusively to the question of resistance. The experiments of Herr Haack, however, illuminate other very important phases of canal navigation, and this brief monograph will be devoted to the consideration of the contribution they make to our knowledge of the negative velocities produced in canal navigation, and the limiting rate of dirigible speed therein. To make this article clearly intelligible it will be necessary to indulge in a repetition of portions of the writer's contribution to "The Barge Canal" report, above mentioned.

* Presented at the meeting of January 8th, 1902.

Herr Haack's description and illustration of his experiments, entitled "Schiffswiederstand und Schiffsbetrieb," were published by A. Asher and Company, Berlin, 1900. His experiments were very numerous, and appear to have been made with characteristic German thoroughness and accuracy. They were made near Lingen, upon a stretch of the Dortmund-Ems Canal, having the usual trapezoidal section, with a mean area of 600 sq. ft. and a mean center depth of about 8½ ft.

The boats used in the experiments consisted of a tug, the *Goedhardt*, and three steel barges, the *Embden*, the *Dortmund* and the *Lloydkahn*.

The *Goedhardt* was 68 ft. long, 16.4 ft. beam, 5 ft. draft, with 3 175 cu. ft., or 90 tons displacement; its coefficient of displacement being 0.587. It was supplied with a compound, condensing engine of 206 I. H.-P. and a marine boiler working under a pressure of 8 atmospheres.

The *Embden* and *Dortmund* were new steel barges, precisely alike, except that the *Dortmund* was supplied with a marine boiler and a non-condensing, compound engine of 60 I. H.-P. The dimensions of the *Dortmund* and the *Embden* were as follows: Length, 215 ft.; beam, 26.9 ft.; area of midship section at 6.6 ft. draft, 173.6 sq. ft.; displacement at 6.6 ft. draft, 33 313 cu. ft., or 1 041 tons; coefficient of displacement at 6.6 ft. draft, 0.887.

The third barge, the *Lloydkahn*, was also a steel boat, somewhat similar to the *Dortmund* and *Embden*, 180 ft. long; 26.3 ft. beam; area of midship section at 6.6 ft. draft; 167.68 sq. ft.; displacement at 6.6 ft. draft, 740 tons; coefficient of displacement, 0.762. These experiments had for their object, not only the determination, by accurate measurements, of the actual resistance in this canal at varying velocities and ratios of prism to boat section, but the determination, by accurate instrumental work, of the precise condition of the prism of water in a canal while a vessel was being forced through it, on the theory that the relative conditions of this prism of water at rest and while a boat was moving in it should afford a measure for determining the entire resistance of the boat navigating it.

The results of these experiments seem to be in agreement with this hypothesis.

This theory, advanced by Haack, briefly stated, is as follows: When a vessel moves in a canal, unlike a vessel moving in open water

where the water moves freely from all directions to fill the void behind it, the water can only reach this void by moving back through the narrow spaces between the boat and the bottom and sides of the canal.

Immediately in front of the wave made by the boat's prow the water is absolutely at rest, while behind the prow wave, the whole length of the boat, and for some distance behind it, the water sinks, below its position before being disturbed, with a back flow seeking to restore the normal condition behind the boat. The extent of this sinking of the water surface and of the velocity of this negative current depend on: The rate of the boat's motion, its model, the friction of its immersed surface, the friction of the canal bed, and of the water particles on one another.

In other words, the amount of this sinking of the water surface and the extent of the negative velocity are functions of the boat's total resistance.

The following is the analytic reasoning:

Let Q = the wet cross-section of the canal;

D = the displacement of the boat;

L = the length of the boat on the waterline;

V = velocity of the boat;

v = the mean negative velocity, or back flow;

$q = \frac{D}{L}$ = the mean cross-section of the boat;

R = the resistance of the boat;

w = the weight of a cubic unit of water;

and $\triangle Q$ = the cross-section between the sunken surface, due to the boat's motion, and the normal water surface.

Then, $Q - \triangle Q - q$ represents the cross-section remaining between the boat and the bed of the canal, through which the displaced water must pass. Now, the volume of water displaced in a unit of time by a boat moving at a velocity, V, is $V(\triangle Q + q)$, and equals the volume flowing at the negative velocity, v, in the section $(Q - \triangle Q - q)$, therefore the mean negative velocity is

$$v = \frac{\triangle Q + q}{Q - \triangle Q - q} V \quad \text{.....(I)}$$

Now, the weight of the volume of water moved is $w(Q - \triangle Q - q)L$, and the work required for moving this weight, with the velocity, v, is $C\, w\, (Q - \triangle Q - q)\, L\, v$, and as, with uniform motion, this equals the resistance of the boat multiplied by the velocity, we have

$$R\, V = C\, w\, (Q - \triangle Q - q)\, L\, v \quad \text{.....(II)}$$

in which C is a cofficient determined by experiment. As the resistance does not vary directly as the change of velocity, this coefficient is necessarily a variable. If we substitute in this equation the value of v shown in Equation (I),

$$v = \frac{\triangle Q + q}{Q - \triangle Q - q}, \text{ we have}$$

$$R = C\, w\, (\triangle Q + q)\, L \ldots\ldots\ldots\ldots\ldots\ldots \text{(III)}$$

Combining Equations (II) and (III) we get

$$\triangle Q = \frac{(Q - q)\, v - q\, V}{v + V}$$

Substituting this value of $\triangle Q$ in Equation (III) we have

$$R = C\, w \left(\frac{(Q - q)\, v - q\, V}{v + V} + q\right) L,$$

$$\text{or, } R = C\, w \left(\frac{Q\, v - q\, v - q\, V}{V + v} + q\right) L,$$

$$\text{or, } R = C\, w\, \frac{v}{V + v}\, Q\, L \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(IV)}$$

Expressions for the values of the coefficient, C, are readily derived from Equations (II), (III) and (IV).

$$\text{As from Equation (II)} \ldots\ldots\ldots\ldots\ldots C = \frac{R\, v}{w\, v\, (Q - \triangle Q - q)\, L}$$

$$\text{Since } q\, L = D, \text{ as from Equation (III)} \ldots\ldots C = \frac{R}{w\, (\triangle\ Q\ L + D)}$$

$$\text{As from Equation (IV)} \ldots\ldots\ldots\ldots\ldots C = \frac{R\, (V + v)}{w\, v\, Q\, L}$$

These formulas furnish the means for determining R for a given value of v, when C has been determined for a like value of v, for a vessel of similar model, in a similar prism, with identical value of $\frac{Q}{q}$.

An important and interesting result of these experiments is their demonstration of the serious error in the ordinary assumption that the velocity of the negative current or back flow induced by the motion of a boat in a narrow channel is measured by the formula $v = \frac{a}{A - a} V$; where v is the velocity sought, V the velocity of the boat, and A and a the areas of the cross-section of the canal and midship section of the boat, respectively. This is illustrated by the disgram, Fig. 1.

Let the full lines of the diagram represent the prism of the canal, its normal water surface, and the midship section of a boat; and let

the dotted lines represent the surface of the water as depressed by the motion of the boat, and the mean section of the boat. Then the space between the dotted lines and the full line representing the normal water surface is the mean section of water displaced, and the space between this dotted and the full lines representing the canal bed is the section through which the displaced water must pass; and they show, for any considerable speed of the boat, so much larger negative velocity of the displaced water than that derived from the old formula as to prove the latter to be not even approximately correct for ordinary ratios of canal section to boat section.

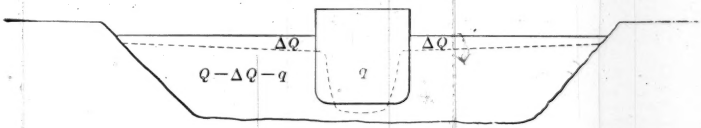


FIG. 1.

Negative Velocities.—The experiments most pertinent to the discussion of this subject consist of the following:

Ten trial trips of the *Goedhardt*, under steam, with varying velocities and constant draft, in which r, the ratio,

$$= \frac{\text{Wet section of canal}}{\text{Immersed mid-section of boat}} = 8.50.$$

Forty-seven trial trips of the *Dortmund*, under her own steam, with varying velocities and depths of draft, as follows:

Depth of draft, in meters.	Ratio, r.
1.50	4.76
1.75	4.15
2.00	3.63
2.25	3.22

Seventy trial trips of the *Embden*, towed by the *Goedhardt*, at varying velocities and depths of draft, as mentioned above for the twin boat *Dortmund*, and identical values of r.

Also, fifteen trial trips of the *Lloydkahn*, towed by the *Goedhardt*, with varying velocities and depths of draft, as follows:

Depth of draft, in meters.	Ratio, r.
1.75	4.30
2.00	3.75
2.25	3.30

The values of the negative velocities, as well as all the other data determined by these experiments, were tabulated and published by Haack, and from these tables the writer has plotted the curves of negative velocity shown in Fig. 2, and has shown in Fig. 3 the curves of resistance, expressed in units of indicated horse-power, for the same groups of experiments as published by Haack.

The abscissas in Figs. 2 and 3 show the speed of the boats, in meters per second, and in miles per hour. The ordinates in Fig. 2 represent the negative velocities, in meters per second and in feet per second. In Fig. 3 the ordinates represent units of indicated horse-power.

It should be noted that the number of trial trips for different values of r differed considerably in these experiments, giving greater weight to some of the curves than to others, and that for small values of r the speeds attained in the experiments were so low as to leave a considerable fraction of their curves of negative velocity and resistance conjectural, but these portions are probably not flatter than shown in the diagrams.

It must also be borne in mind that the mean negative velocities are probably as much less than the corresponding maximum negative velocities as the mean immersed section of the boat is less than its immersed midship section. It will be observed that the resistance curves in Fig. 3 indicate considerable sharper curvature for the *Dortmund* going under steam than for the twin boat *Embden* towed by the *Goedhardt*. The data for the negative velocity curves for these two cases show small divergencies in the same direction, but these are not marked enough to make separate curves necessary. Though the action of the *Dortmund* screw doubtless had some effect, the difference in the resistance curves in these two cases is probably principally due to the necessarily inexact result of reducing to indicated horse-power the resistance of the *Embden*, which was measured in the tow line.

There is perhaps no phenomenon, incident to canal navigation, where considerable speed is desired, so important in determining the proper section of the canal prism as this of the negative current induced by its operation, as such current cannot be permitted to acquire a velocity destructive to the integrity of the earthwork, and these experiments afford the first and only important contribution to our knowledge on this subject.

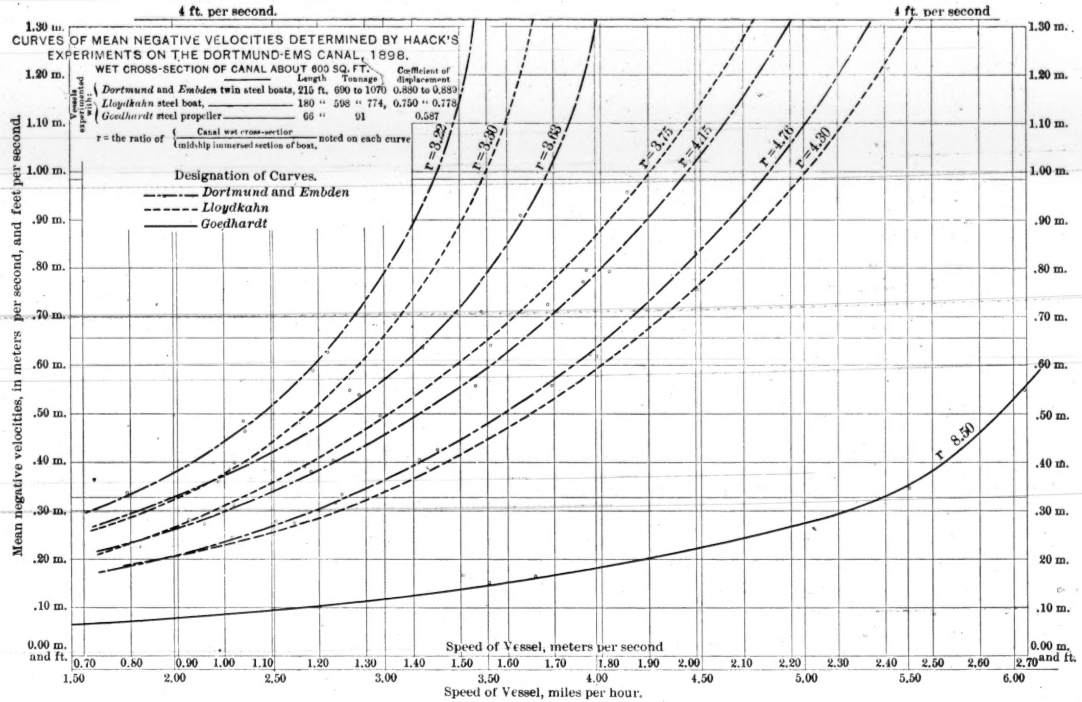

4 ft. per second.
4 ft. per second
CURVES OF MEAN NEGATIVE VELOCITIES DETERMINED BY HAACK'S EXPERIMENTS ON THE DORTMUND-EMS CANAL, 1898.
WET CROSS-SECTION OF CANAL ABOUT 800 SQ. FT.
Vessels experimented with
Length
Tonnage
Coefficient of displacement
Dortmund and Embden twin steel boats, 215 ft. 690 to 1070 0.880 to 0.889
Lloydkahn steel boat, 180 " 598 " 774, 0.750 " 0.778
Goedhardt steel propeller 66 " 91 0.587
r = the ratio of Canal wet cross-section / midship immersed section of boat, noted on each curve
Designation of Curves.
Dortmund and Embden
Lloydkahn
Goedhardt
r = 3.22
r = 3.30
r = 3.65
r = 3.75
r = 4.15
r = 4.70
r = 4.80
r = 8.50
Mean negative velocities, in meters per second, and feet per second.
1.30 m.
1.20 m.
1.10 m.
1.00 m.
.90 m.
.80 m.
.70 m.
.60 m.
.50 m.
.40 m.
.30 m.
.20 m.
.10 m.
0.00 m. and ft.
0.70 0.80 0.90 1.00 1.10 1.20 1.30 1.40 1.50 1.60 1.70 1.80 1.90 2.00 2.10 2.20 2.30 2.40 2.50 2.60 2.70
Speed of Vessel, meters per second
1.50 2.00 2.50 3.00 3.50 4.00 4.50 5.00 5.50 6.00
Speed of Vessel, miles per hour.

FIG. 2.

Let us consider the applicability of the results to canals of larger section, in which the limit of allowable speed is a question of the first importance. The difference of conditions which might affect the negative velocity in a ship canal, compared with this Dortmund-Ems barge canal, navigated by steel boats of like model, with equal values of r, affect only that element of the total resistance represented by the friction of the disturbed particles of water upon one another on account of the greater distance between the wet surface of the boat and that of the canal in the former case.

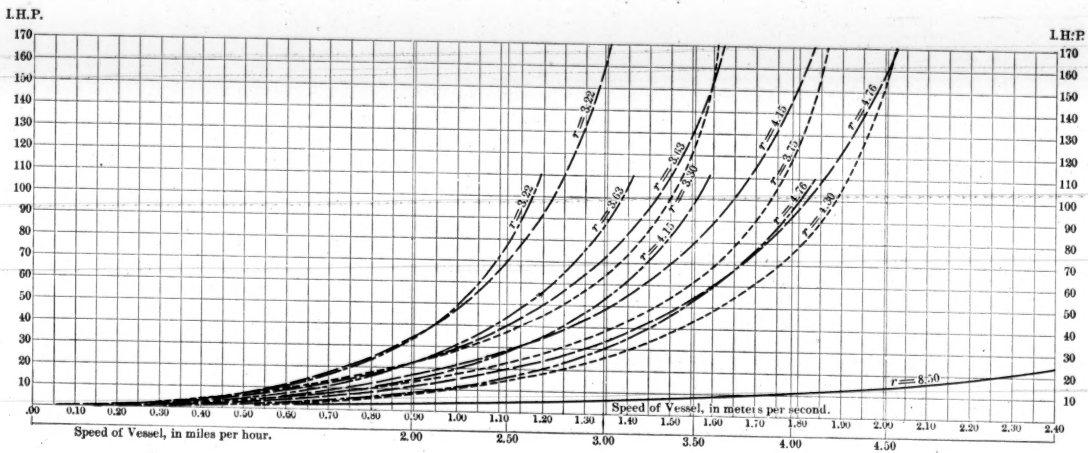
It is well known that when a boat moves in shallow water the proximity of the boat's bottom to the ground retards the speed of the boat, owing to the friction among the particles of water lying between the boat and the ground induced by the friction of the boat's surface carrying along the water adhering to it, while the friction of the ground retards the water adjacent to its surface, thus setting up motion and friction among the particles of water between these surfaces.

This retardation diminishes rapidly as the distance between the boat and the ground increases, and this element of resistance is doubtless negligible in seeking the relation between the negative velocities in the Dortmund-Ems canal and those in a larger canal, except that due to the depth of water under the boat. It will be remembered that the larger values of r, in Haack's experiments were obtained with the boats at light draft, leaving 2 to 3 ft. of water under them, and are therefore directly comparable with the boats of like model running equally near the bottom in large canals, where the values of r are identical.

The effect of model, on both resistance and negative velocity, is shown by comparing the curves for the *Dortmund* and *Embden*, having a coefficient of displacement of 0.88, with those of the *Lloydkahn*, having a coefficient of 0.76, the resistance and negative velocity both being less in the case of the *Lloydkahn*, with $r = 4.30$, than in the case of the *Dortmund* and the *Embden*, with $r = 4.76$.

The writer is convinced that these curves can be used without serious error in determining the approximate mean negative velocities of boats having block displacements of 0.75 to 0.90 for the values of r which they cover, from which the maximum negative velocities can be derived from the coefficient of the boat's displacement.

Limits of Dirigibility in Canals.—The observations, made during the progress of Haack's experiments, upon the steering qualities of the ex-



CURVES OF INDICATED HORSE POWER, DETERMINED BY HAACK'S EXPERIMENTS ON THE DORTMUND-EMS CANAL, WITH THE BOATS *Goedhardt, Dortmund, Embden* AND *Lloydkahn*, DESCRIBED ON FIG. 2.

Designation of Curves:

- - - - - - *Embden* (Towed).
— — — *Dortmund*, under own steam.
- - - - - - - *Lloydkahn*.
——— *Goedhardt*.

FIG. 3.

perimental boats, at different depths of draft and different rates of speed, are interesting and important. They show that with 2 ft. of water, or more, under the boats when at rest, and the value of r more than 4, the difficulties of steering did not become serious until the negative velocity approximated 3 ft. per second, but that with little more than 1 ft. of water under them, and the value of r about 3.25, the boats became difficult to steer when the mean negative velocity was only 2 ft. per second.

The *rationale* of these steering difficulties is not far to seek.

It is clearly impossible to propel a vessel accurately upon the center line of a narrow channel, and if it is moving off this center line the sinking of the water surface will be unequal at its sides, because the void created by its motion is equal on both sides and must be filled by the water adjacent, necessarily resulting in a greater depression of the water surface between the vessel and the contiguous shore than on the opposite side; the difference of head creating a pressure tending to move the boat toward the near shore. This tendency is increased greatly by the turning moment induced by the greater negative current on the narrower side of the channel, obviously tending to turn the bow of the vessel toward the same shore. It is true that these effects are diminished because the change of head on opposite sides of the boat induces a current under it, the volume of which depends on the depth of water existing there. It is the relative efficiency of this current, which depends upon the depth of water under the boat, that explains the greater difficulty of steering in a canal which is shallow as well as narrow, than in one of like width, but deeper.

As a concrete example, Herr Haack's analysis gives the following conditions for the *Lloydkahn* moving at the rate of 1.60 m. per second at a draft of 1.65 m., 2 m. to starboard of the middle of the Dortmund-Ems Canal: Sinking of water surface 0.089 m. greater on starboard than on port side; mean negative velocity 0.285 m. per second greater on that side; and turning moment to starboard 135 kgr.

The light which these experiments have thrown upon these problems of negative velocities and dirigibility emphasizes the importance of assigning a liberal value to r, in the design of canals where speed is highly desirable; and also the fact that width may with advantage be sacrificed for depth in realizing a given value for r.

Such a modification in the prevailing ratios of canal depth to width would diminish the resistance to propulsion, as well as the back flow and the steering difficulties.

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